Compound CVT with K-H-V Differential Gear and V-belt Drive

Yeon-Su Kim[1] and Sang-Hoon Choi[2, #]

[1] LRT System Development Team, Korea Railroad Research Institute, Gyongki-Do, Korea,
[2] Department of Mechanical Design and Production Engineering, Konkuk University, Seoul, Korea

ABSTRACT

Continuously variable transmission (CVT) mechanisms combine the functions of a K-H-V type differential gear unit and a V-belt type continuously variable unit (CVU). For the 24 different mechanisms, 12 of them are power circulation modes while the other 12 are power split modes. Some useful theoretical formulas related to speed ratio, power flow and efficiency were derived and analyzed. These mechanisms have many advantages: they decrease CVT size and weight, increase overall efficiency, extend speed ratio range, and generate geared neutral. Compound CVTs were developed by combining the power circulation mode and power split mode, which can offer backward motion, geared neutral, underdrive and overdrive.

Key Words : Continuously variable transmission, input coupled, power circulation, power split, compound CVT

1. Introduction

Most continuously variable transmission mechanisms have disadvantages that make it impossible to realize geared neutral by themselves, while their power transmission efficiency and lifecycle are inferior to that of a conventional gear train.[1] Continuously variable transmissions that have a differential gear unit can remove the disadvantages stated above, make it possible to decrease their size and reduce their weight. In addition, it is not necessary for starting devices like a torque converter to make geared neutral, and can increase power transmission efficiency and speed ratio ranges.[2-10]

Recently, we have suggested various power circulation modes and power split modes for the input coupled type and the output coupled type. CVT mechanisms have combined a 2K-H type I and type II differential gear unit with a V-belt continuously variable unit, while deriving useful formulas for efficiency, power flow, power ratio and speed ratio for them.[8-11] In combining the power circulation mode and power split mode, we proposed compound CVTs that can improve the characteristics of efficiency and power ratio, realize backward motion, geared neutral and forward motion by themselves.[11-14]

Twenty-four CVT mechanisms which combine the K-H-V differential gear unit and the V-belt continuously variable units, were developed in this study; with 12 of these falling under the input coupled types, and the other 12 under output coupled types. Theoretical formulas for speed ratio, power transmission efficiency, power flow and power ratio were also derived. In addition, two compound CVTs were developed by the performance characteristics of the CVT mechanisms.

2. General definitions

2.1 V-belt continuously variable unit

The continuously variable unit considered in this study is the V-belt drive with two variable-diameter pulleys and effective diameters (Fig. 1). One pulley was set by a mechanical linkage while the other was spring-loaded to provide automatic correspondence. The center distance between the two variable-diameter pulleys was fixed.

Manuscript received: July 29, 2003 ;
Accepted: March 16, 2004
Corresponding Author:
Email: shchoi@konkuk.ac.kr
Tel: +82-2-450-3476, Fax: +82-2-444-1044

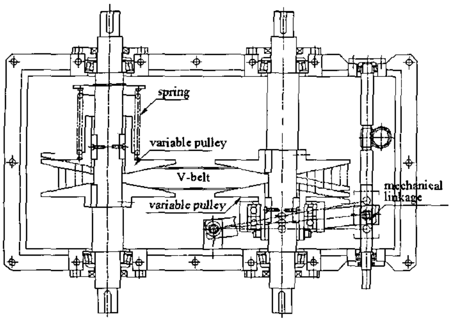


Fig. 1 V-belt type continuously variable unit

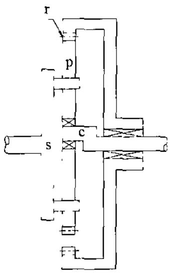


Fig. 2 K-H-V type differential gear unit

2.2 K-H-V type differential gear unit

The K-H-V type differential gear unit in Fig. 2 consists of an internal ring gear(r), a planet gear (p), a carrier(c), and a concentric shaft(s) with a ring gear(r). In this paper, the mating efficiency (η_{rp}) between the ring gear(r) and the planet gear (p) is defined as basic efficiency (η_o).[15]

$$\eta_o = \eta_{rp} \tag{1}$$

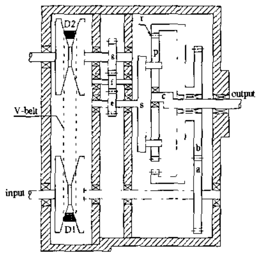
2.3 Power flows

For input coupled types, a CVU is coupled directly to a differential gear unit with the remaining shaft of a CVU linked to the input shaft. The output coupled type is an inversion of the input coupled type.[1,3,11] There are two power flow modes in the CVT mechanism, which are the power circulation mode and the power split mode. The power circulation mode has positive circulation and negative circulation. In the power split mode, there is no power circulation but the input power is divided into two branch streams through the CVU and a differential gear.

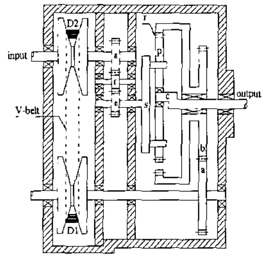
3. Basic configurations and theory

3.1 Basic configurations

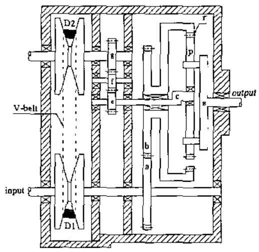
There are six possible ways in combining the V-belt type CVU with the K-H-V type differential gear unit, with either the input coupled type or the output coupled type. The basic configurations of the input coupled type are shown in Fig. 3, which are actually 12 basic configurations because power flow changes as weather an idler gear (f) is or not. Configuration 1 without an idler gear (f) is the power circulation mode, but configuration 1 with an idler gear (f) is the power split mode(see 3.2). For the output coupled types, the input shaft and output shaft of the input coupled type were converted into each other. Therefore, there were also 12 basic configurations for the output coupled type.



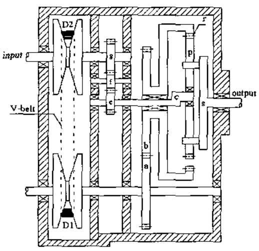
(a) configuration 1



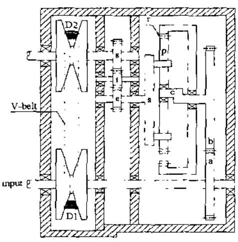
(b) configuration 2



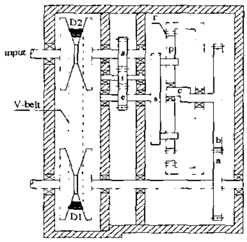
(c) configuration 3



(d) configuration 4



(e) configuration 5



(f) configuration 6

Fig. 3 Basic configurations of input coupled type

3.2 Input coupled type

Deriving the theoretical formulas for speed ratio, efficiency, power flow and power ratio was similar to the method of the preceding studies that had been previously verified by experimental studies.[8-11]

3.2.1 Power circulation mode

The ring gear(r) and the planet gear (p) without an idler gear (f) in configuration 1, as shown in Fig. 3(a) have the same revolution direction. This configuration can be substituted for the equivalent differential gear unit as shown in Fig. 4, where element 1 substitutes for the K-H-V type differential gear unit and element 2 replaces the continuously variable unit. The carrier (c') in the equivalent differential gear unit was fixed.

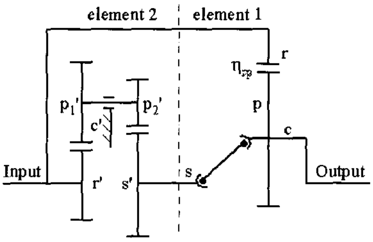


Fig. 4 Equivalent differential gear unit for input coupled configuration 1 without an idler gear (f)

The following formula should be satisfied so that Fig. 4 will be equivalent to configuration 1 without an idler gear (f):

$$i_{eq} = \frac{z_r' z_{p2}'}{z_s' z_{p1}'} = \frac{D_1}{D_2}\frac{z_g}{z_e}\frac{z_b}{z_a} \tag{2}$$

Where i_{eq} is the basic gear ratio of element 2, z_r', z_s', z_{p1}' and z_{p2}' the numbers of teeth of the planet gear r', s', p_1 and p_2. D_1 and D_2 are effective diameters of variable pulleys, while z_a, z_b, z_e, z_g are the numbers of the teeth of gear a, b, e, g in Fig. 3(a).

The operation of element 1 is equivalent to the sum of two operation constituents. In constituent 1, the planet gear (p) is driven, the carrier(c) is driving, and the ring gear(r) is fixed. In constituent 2, the ring gear(r) is driving, the carrier(c) is driven, and the planet gear (p) is fixed. The output power P_{01}, P_{02} of each constituent can be defined by the overall output power (P_0), the basic gear ratio (i_o) of the differential gear unit, transmitted torque ratio and the components' gear ratios.

The output power of configuration 1 without an idler gear (f) always circulates because " $P_{02}/P_{01}<0$" is always satisfied. Appendix 1 shows an overall speed ratio (i), theoretical efficiency (η), CVU power ratio (P_{CVU}/P_i), and power ratio (P_{dif}/P_i) of the differential power unit for configuration 1. Configuration 2, 3, 4 without an idler gear (f) and configuration 5, 6 with an idler gear (f) are also power circulation modes. Using the similar method, theoretical formulas are derived and shown in Appendix 1. In Appendix 1, P_i is the input power, η_{CVU} is the efficiency of the CVU, η_{ab} is the mating efficiency between gear a and b, η_{ef} is the mating efficiency between gear e and f, η_{fg} is the mating efficiency between gear f and g, η_{eg} is the mating efficiency between e and g. η_o' is the efficiency of element 2 in the equivalent differential gear unit:

Configuration 1, 2, 3, 4 without an idler gear (f)

$$\eta_o' = \eta_{ab}\eta_{eg}\eta_{CVU} \tag{3}$$

Configuration 5, 6 with an idler gear (f)

$$\eta_o' = \eta_{ab}\eta_{eg}\eta_{fg}\eta_{CVU} \tag{4}$$

3.2.2 Power split mode

The ring gear(r) and the planet gear (p) of configuration 1 with an idler gear (f) as shown in Fig. 3(a) have opposite revolution directions. This configuration can be substituted by the equivalent differential gear unit (Fig. 5), where the following formula should be satisfied:

$$i_{eq} = \frac{z_r'}{z_s'} = \frac{D_1}{D_2}\frac{z_g}{z_e}\frac{z_b}{z_a} \tag{5}$$

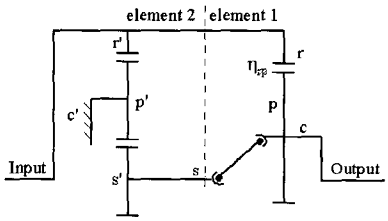


Fig. 5 Equivalent differential gear unit for input coupled configuration 1 with an idler gear (f)

Element 1 can be divided into two operational constituents as it was in the power circulation mode in 3.2.1. This configuration is a power split mode because " $P_{02}/P_{01}>0$" is always satisfied. Appendix 2 shows theoretical formulas of configuration 5, 6 without an idler gear (f), and configurations 1, 2, 3, 4 with an idler gear (f). The η_o', efficiency of element 2, is identical to formula (4) in case of configuration 1, 2, 3, 4 with an idler gear (f), and formula (3) in case of configuration 5, 6 without an idler gear (f).

3.3 Output coupled type

The power flow of the output coupled type is the same as that of the input coupled type. Therefore, theoretical formulas can be derived by the method in 3.2.1. Appendix 3 shows theoretical formulas of power split modes for the output coupled type.

4. Performance characteristics

4.1 Design parameters

Speed ratios, power ratios, efficiencies of basic configurations in Appendix 1-3 vary by basic gear ratio ($i_o=z_r/z_p$) of the K-H-V type differential gear unit, z_b/z_a and z_g/z_e. Therefore, three variables were established as design parameters of basic configurations, while parametric studies were performed to analyze their performance characteristics as shown in Table 1.

Table 1 Parameter ranges for parametric analysis

Parameters	Ranges	Remarks
z_r/z_p	2.0 ~ 4.0	z_b/z_a= 2.0 z_g/z_e=1.0
z_b/z_a	1/3 ~ 3.0	z_r/z_a=3.0 z_g/z_e=1.0
z_g/z_e	1/3 ~ 3.0	z_r/z_s=3.0 z_g/z_e=2.0

Table 2 Efficiencies of gears and the CVU

K-H-V differential gear unit & gear trains	
$\eta_{rp}=\eta_{ef}=\eta_{fg}=\eta_{ab}=0.982$	
V-belt continuously variable unit	
Speed ratios	Efficiencies of the CVU
0.50	0.938
0.66	0.904
1.00	0.870
2.00	0.824

The efficiencies of the CVU, the differential gear unit and gear trains are shown in Table 2.[8-11] For the determination of the CVU efficiency, many experimental studies were performed for speed ratio 0.5, 0.66, 1.0 and 2.0. Efficiencies between measured efficiencies for the four speed ratios were determined by linear interpolation. For the mating efficiencies of the differential gear and the gear trains in Fig.3, some assumptions were defined; all gears are standard spur gear without backlash, bearing loss while the churning loss of lubricant are negligible. That is, friction loss of tooth surfaces is the only power loss in the differential gear and gear trains.[16-17]

4.2 Characteristics of the input coupled type

For the six power circulation modes and the six power split modes of the input coupled type, performance characteristics analysis were performed as changing the three design parameters (Table 1).

4 .2 .1 Power circulation mode

Neutral points (geared neutral) in the speed ratios of configuration 1, 4, 5 moved to the speed ratio 2.0 of the CVU as the basic gear ratios ($i_0=z_r/z_p$) of the differential gear unit increased, but that of configuration 2, 3, 6 moved to the speed ratio 0.5 of the CVU. For the gear ratio z_b/z_a and z_g/z_e, the trend of configuration 3, 4 was similar to the case of basic gear ratios ($i_0=z_r/z_p$), but configuration 1, 2, 5, 6 had opposite trends. Fig. 6 shows efficiencies of the configuration 1 without an idler gear (power circulation mode) as changing basic gear ratio ($i_0=z_r/z_p$) of the differential gear unit.

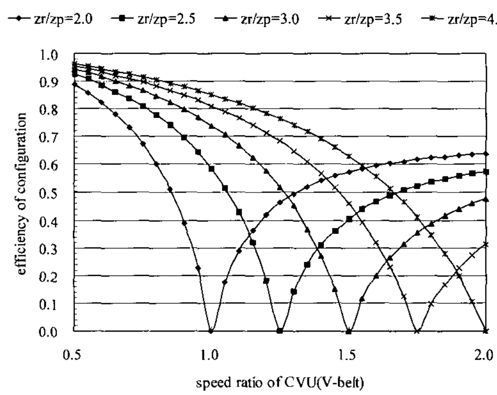


Fig. 6 Efficiencies of input coupled type configuration 1(power circulation) as the gear ratio z_r/z_p

The slopes of the overall speed ratios versus the CVU speed ratios in configuration 1, 3, 5 were positive, but that of configuration 2, 4, 6 were negative. The more basic gear ratio ($i_0=z_r/z_p$) increased, while the slopes of the overall speed ratios versus the CVU speed ratios in configuration 3,4,5,6 increased. However, configuration 1, 2 had opposite trends. For the gear ratio z_b/z_a, configuration 1, 2, 5, 6 had similar trends in case of basic gear ratios ($i_0=z_r/z_p$), but configuration 3, 4 had opposite trends. For the gear ratio z_g/z_e, configuration 3, 4, 5, 6 had similar trends for basic gear ratios ($i_0=z_r/z_p$), but configuration 1, 2 had opposite trends. Fig. 7 shows the overall speed ratios of configuration 1 without an idler gear as changing the basic gear ratio ($i_0=z_r/z_p$).

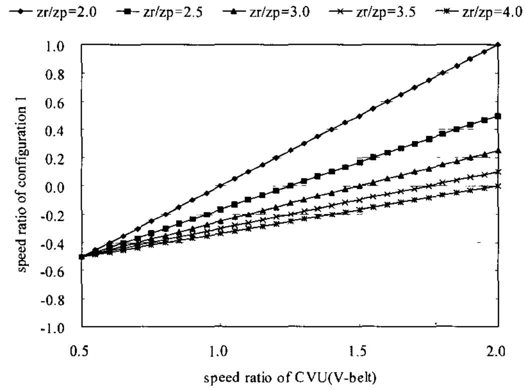


Fig. 7 Speed ratios of input coupled type configuration 1(power circulation) as the gear ratio z_r/z_p

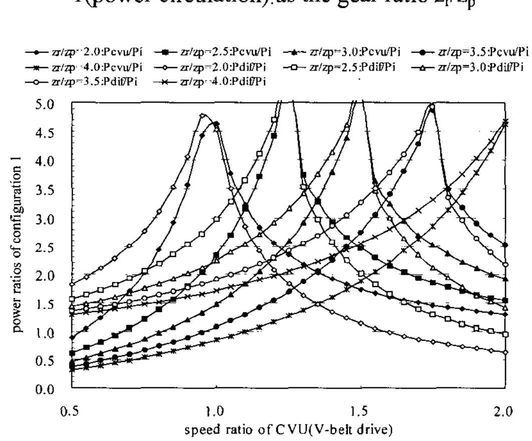


Fig. 8 Power ratios of input coupled type configuration 1(power circulation) as the gear ratio z_r/z_p

There is no output power at the geared neutral (speed ratio 0.0) in power circulation mode, that is, all power circulates in the CVT mechanism. Therefore, the CVU power ratio (P_{CVU}/P_i) and that of the differential gear unit (P_{dif}/P_i) are maximum. They have similar trends as changing the design parameters in case of overall efficiency. Fig.8 shows the power ratios of the CVU and the differential gear unit as changing the basic gear ratio ($i_0=z_r/z_p$) in configuration 1 without an idler gear.

Power circulation modes have backward motion, geared neutral and forward motion in overall speed ratios because the revolution direction of the output shaft changes when passing through neutral point. As shown in Fig. 7, the overall speed ratio is zero in the neutral point (geared neutral). However, it is difficult to reduce the size and weight of the power circulation mode because the power ratios of its components are large (Fig.8).

4.2.2 Power split mode

The efficiencies of configuration 1, 4, 5 were proportional to the increase of the basic gear ratio ($i_0=z_r/z_p$), but the efficiencies of configuration 2, 3, 5 were inversely proportional to that. For the gear ratio z_b/z_a, configuration 1, 2, 4, 6 had opposite trends for basic gear ratios ($i_0=z_r/z_p$), but configuration 3, 4 had similar trends. Fig. 9 shows efficiencies and overall speed ratios of configuration 1 with an idler gear (power split mode) as the changing basic gear ratio ($i_0=z_r/z_p$).

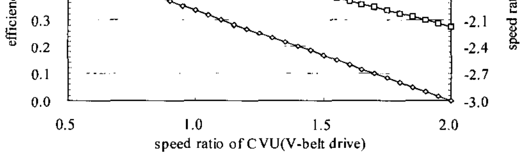


Fig. 9 Efficiencies and speed ratios of input coupled type configuration 1 as the gear ratio z_r/z_p

The overall speed ratios and their slopes are negative values in all configurations. For configuration 1, 2, 5, 6, the overall speed ratios and their slopes decreased as the basic gear ratio ($i_0=z_r/z_p$) and z_b/z_a increased. But increased as the gear ratio z_g/z_e increased. Configuration 3, 4 had opposite trends.

Power ratios of the differential gear unit (P_{dif}/P_i) had similar trends in case of overall efficiencies as changing the design parameters, but CVU power ratios (P_{CVU}/P_i)

had opposite trends because the efficiency of the differential gear unit was higher than that of the CVU. Fig. 10 shows the power ratios of the differential gear unit (P_{dif}/P_i) and that of the CVU (P_{CVU}/P_i) as the changing basic gear ratio ($i_0=z_r/z_p$) for configuration 1 with an idler gear (power split mode). Power split modes can maintain higher efficiency and lower power ratios than power circulation modes (Fig.10). However, power split modes cannot realize geared neutral.

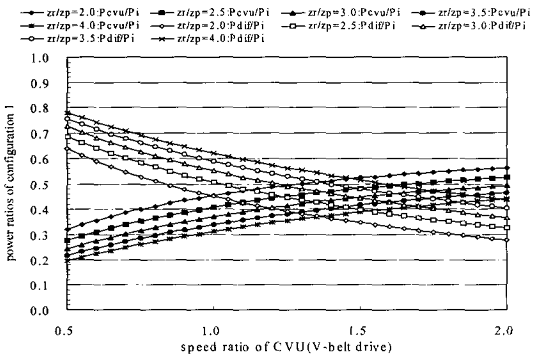


Fig. 10 Power ratios of input coupled type configuration 1(power split) as the gear ratio z_r/z_p

4 .3 Characteristics of the output coupled type

4 .3 .1 Power circulation mode

Power circulation modes of the output coupled type have speed ratios, which are hyperbolic curves that diverge at a neutral point. The revolution direction of the output shaft is reverted with efficiency at zero around the neutral point. Therefore, power circulation modes of the output coupled type cannot be used for power transmission device.

4 .3 .2 Power split mode

The efficiencies of the power split modes in the output coupled type have similar trends in the input coupled type as changing the design parameters. These, however, can be maintained constantly as changing the CVU speed ratio. The output coupled type cannot be used for the compound CVT because power circulation modes cannot be used for the power transmission device. However, the power split modes of the output coupled type are more beneficial than that of the input coupled type. The efficiencies of the power split modes are proportional power ratios of the differential gear unit (P_{dif}/P_i), and are inversely proportional to that of the CVU (P_{CVU}/P_i).

5. Compound CVT

5. 1 Design requirements

Compound CVTs were developed by combining the power circulation mode and the power split mode, which can offer backward motion, geared neutral, underdrive and overdrive (Fig. 11). These can also improve their efficiencies and power ratios. Only the input coupled type can be used for the compound CVT. The following requirements should be satisfied so that compound CVTs will operate smoothly.

- The power circulation mode and the power split modes should have an opposite slope for the overall speed ratio at the transition point.
- The power circulation mode and the power split mode should have an identical speed ratio at the transition point. In this study, the speed ratio at the point is 0.5.
- The power circulation mode should offer geared neutral, while the power split mode should realize speed ratios of more than 1.0 so that the compound CVTs can perform backward motion, geared neutral and forward motion.
- All gear ratios of the compound CVTs should reduce their size and weight.

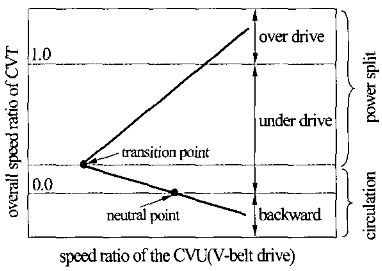
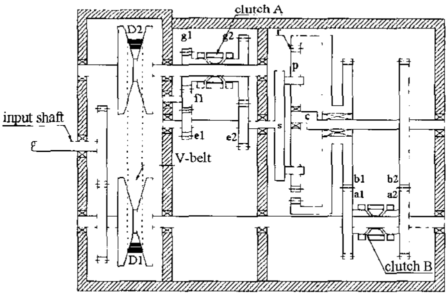


Fig. 11 Power flow modes and speed ratio ranges of the compound CVT

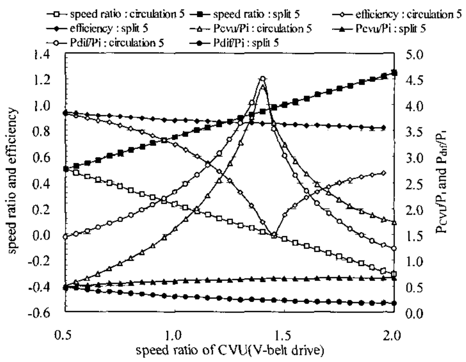
The speed ratio at the transition point, and component gears' ratios can be determined by considering vehicle performance (the maximum speed, acceleration and deceleration, the occupied space of power transmission device, weight and fuel economy). However, we cannot consider the requirements of any specific vehicle at the present time. Four design requirements, as stated above, were presented to design the compound CVT in this paper. Two compound CVTs have been developed, based on the characteristics of the input coupled type (6 power circulation modes, 6 power split modes) as changing three design parameters. These require only two clutches respectively, while all gear ratios were kept small to satisfy their performance and design requirements.

5.2 Type I

This compound CVT combines configuration 5 with an idler gear (power circulation mode) and configuration 5 without an idler gear (power split mode) in Fig. 3(e). Fig.12 (a) shows the structure of the compound CVT which requires only two clutches (A, B) to realize two operation modes. If all clutches move to the right, power circulation will occur. If all clutches move to the left, power split will occur. Fig. 12(b) shows the performance characteristics of the compound CVT with z_r/z_p=2.0, z_{b1}/z_{a1}=0.65, z_{g1}/z_{e1}=1.08, z_{b2}/z_{a2}=2.0, z_{g2}/z_{e2}=1.0, where z_{b1}, z_{a1}, z_{g1}, z_{e1}, z_{b2}, z_{a2}, z_{g2}, z_{e2} are the numbers of teeth of gear b1, a1, g1, e1, b2, a2, g2, e2 in Fig.12 (a).



(a) Structure of the compound CVT

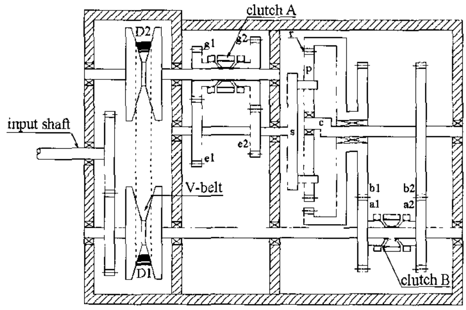


(b) Characteristics on the compound CVT

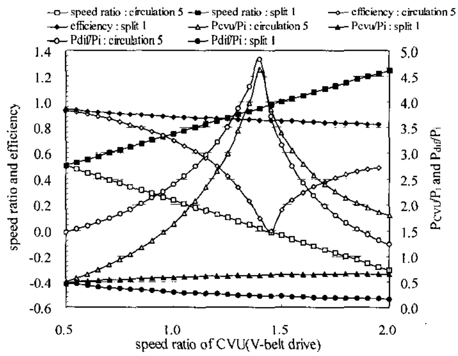
Fig. 12 Compound CVT composed of power circulation mode 5 and power split mode 5

5.3 Type II

This compound CVT combines configuration 1 without an idler gear (power circulation mode) in Fig. 3(a) and configuration 5 without an idler gear (power split mode) in Fig.3 (e). Fig. 13 (a) shows the structure of the compound CVT which requires only two clutches (A, B) to operate smoothly. If all clutches move to the left, power circulation will occur. If all clutches move to the right, power split will occur. Fig. 13 (b) shows the performance characteristics of the compound CVT with z_r/z_p=2.0,z_{b1}/z_{a1}=2.6, z_{g1}/z_{e1}=0.54, z_{b2}/z_{a2}=2.0, z_{g2}/z_{e2}=1.0.



(a) Structure of the compound CVT



(b) Characteristics on the compound CVT

Fig. 13 Compound CVT composed of power circulation mode 5 and power split mode 1

6. Conclusions

This paper combined the V-belt type CVU with the K-H-V type differential gear unit, and proposed 24 basic configurations: 12 power circulation modes and 12 power split modes. For the basic configurations, theoretical formulas were derived and their characteristics analyzed. Based on those results, two compound CVTs were developed by combining the power circulation mode and the power split mode. The conclusions were:

- Power circulation modes of the input coupled type can make backward motion, geared neutral and forward motion, but cannot maintain high efficiency and low power ratio over the operating speed ratio range. Power split modes of the input coupled type have low power ratio and high efficiency, but cannot make geared neutral and backward motion.
- The power circulation modes of the output coupled type cannot be used for CVT mechanisms.
- The power split modes of the output coupled type are more beneficial than those of the input coupled type.
- The compound CVTs developed in this paper can improve the efficiency and power ratios when compared with the case where power circulation modes and power split modes were used individually.

References

1. Beachley, N. H. and Frank, A. A., "Principles and Definitions for Continuously Variable Transmissions with Emphasis on Automotive Application," ASME Paper 80-C2/DET-95, 1980.
2. Macmillan, R.H. and Davies, P.B., "Analytical Study of Systems for Bifurcated Power Transmission," Journal of Mechanical Engineering Science, Vol. 7, No. 1, pp. 40-47, 1965.
3. White, G., "Properties of Differential Transmission," The Engineer, pp.105-111, 1967.
4. Yu, D. and Beachley, N.H., "On the Mechanical Efficiency of Differential Gearing," ASME Journal of Mechanisms, Transmissions and Automation in Design, Vol.107, pp.61-67, 1985.
5. Zhang, Y. and Leduc, B., "Efficiency predetermination of planetary trains used as continuously variable power transmission," European Journal of Mech. Eng., Vol.37, No.3, pp.169-173, 1992.
6. Wohl, R., Long, T., Mucino, V.H. and Smith, J.E., "A Model for a Planetary-CVT Mechanism: Analysis and Synthesis," SAE Paper 930065, 1993.
7. Morozumi, M. and Kishi, S., "A Mechanical Consideration on CVT with Differential Gearing (1), " Mechanical Research (Japanese), Vol. 49, No. 5, pp. 50-58., 1997.
8. Kim, Yeon-Su and Choi, Sang-Hoon, "Power Flow and Efficiency of the Input Coupled Type CVT Combined Differential Gear Unit, " Journal of KSPE, Vol. 17, No. 11, pp. 141-150, 2000.
9. Choi, Sang-Hoon and Kim, Yeon-Su, "Characteristics on the Output Coupled Type CVT Combined 2K-H Differential Gear Unit and V-belt Drive," Journal of the KSPE, Vol. 18, No 3, pp. 205-216, 2000.
10. Kim, Yeon-Su and Choi, Sang-Hoon, "Experimental Study on the Input Coupled Type CVT Combined a Differential Gear and a V-Belt type CVU," Int'l. J. of the KSPE, Vol. 2, No. 1, pp. 43-55, 2001.
11. Kim, Yeon-Su, "Design of compound CVTs using a V-belt Drive and Differential gear units. " Doctoral Dissertation of Konkok University, 2002.
12. Roberts, R. W., "Continuously Variable Transmission with Synchronous shift," US Patent No. 4589303, 1984.
13. Hanachi, S., "A Study of the Dynamics of a Split-Torque, Geared-Neutral Transmission Mechanism," Journal of Mechanical Design, Vol.112, pp.261-270, 1990.
14. Morozumi, M., Kishi, S., Furukawa, Y. and Misawa, H., "A Study of Variable Speed Transmission with Differential Gear," journal of Japan Automotive technology Association, No. 46, pp.45-49, 1990.
15. Morozumi, M., Theory and Design of Planetary Gears and Differential Gears (Japanese), Tokyo, Japan, pp. 8-31, 1989.
16. Kim, Yeon-Su, Choi, Sung-Uk and Choi, Sang-Hoon, "A Study on the Efficiency Analysis of 2K-H Type Planetary Gear Train," Journal of the KSPE , Vol. 17, No. 3, pp. 200-207, 2000.
17. Kim, Yeon-Su and Choi, Sang-Hoon, "Interference and Efficiency Analysis of 2K-H I Type Differential Gear Unit," Int'l. J. of the KSPE, Vol. 1, No. 1, pp. 5-14, 2000.

Appendix 1 Speed ratios, power ratios and efficiencies for six power circulation modes in the input coupled system

Configuration	Criteria for the power flow direction	Equivalent relations (i_{eq})	Speed ratios (i)	Overall efficiencies for CVT (η)	CVU power ratio (P_{CVU}/P_i)	Power ratio for differential gear units (P_{dif}/P_i)
1	$i_{eq} < i_o$	$\frac{D_1 z_g z_b}{D_2 z_e z_a}$	$-\frac{i_o - i_{eq}}{i_o - 1}\frac{z_a}{z_b}$	$\frac{(\eta_o i_o - 1)(i_o - \eta_o)(i_o - i_{eq})}{\eta(i_o - 1)[i_o(i_o - \eta_o) - \eta_o' i_{eq}(\eta_o i_o - 1)]}$	$\frac{i_{eq}(\eta_o i_o - 1)}{i_o(i_o - 1) - \eta_o' i_{eq}(\eta_o i_o - 1)}$	$\frac{i_o(i_o - \eta_o)}{i_o(i_o - 1) - \eta_o' i_{eq}(\eta_o i_o - 1)}$
	$i_{eq} > i_o$		$\frac{i_{eq} - i_o}{i_o - 1}\frac{z_a}{z_b}$	$\frac{\eta_o'(\eta_o i_o - 1)(i_o - \eta_o)(i_{eq} - i_o)}{(i_o - 1)[i_{eq}(i_o - \eta_o) - \eta_o' i_{eq}(\eta_o i_o - 1)]}$	$\frac{\eta_o' i_{eq}(i_o - \eta_o)}{i_{eq}(i_o - \eta_o) - \eta_o' i_o(\eta_o i_o - 1)}$	$\frac{\eta_o' i_o(\eta_o i_o - 1)}{i_{eq}(i_o - \eta_o) - \eta_o' i_o(\eta_o i_o - 1)}$
2	$i_{eq} < i_o$	$\frac{D_1 z_g z_b}{D_2 z_e z_a}$	$-\frac{i_o - i_{eq}}{i_{eq}(i_o - 1)}\frac{z_g}{z_e}$	$\frac{\eta'(\eta_o i_o - 1)(i_o - \eta_o)(i_o - i_{eq})}{\eta(i_o - 1)[i_o(i_o - \eta_o) - \eta_o' i_{eq}(\eta_o i_o - 1)]}$	$\frac{\eta_o' i_o(i_o - \eta_o)}{i_o(i_o - \eta_o) - \eta_o' i_{eq}(\eta_o i_o - 1)}$	$\frac{\eta_o' i_{eq}(\eta_o i_o - 1)}{i_o(i_o - 1) - \eta_o' i_{eq}(\eta_o i_o - 1)}$
	$i_{eq} > i_o$		$\frac{i_{eq} - i_o}{i_{eq}(i_o - 1)}\frac{z_g}{z_e}$	$\frac{(\eta_o i_o - 1)(i_o - \eta_o)(i_{eq} - i_o)}{(i_o - 1)[i_{eq}(i_o - \eta_o) - \eta_o' i_{eq}(\eta_o i_o - 1)]}$	$\frac{i_o(\eta_o' i_o - \eta_o)}{i_{eq}(i_o - \eta_o) - \eta_o' i_o(\eta_o i_o - 1)}$	$\frac{i_{eq}(i_o - \eta_o)}{i_{eq}(i_o - \eta_o) - \eta_o' i_o(\eta_o i_o - 1)}$
3	$i_o > i_{eq}(i_o - 1)$	$\frac{D_1 z_g z_b}{D_2 z_e z_a}$	$-[i_o - i_{eq}(i_o - 1)]\frac{z_a}{z_b}$	$\frac{\eta_o[i_o - i_{eq}(i_o - 1)]}{i_o - \eta_o' \eta_o i_{eq}(\eta_o i_o - 1)}$	$\frac{\eta_o i_{eq} - (\eta_o i_o - 1)}{i_o - \eta_o' \eta_o i_{eq}(\eta_o i_o - 1)}$	$\frac{i_o}{i_o - \eta_o' \eta_o i_{eq}(\eta_o i_o - 1)}$
	$i_o < i_{eq}(i_o - 1)$		$[i_{eq}(i_o - 1) - i_o]\frac{z_a}{z_b}$	$\frac{(\eta_o i_o - 1)(i_o - \eta_o)(i_{eq} - i_o)}{i_{eq}(i_o - \eta_o) - \eta_o \eta_o'^2 i_o}$	$\frac{\eta_o' i_{eq}(i_o - \eta_o)}{i_{eq}(i_o - \eta_o) - \eta_o \eta_o'^2 i_o}$	$\frac{\eta_o' \eta_o^2 i_o}{i_{eq}(i_o - \eta_o) - \eta_o \eta_o'^2 i_o}$
4	$i_o > i_{eq}(i_o - 1)$	$\frac{D_1 z_g z_b}{D_2 z_e z_a}$	$-\frac{i_o - i_{eq}(i_o - 1)}{i_{eq}}\frac{z_g}{z_e}$	$\frac{\eta_o' \eta_o[i_o + i_{eq}(i_o - 1)]}{i_o - \eta_o' \eta_o i_{eq}(\eta_o i_o - 1)}$	$\frac{\eta_o' i_o}{i_o - \eta_o' \eta_o i_{eq}(\eta_o i_o - 1)}$	$\frac{\eta_o' \eta_o i_{eq}(\eta_o i_o - 1)}{i_o - \eta_o' \eta_o i_{eq}(\eta_o i_o - 1)}$
	$i_o < i_{eq}(i_o - 1)$		$\frac{i_{eq} - (i_o - 1) - i_o}{i_{eq}}\frac{z_g}{z_e}$	$\frac{\eta_o[i_{eq} i_o - 1) - i_o]}{i_{eq}(i_o - \eta_o) - \eta_o' \eta_o'^2 i_o}$	$\frac{\eta_o^2 i_o}{i_{eq}(i_o - \eta_o) - \eta_o' \eta_o'^2 i_o}$	$\frac{i_{eq}(i_o - \eta_o)}{i_{eq}(i_o - \eta_o) - \eta_o' \eta_o'^2 i_o}$
5	$i_{eq} > i_o - 1$	$\frac{D_1 z_g z_b}{D_2 z_e z_a}$	$\frac{i_{eq} - (i_o - 1)}{i_o}\frac{z_a}{z_b}$	$\frac{(\eta_o' \eta_o[i_{eq} - (i_o - 1)]}{i_{eq} - \eta_o'(\eta_o i_o - 1)}$	$\frac{\eta_o' i_{eq}}{i_{eq} - \eta_o'(\eta_o i_o - 1)}$	$\frac{\eta_o'(\eta_o i_o - 1)}{i_{eq} - \eta_o'(\eta_o i_o - 1)}$
	$i_{eq} < i_o - 1$		$-\frac{(i_o - 1) - i_{eq}}{i_o}\frac{z_a}{z_b}$	$\frac{(i_o - 1) - i_{eq}}{(i_o - \eta_o) - \eta_o' \eta'_o i_{eq}}$	$\frac{\eta_o i_{eq}}{(i_o - \eta_o) - \eta_o' \eta'_o i_{eq}}$	$\frac{i_o - \eta_o}{(i_o - \eta_o) - \eta_o' \eta'_o i_{eq}}$
6	$i_{eq} > i_o - 1$	$\frac{D_1 z_g z_b}{D_2 z_e z_a}$	$\frac{i_{eq} - (i_o - 1)}{i_o i_{eq}}\frac{z_g}{z_e}$	$\frac{\eta_o[i_{eq} - (i_o - 1)]}{i_{eq} - \eta_o'(\eta_o i_o - 1)}$	$\frac{\eta_o i_o - 1}{i_{eq} - \eta_o'(\eta_o i_o - 1)}$	$\frac{i_{eq}}{i_{eq} - \eta_o'(\eta_o i_o - 1)}$
	$i_{eq} < i_o - 1$		$-\frac{(i_o - 1) - i_{eq}}{i_o i_{eq}}\frac{z_g}{z_e}$	$\frac{\eta_o'[(i_o - 1) - i_{eq}]}{(i_o - \eta_o) - \eta_o' \eta'_o i_{eq}}$	$\frac{\eta_o'(i_o - 1)}{(i_o - \eta_o) - \eta_o' \eta'_o i_{eq}}$	$\frac{\eta_o' \eta_o i_{eq}}{(i_o - \eta_o) - \eta_o' \eta'_o i_{eq}}$

Appendix 2 Speed ratios, power ratios, and efficiencies for the six power-split modes in the input couple system.

Configuration	Equivalent relations (i_{eq})	Speed ratios (i)	Overall efficiencies of CVT(η)	CVU power ratios (P_{CVU}/P_i)	Power ratios for differential gear units (P_{dif}/P_i)
1	$\frac{D_1 z_g z_b}{D_2 z_e z_a}$	$-\frac{i_o+i_{eq}}{i_o-1}\frac{z_a}{z_b}$	$\frac{\eta_o'(\eta_o i_o-1)(i_o+i_{eq})}{(i_o-1)(i_{eq}+\eta_o'\eta_o i_o)}$	$\frac{\eta_o' i_{eq}}{(i_{eq}+\eta_o'\eta_o i_o)}$	$\frac{\eta_o'\eta_o i_o}{(i_{eq}+\eta_o'\eta_o i_o)}$
2	$\frac{D_1 z_g z_b}{D_2 z_e z_a}$	$-\frac{i_o+i_{eq}}{i_{eq}(i_o-1)}\frac{z_g}{z_e}$	$\frac{\eta_o'(\eta_o i_o-1)(i_o+i_{eq})}{(i_o-1)[\eta_o' i_{eq}+\eta_o i_o)}$	$\frac{\eta_o'\eta_o i_o}{\eta_o' i_{eq}+\eta_o i_o}$	$\frac{\eta_o' i_{eq}}{\eta_o' i_{eq}+\eta_o i_o}$
3	$\frac{D_1 z_g z_b}{D_2 z_e z_a}$	$-[i_o+i_{eq}(i_o-1)]\frac{z_a}{z_b}$	$\frac{\eta_o'\eta_o[i_o+i_{eq}i_o(i_o-1)}{i_{eq}(i_o-\eta_o)-\eta_o i_o}$	$\frac{\eta_o' i_{eq}(i_o-\eta_o)}{i_{eq}(i_o-\eta_o)+\eta_o' i_o}$	$\frac{\eta_o' i_o}{i_{eq}(i_o-\eta_o)+\eta_o' i_o}$
4	$\frac{D_1 z_g z_b}{D_2 z_e z_a}$	$-\frac{i_o+i_{eq}(i_o-1)}{i_{eq}}\frac{z_g}{z_e}$	$\frac{\eta_o'\eta_o[i_o+i_{eq}(i_o-1)]}{\eta_o' i_{eq}(i_o-\eta_o)+i_o}$	$\frac{\eta_o' i_{eq}}{\eta_o i_{eq}(i_o-\eta_o)+i_o}$	$\frac{\eta_o' i_{eq}(i_o-\eta_o)}{\eta_o i_{eq}(i_o-\eta_o)+i_o}$
5	$\frac{D_1 z_g z_b}{D_2 z_e z_a}$	$-\frac{i_{eq}+(i_o-1)}{i_o}\frac{z_a}{z_b}$	$\frac{\eta_o'\eta_o[i_{eq}+(i_o-1)]}{i_{eq}-\eta_o'\eta_o i_o-\eta_o)}$	$\frac{\eta_o' i_{eq}}{i_{eq}+\eta_o'\eta_o(i_o-\eta_o)}$	$\frac{\eta_o'\eta_o(i_o-\eta_o)}{i_{eq}+\eta_o'\eta_o(i_o-\eta_o)}$
6	$\frac{D_1 z_g z_b}{D_2 z_e z_a}$	$-\frac{i_{eq}(i_o-1)}{i_o i_{eq}}\frac{z_g}{z_e}$	$\frac{\eta_o'\eta_o[i_{eq}(i_o-1)]}{\eta_o' i_{eq}+\mu_o(i_o-\eta_o)}$	$\frac{\eta_o'\eta_o(i_o-\eta_o)}{\eta_o i_{eq}+\eta_o(i_o-\eta_o)}$	$\frac{\eta_o' i_{eq}}{\eta_o i_{eq}+\eta_o(i_o-\eta_o)}$

Appendix 3 Speed ratios, power ratios, and efficiencies for six power split modes in output coupled system

Configuration	Equivalent Relations (i_{eq})	Speed ratios (i)	Overall efficiencies of CVT (η)	CVU power ratios CVU (P_{CVU}/P_i)	Power ratios for differential gear units (P_{dif}/P_i)
1	$\frac{D_1 z_g z_b}{D_2 z_e z_a}$	$-\frac{i_o-1}{i_o+i_{eq}}\frac{z_b}{z_a}$	$\frac{(i_o-1)(\eta_o'\eta_o i_{eq}+i_o)}{(i_o-\eta_o)(i_o+i_{eq})}$	$\frac{\eta_o' i_{eq}(i_o-1)}{(i_o-1)(i_o+i_{eq})}$	$\frac{i_o(i_o-1)}{(i_o-1)(i_o+i_{eq})}$
2	$\frac{D_1 z_g z_b}{D_2 z_e z_a}$	$-\frac{i_{eq}(i_o-1)}{i_{eq}(i_o-1)}\frac{z_g}{z_e}$	$\frac{(i_o-1)(\eta_o' i_{eq}+\eta_o i_o)}{(i_o-\eta_o)(i_o+i_{eq})}$	$\frac{i_o(i_o-1)}{(i_o-1)(i_o+i_{eq})}$	$\frac{\eta_o i_{eq}(i_o-1)}{(i_o-\eta_o)(i_o+i_{eq})}$
3	$\frac{D_1 z_g z_b}{D_2 z_e z_a}$	$-\frac{1}{i_o+i_{eq}(i_o-1)}\frac{z_b}{z_a}$	$\frac{\eta_o' i_{eq}(\eta_o i_o-1)+\eta_o i_o}{i_o+i_{eq}(i_o-1)}$	$\frac{i_{eq}(\eta_o i_o-1)}{i_o+i_{eq}(i_o-1)}$	$\frac{\eta_o i_o}{i_o+i_{eq}(i_o-1)}$
4	$\frac{D_1 z_g z_b}{D_2 z_e z_a}$	$-\frac{i_{eq}}{i_o+i_{eq}(i_o-1)}\frac{z_e}{z_g}$	$\frac{i_{eq}(\eta_o i_o-1)+\eta_o'\eta_o i_o}{i_o+i_{eq}(i_o-1)}$	$\frac{\eta_o i_o}{i_o+i_{eq}(i_o-1)}$	$\frac{i_{eq}(\eta_o i_o-1)}{i_o+i_{eq}(i_o-1)}$
5	$\frac{D_1 z_g z_b}{D_2 z_e z_a}$	$-\frac{i_o}{i_{eq}+(i_o-1)}\frac{z_b}{z_a}$	$\frac{\eta_o'\eta_o 2i_{eq}+(\eta_o i_o-1)}{\eta_o[i_{eq}+(i_o-1]}$	$\frac{\eta_o i_{eq}}{i_{eq}(i_o-1)}$	$\frac{\eta_o i_o-1}{\eta_o[i_{eq}+(i_o-1]}$
6	$\frac{D_1 z_g z_b}{D_2 z_e z_a}$	$-\frac{i_o i_{eq}}{i_{eq}+(i_o-1)}\frac{z_e}{z_g}$	$\frac{\eta_o 2i_{eq}+\eta_o'(\eta_o i_o-1)}{\eta_o[i_{eq}+(i_o-1]}$	$\frac{\eta_o i_o-1}{\eta_o[i_{eq}+(i_o-1]}$	$\frac{\eta_o i_{eq}}{i_{eq}+(i_o-1)}$