Journal of the Korean Society for Nondestructive Testing
Vol. 25, No. 3 (2005. 6)

Application of Phase-shifting Method using Fourier Transform to Measurement of In-plane Displacement by Speckle Interferometry

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Abstract Phase-shifting method using Fourier transform (PSM/FT) has been applied to measurement of in-plane displacement of a specimen. Thirty-two interference fringe patterns each of which has different phase of $\pi/16$ radian have been gathered from a specimen with in-plane displacement. Low-pass filtering by 2-D Fourier transform is used to suppress spatial noise of the fringe patterns. α-directional Fourier transform for PSM/FT is performed by use of the low-pass filtered 32 fringe patterns. Two kinds of specimens are used for experiment. One is a rectangular steel plate and the other one is a rectangular steel plate containing a circular hole at the center. In-plane displacement of each specimen is measured by PSM/FT, and calculated by finite element method (ANSYS) for comparison. The results are quite comparable, so that PSM/FT can be applied to measurement of in-plane displacement.

Keywords: speckle interferometry, phase shifts, Fourier transform, fringe analysis

1. Introduction

Optical signal processing has been applied widely to science and engineering areas. Speckle interferometry (Dainty, 1984 and Cloud, 1995) is one of those applications to measure small displacement. Speckle interferometry has characteristics of non-contact and full-field measurement of displacement. Phase shifting method used in speckle interferometry (Creath, 1985, Scalea et al., 1998 and Baek et al., 2003) is optical technique in which known phases are added into one of optical signals by control of optical path length and resultant interference fringe patterns are analyzed. Phase-shifting method using Fourier transform (PSM/FT) (Morimoto et al., 1994) has been developed for analysis of interference fringe patterns. In PSM/FT, the interference fringe patterns are obtained from optical experiment by use of phase shifting method and α-directional Fourier transform is performed for the patterns. Advantage of PSM/FT can eliminate noises caused by phase-shifting error because it uses only one of frequency components in the patterns by α-directional Fourier transform. In this paper, PSM/FT is applied to speckle interferometry with optical system in order to measure small in-plane displacement of two specimens. Results from optical experiment are compared with results from finite element method (FEM).

2. Analysis of phase-shifting method using Fourier transform

In speckle interferometry using phase-shifting method, interference fringe pattern is obtained by subtracting the pattern with the specimen loaded

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from the pattern with the specimen not loaded as follows;

$$I(x,y:\alpha)=A(x,y)\cos[\phi(x,y)-\alpha]+B(x,y) \quad (1)$$

where $A(x,y)$ is the amplitude of the brightness in the pattern and $B(x,y)$ is the average brightness. Piezoelectric transducer (PZT) can control an optical path length. α is the known phase which is added into one of the two optical beams through controlling PZT. The known phase, α, covers from 0 to 2π radian at equal intervals. $\phi(x,y)$ is the phase of fringe pattern caused by the in-plane displacement of a specimen. For development of PSM/FT, $I(x,y:\alpha)$ is expressed by Fourier series with respect to α because it is a periodic function with a period $T=2\pi$. That is,

$$I(x,y:\alpha)=\sum_{n=-\infty}^{\infty} c_n \exp(jn\omega_0\alpha) \quad (2)$$

where

$$c_n=\frac{1}{T}\int_{-T/2}^{T/2} I(x,y:\alpha)\exp(-jn\omega_0\alpha)d\alpha \quad (3)$$

j is the imaginary unit, and $\omega_0(=1)$ is the fundamental frequency. When α-directional Fourier transform of Eqn. (2), $F_\alpha(x,y:\omega)$, is taken,

$$\begin{aligned}F_\alpha(x,y:\omega)&=\int_{-\infty}^{\infty} I(x,y:\alpha)\exp(-j\omega\alpha)d\alpha\\&=\sum_{n=-\infty}^{\infty} c_n\int_{-\infty}^{\infty}\exp(jn\omega_0\alpha)\exp(-j\omega\alpha)d\alpha\\&=\sum_{n=-\infty}^{\infty} 2\pi c_n\delta(\omega-n\omega_0)\end{aligned} \quad (4)$$

where ω is frequency and δ is the Dirac delta function. By substituting Eqn. (1) into Eqn. (4) and taking $\omega=\omega_0$, Eqn. (4) becomes

$$\begin{aligned}F_\alpha(x,y:\omega_0)&=\int_{-\pi}^{\pi}[A\cos(\phi(x,y)-\alpha)+B]\exp(-j\alpha)d\alpha\\&=\pi A\exp(-j\phi(x,y))\end{aligned} \quad (5)$$

When the arctangent of the ratio of the imaginary part $\mathrm{Im}[F_\alpha(x,y:\omega_0)]$ to the real part $\mathrm{Re}[F_\alpha(x,y):\omega_0)]$ is taken, the phase ϕ can be calculated as follows:

$$\phi(x,y)=-\tan^{-1}\left(\frac{\mathrm{Im}[F_\alpha(x,y:\omega_0)]}{\mathrm{Re}[F_\alpha(x,y:\omega_0)]}\right) \quad (6)$$

By use of the calculated phase, ϕ, the displacement of a specimen can be obtained. Only fundamental frequency component ($\omega=1$) is used for the calculation of phase, so that the noises in the high frequency components caused by phase-shifting error can be eliminated (Morimoto and Fujisawa, 1994).

3. Optical experiment

3.1. Experimental set-up

Speckle interferometry consists of an optical system that uses speckle effect and interference characteristics of light from a laser and a computing system that processes data gathered by the optical system. The optical system in speckle interferometry depends on what physical quantity is measured. In this paper, in-plane displacement of a specimen is measured with speckle interferometry by PSM/FT. Fig. 1 is the schematic diagram of optical experiment system for speckle interferometry by PSM/FT. In Fig. 1, LA is a laser, PA is a pin-hole assembly, CL is a collimating lens, BS is a non-polarizing beam splitter, MR1 and MR2 are mirrors. SL is a specimen installed in a tensile loading device, CCD is a CCD camera, and PC is a personal computer. BS is placed on PZT control stage that is controlled by CNT (PZT controller). θ is an angle between incident light and vertical line to the specimen. As shown in Fig. 1, BS splits optical light from the laser into two lights. PZT moves BS toward MR1, so that optical path length of the light from the laser to the specimen through MR1 becomes different slightly from that of the light from the laser to the specimen through MR2. PC connected to CNT controls the movement of PZT. The slight difference of the optical path length is calculated by geometry of set-up in optics and used in phase shifting method of PSM/FT.

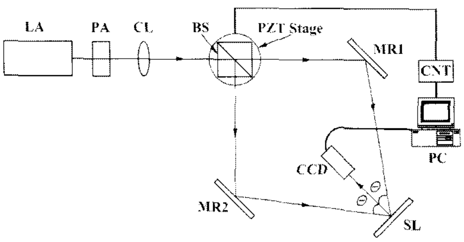


Fig. 1 Schematic diagram of optical experiment system for speckle interferometry by PSM/FT

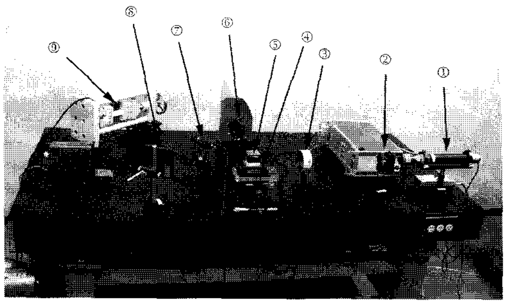
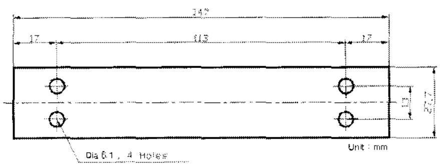


Fig. 2 Picture of optical experiment system for speckle interferometry by PSM/FT. (① He-Ne CW laser, ②pin-hole assembly, ③ collimating lens, ④ PZT control stage, ⑤ beam splitter, ⑥ mirror, ⑦ CCD camera, ⑧ mirror, ⑨ specimen in loading device)

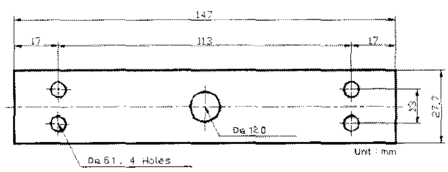
The minimum travel length of PZT stage is a few nanometers, so that it is fine enough to achieve desired phase shifting. The two lights illuminate the specimen and resultant fringe patterns in the specimen with tensile load are collected through CCD camera and saved in PC. α-directional Fourier transform is performed and the phase ϕ in Eqn. (6) is obtained in PC by use of the collected fringe patterns. Fig. 2 shows the picture of optical experiment system for speckle interferometry by PSM/FT.

3.2. Results and discussions

Optical experiments are performed to measure in-plane displacement in two kinds of specimens with speckle interferometry by PSM/FT. The first specimen shown in Fig. 3 (a) is a rectangular steel plate of which size is 113 mm x 27.7 mm with 1.15 mm of thickness. The second one of Fig. 3 (b) is the same size of the rectangular steel plate containing a circular hole at the center. The diameter of hole is 12 mm.



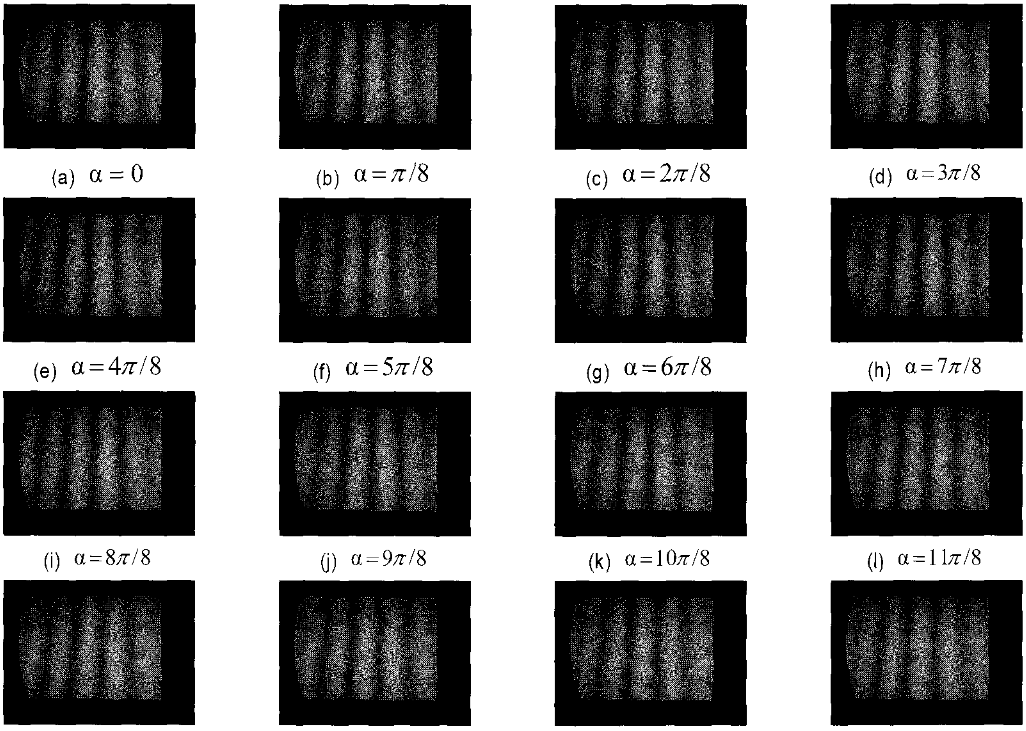
(a)



(b)

Fig. 3 Finite width uniaxially loaded tensile plate. (a) Rectangular steel plate, (b) rectangular steel plate containing a circular hole (thickness of both plates is 1.15 mm)

Each of the specimens is sprayed with white paint for speckle interferometry. The specimen is installed in a loading device that applies tensile load to the specimen in order to make in-plane displacement. Fringe patterns of $I(x,y:\alpha)$ are taken through CCD camera in Fig. 1. The fringe patterns consist of 32 patterns which are sequentially phase-shifted by PZT stage and are



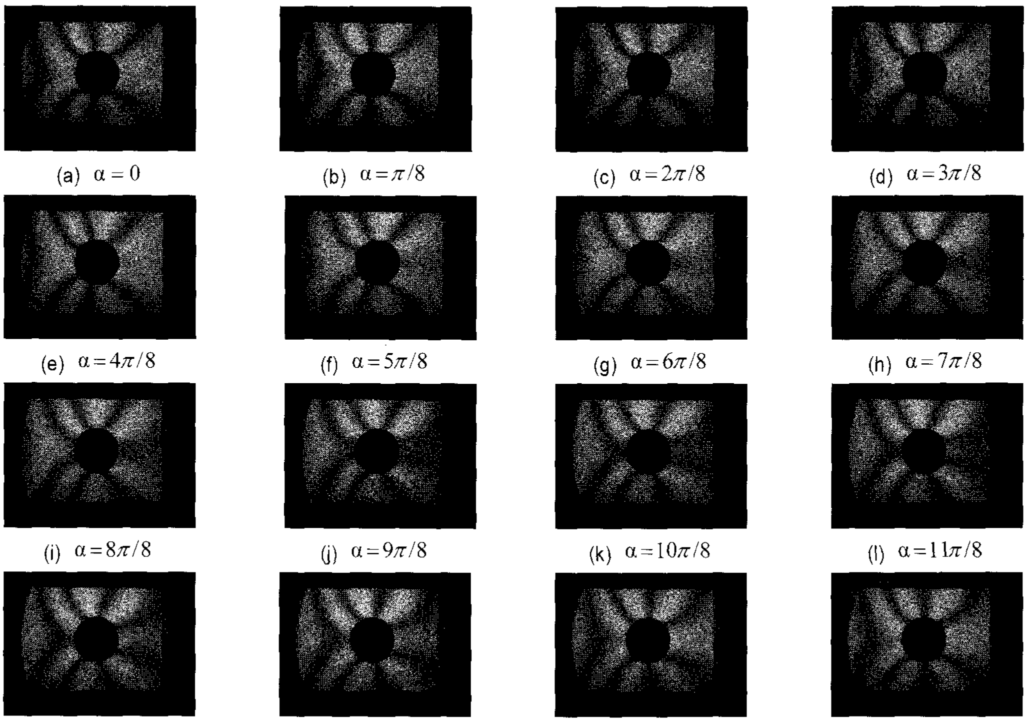
(a) $\alpha=0$ (b) $\alpha=\pi/8$ (c) $\alpha=2\pi/8$ (d) $\alpha=3\pi/8$

(e) $\alpha=4\pi/8$ (f) $\alpha=5\pi/8$ (g) $\alpha=6\pi/8$ (h) $\alpha=7\pi/8$

(i) $\alpha=8\pi/8$ (j) $\alpha=9\pi/8$ (k) $\alpha=10\pi/8$ (l) $\alpha=11\pi/8$

(m) $\alpha=12\pi/8$ (n) $\alpha=13\pi/8$ (o) $\alpha=14\pi/8$ (p) $\alpha=15\pi/8$

Fig. 4 16 fringe patterns of rectangular steel plate at every interval of $\alpha=\pi/8$ radian



(a) $\alpha=0$ (b) $\alpha=\pi/8$ (c) $\alpha=2\pi/8$ (d) $\alpha=3\pi/8$

(e) $\alpha=4\pi/8$ (f) $\alpha=5\pi/8$ (g) $\alpha=6\pi/8$ (h) $\alpha=7\pi/8$

(i) $\alpha=8\pi/8$ (j) $\alpha=9\pi/8$ (k) $\alpha=10\pi/8$ (l) $\alpha=11\pi/8$

(m) $\alpha=12\pi/8$ (n) $\alpha=13\pi/8$ (o) $\alpha=14\pi/8$ (p) $\alpha=15\pi/8$

Fig. 5 16 fringe patterns of rectangular steel plate containing a circular hole at every interval of $\alpha=\pi/8$ radian saved in PC. Phase shifting at each step is $\pi/16$ radian. Size of each fringe pattern is 640 x 480 with 8 bits brightness. 16 fringe patterns out of the 32 fringe patterns are shown as examples at every interval of $\pi/8$ radian for the rectangular steel plate and the rectangular steel plate containing a hole, respectively, in Figs. 4 and 5. $I(x,y:\alpha)$ contains lots of spatial noises caused in speckle interferometry, and these spatial noises can be suppressed through two-dimensional (2-D) Fourier filtering. To do it, 2-D Fourier transform, low pass filtering, and 2 D inverse Fourier transform are performed for each of the 32 fringe patterns. α – directional Fourier transform of Eqn. (4), is performed with low pass filtered fringe pattern $I_L(x,y:\alpha)$. A mixed radix fast Fourier transform (MRFFT) (Morimoto et al., 1988) is used for 2-D Fourier filtering and α -directional Fourier transform. Only $F_\alpha(x,y:\omega)$ at $\omega=\omega_0=1$ is taken to calculate the wrapped phase ϕ in Eqn. (6).

$$F_\alpha(x,y:\omega)=\int_{-\infty}^{\infty} I_L(x,y:\alpha)\exp(-j\omega\alpha)d\alpha$$

The images of wrapped phases ϕ for the rectangular steel plate and the rectangular steel plate containing a hole are shown, respectively, in Figs. 6 and 7. Unwrapped phase ϕ_u is obtained by adding $\pm 2\pi$ radian at discontinuity of the wrapped phase. The in-plane displacement of the specimen, u, along the longitudinal direction is obtained through

$$u=\frac{\lambda}{4\pi\sin\theta}\phi_u \qquad (7)$$

where u is the longitudinal displacement and λ is the wavelength of light from a laser. The laser used in the experiment is He-Ne laser and λ is 633 nm. The angle between incident light and vertical line to the specimen, θ, is around 26°. Fig. 8 plots wrapped and unwrapped light intensity distribution along the line A-A indicated in Fig. 6 of the rectangular steel plate. In Fig. 9 (a), (b) and (c), wrapped and unwrapped light intensity distributions are plotted along the specified lines indicated in Fig. 7 of the rectangular steel plate containing a hole.

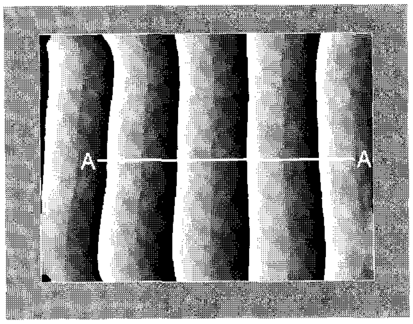


Fig. 6 Wrapped phase ϕ of rectangular steel plate

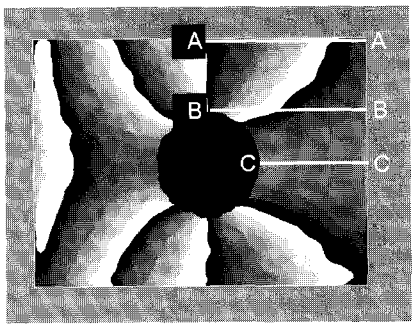


Fig. 7 Wrapped phase ϕ of rectangular steel plate containing a circular hole at the center

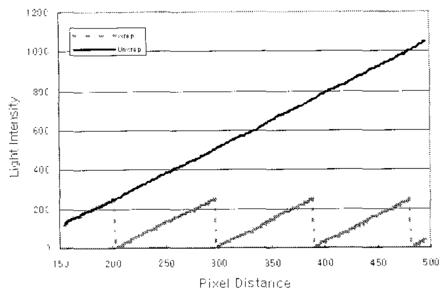
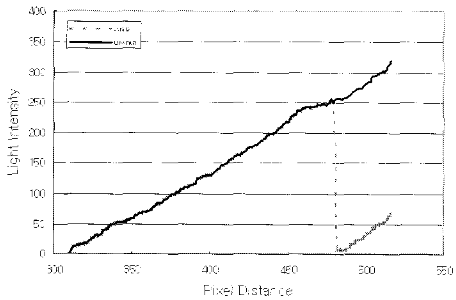
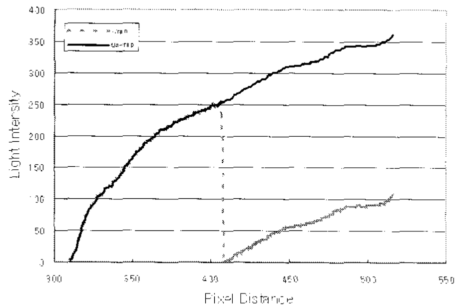


Fig. 8 Wrapped and unwrapped light intensity distribution along the line A-A indicated in Fig. 6 of the rectangular steel plate

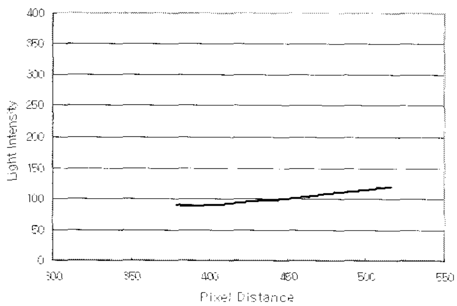
There is no phase jump along the line C-C in Fig. 7, so that the unwrapped light distribution is the same as the wrapped light distribution in Fig. 9 (c).



(a) Line A-A



(b) Line B-B



(c) Line C-C

Fig. 9 Wrapped and unwrapped light intensity distribution along the specified lines indicated in Fig. 7 of the rectangular steel plate containing a circular hole at the center. (a) Line A-A, (b) Line B-B, (c) Line C-C

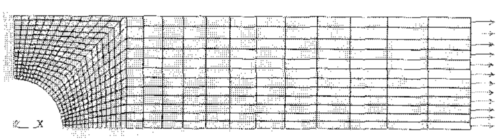


Fig. 10 ANSYS discretization of a quarter plate with a circular hole

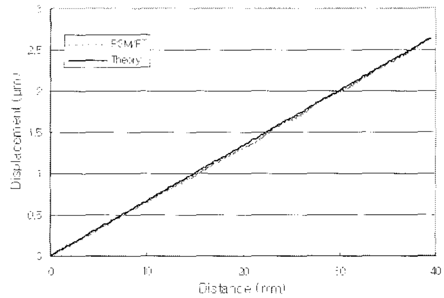
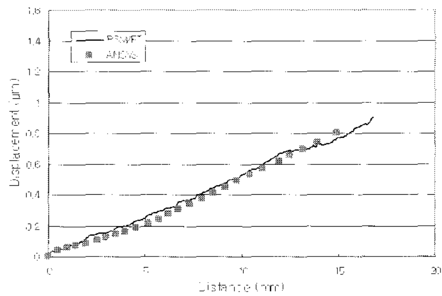
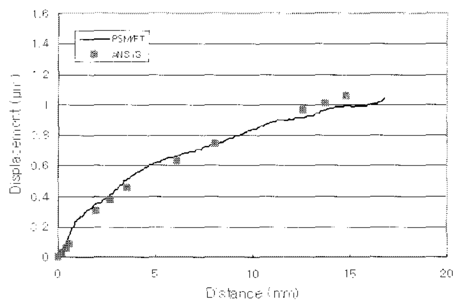


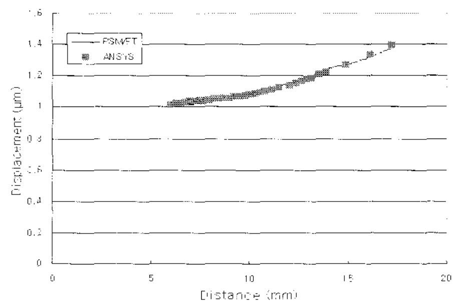
Fig. 11 In-plane displacement along line A-A indicated in Fig. 6 of rectangular steel plate by experiment and theory



(a) Line A-A



(b) Line B-B



(c) Line C-C

Fig. 12 In-plane displacement along the specified lines indicated in Fig. 7 of rectangular steel plate containing a circular hole by experiment and ANSYS. (a) Line A-A , (b) Line B-B, (c) Line C-C

For comparative purpose, the plate of Fig. 3 (b) is analyzed by ANSYS. The ANSYS discretization for one-quarter of the tensile plate of Fig. 3 (b) is shown in Fig. 10. It uses 416 eight-node isoparametric elements and 1349 nodes. In the vicinity of the circular hole, the ANSYS model of Fig. 10 utilizes elements on the edge of the hole as small as 1.5° by 0.013r, where "r" is the radius of the circular hole.

Fig. 11 compares the experimental results obtained from the speckle interferometry by PSM/FT with theoretical results from the linear elasticity of the rectangular steel plate along the line A-A indicated in Fig. 6 (Dally and Riley, 1991). For the rectangular steel plate containing a hole at the center, the results of the speckle experiment with PSM/FT, as shown in Fig. 12, are compared with those of ANSYS along the specified lines of A-A, B-B and C-C, respectively, as indicated in Fig. 7. As shown in Figs. 11 and 12, the results from the speckle experiment by PSM/FT agree well with those of the linear elasticity and ANSYS.

4. Conclusion

Phase shifting method used in speckle interferometry is optical technique in which known phases are added into optical light and resultant interference fringe patterns are analyzed to obtain information about small displacement. In this paper, phase-shifting method using Fourier transform has been applied to measurement of in-plane displacement of a specimen. Thirty-two fringe patterns each of which has different phase of $\pi/16$ radian have been gathered from a specimen with tensile load by use of CCD camera. PZT has been used for phase-shifting method. Low-pass filtering by 2-D Fourier transform is used to suppress spatial noise of the 32 fringe patterns. α -directional Fourier transform is performed by use of the low-pass filtered 32 fringe patterns. Two kinds of specimens are used for optical experiment. One is a rectangular steel plate and the other one is a rectangular steel plate containing a circular hole at the center. In-plane displacement of each specimen is measured by PSM/FT and calculated by FEM (ANSYS) for comparison. The results are quite comparable, so that PSM/FT can be applied to measurement of in-plane displacement.

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