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Modeling of a Functional Surface using a Modified B-spline

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ABSTRACT

This research presents modeling of a functional surface that is constructed with a free-formed surface. The modeling of functional surfaces, being introduced in this paper, adopts a modified B-spline that utilizes an approximating technique. The modified B-Spline is constructed with altered control vertices. It is applied to measure the surface of an impeller blade. This research builds an algorithm accepting inputs of measured points. Generating the cutter-paths for NC machining employs the model of the constructed surfaces. The machined surfaces that are generated in several cases are compared with each other in the aspect of machining accuracy.

Key Words : Reverse Engineering, CAD/CAM, B-spline, Modified B-spline, NURBS, Interpolation, Approximation

1. Introduction

There is a considerable progress in the technique of reverse engineering. It is an effective method in developing new products without a CAD model that is formed with sculptured surfaces.[1-10] Non-contact measuring instruments are used vigorously in the reverse engineering because of high speed and capability of handing numerous data in measurement.[6,7] But there is a drawback that an object with a caved-in or highly twisted shape cannot be measured because the light can not be transmitted into it. Therefore, the functional surface with a complex shape such as a blade, a propeller, an impeller, and a rotor cannot be applied in the reverse engineering utilizing non-contact measuring instruments. Thus, there have been insufficient researches in this area. To the contrary, contacting measuring instruments that have high accuracy can measure twisted shape using a rotational probe. This causes less measurement errors during the registration of each coordinate.

However, measuring accuracy is not assured in measuring highly twisted blades like an impeller especially when surfaces in narrow space between objects are measured by manual operation of a measuring probe with a non-automatic scanning process. The fairing of a curve is recommended because a curve constructed from a fitting method includes dimensional errors, which degrades the quality of a surface.

Nowacki[12,13] suggested a modeling method for a smooth curve which passes through given data. The method switches a curve-modeling problem into a nonlinear equation form with the fairing of interpolating condition, end condition, and integral condition. Goodman[14] suggested a modeling method of a rational third-order curve with the Geometric-2 continuity condition by torsion condition preserving the shape of the given data. Besides, to purify the modeled curve, there is research on the changing method for the shape of a NURBS curve. Farin[15], Sapidis[16] and Pigounakis[17] suggested the fairing method of a NURBS curve through controlling its knot vectors. Piegl[18], Au[19] and Rando[20] suggested the fairing method of a NURBS curve that alters

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its control vertices and weighting factors.

When the deviation of given data is relatively small, the curves generated from the above approaches which construct smooth curves pass through given data. But, these research cannot be directly applied to this paper because the deviation of given data is neither expectable nor reliable.

A least square method, approximating given data in second order, is a polynomial regression21. The method minimizes the summation of the square of the deviation between measured points and constructed points on the approximating curve. But a polynomial regression curve is in the form of an explicit equation, which is not suitable to fit a three-dimensional curve because of tending to overshoot when the order is increased. On the other hand, it can be applied with the least square method to fit a parametric curve. Piegl6 suggested a parametric curve fitting method through approximating points that are obtained from the application of the least square method.

An approximating method to generate a smooth curve with the fairing of the point data constructs a NURBS curve expressed by a parameter. This derives the solutions of non-linear optimization problems with some restrictions.19-20,22-24 An objective function of non-linear optimization is used in the scale of a fairing derived from the geometrically intrinsic elements such as curvature, radius of curvature, tangent vector, normal vector, etc. A limited equation uses a deviation of distance between given data and modified data.

But these methods are difficult to use since the boundary of a curve should be adjusted. Even though a fairing has been done, the shape of a curve should be maintained. Thus one has to choose a proper fairing method in order to keep designer's aim for the shape of a curve.

This paper suggests a method where it is not necessary to derive approximating points as in the methods above. Because we can construct an approximating curve through the correction of a B-spline algorithm, after altering measured points into the control points of a B-spline curve, unlike the existing method that constructs an approximating curve through the fairing of measured points.

The main objective of this research is to improve the quality in terms of the accuracy of modeling in the reverse engineering. This research obtains a proper approximating curve and measures points on a curve with a programmable measurement process.

2. Proper Process for a Section Curve

2.1 B-spline and B-spline Approximating Curve

Fig. 1 shows a B-spline curve that interpolates all the given points by passing them.

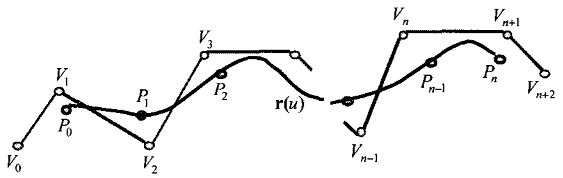


Fig. 1 Construction of a composite B-spline curve with control vertices

The segment of the curve is formulated in a matrix form as

$$\mathbf{r}^i(u) = \mathbf{U}\ \mathbf{N}\ \mathbf{R}^i \quad \text{for } i=0,1,2,\ldots,n-1$$
$$\text{where } \mathbf{U} = [1\ u\ u^2\ u^3\ ],$$
$$\mathbf{N} = \frac{1}{6}\begin{bmatrix} 1 & 4 & 1 & 0 \\ -3 & 0 & 3 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix}$$
$$\mathbf{R}^i = [\,V_i\ V_{i+1}\ V_{i+2}\ V_{i+3}\,]^T \tag{1}$$

We construct an approximating curve that connects all the measured points with smoothness. The measured points are unevenly scattered on the surface of a blade. This research suggests an approximating method for a B-spline curve, which uses measured points for control vertices instead of directly using original control vertices.

The composite B-spline curve that is derived from Eq. (1) needs control vertices transformed from the measured points and the tangents of the curve at both ends. The end tangents can be found by applying the free end condition. In order to approximate a B-spline curve, the measured points, $P^i = [\,P_i\ P_{i+1}\ P_{i+2}\ P_{i+3}\,]^T$ should be transformed to the matrix form of control vertices, $R^i = [\,V_i\ V_{i+1}\ V_{i+2}\ V_{i+3}\,]^T$.

2.2 Modified B-spline Approximating Curve

The B-spline approximating curve that uses the measured points for control vertices is shown in Fig. 2. The starting point is at 1/3 of the length of the line connecting P_1 and M_0 . The latter is the mid-point between P_0 and P_2.

The same as the starting point the curve ends at a point that is at 1/3 of the length of the line joining P_2 and M_1.

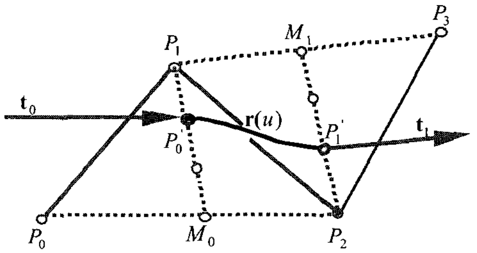


Fig. 2 Construction of a B-spline approximating curve with measured points

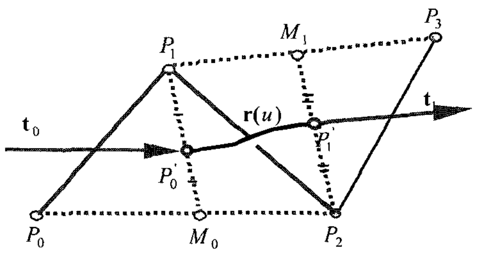


Fig. 3 Construction of a Modified B-spline approximating curve with measured points

Fig. 3 shows the suggested approximation of a B-spline and the curve passes the point that is a mid point on the line connecting P_1 and M_0. This can be a virtue for the purpose of the fairing of the curve. Boundary conditions of a Modified B-spline curve are as the followings.

a) $M_0=(P_0+P_2)/2$; $M_1=(P_1+P_3)/2$;
$P_0'=(P_1+M_0)/2$; $P_1'=(P_2+M_1)/2$

b) It starts from P_0' *and ends at* P_1'

c) Start tangent vector $\mathbf{t}_0$ *at* P_0' *is equal to* (M_0-P_0)

d) End tangent vector $\mathbf{t}_1$ *at* P_1' *is equal to* (M_1-P_1)

The boundary conditions of the Modified B-spline are expressed as in the Eq. (2) and rewritten in the matrix form as in the Eq. (3). On substituting the Eq. (3) into the Ferguson curve equation such as Eq. (4), we obtain the following expression for the Modified B-spline curve as in the Eq. (5).

$$
\begin{aligned}
P_0' &= \mathbf{r}(0) = [2P_1+(P_0+P_2)]/4 \\
P_1' &= \mathbf{r}(1) = [2P_2+(P_1+P_3)]/4 \\
\mathbf{t}_0 &= \dot{\mathbf{r}}(0) = (P_2-P_0)/2 \\
\mathbf{t}_1 &= \dot{\mathbf{r}}(1) = (P_3-P_1)/2
\end{aligned} \tag{2}
$$

$$
\mathbf{S} = \begin{bmatrix} P_0' \\ P_1' \\ \mathbf{t}_0 \\ \mathbf{t}_1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ -2 & 0 & 2 & 0 \\ 0 & -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix} = \mathbf{K_m R} \tag{3}
$$

$$
\begin{aligned}
\mathbf{r}(u) &= \mathbf{UA} = \mathbf{UCS} \quad with \ 0 \le u \le 1 \\
&= \begin{bmatrix} 1 & u & u^2 & u^3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -3 & 3 & -2 & -1 \\ 2 & -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_0' \\ P_1' \\ \mathbf{t}_0 \\ \mathbf{t}_1 \end{bmatrix}
\end{aligned} \tag{4}
$$

$$
\begin{aligned}
&\mathbf{r}(u) = \mathbf{UCS} = \mathbf{UC}(\mathbf{K_m R^i}) = \mathbf{U}(\mathbf{CK_m})\mathbf{R^i} = \mathbf{UN_m R^i} \\
&with \ 0 \le u \le 1 \qquad where \\
&\mathbf{U} = \begin{bmatrix} 1 & u & u^2 & u^3 \end{bmatrix} \\
&\mathbf{N_m} = \frac{1}{4} \begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 0 & 2 & 0 \\ 1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\
&\mathbf{R^i} = \begin{bmatrix} P_0 & P_1 & P_2 & P_3 \end{bmatrix}
\end{aligned} \tag{5}
$$

2.3 Verification of Modified B-spline Approximating Curve

This paper shows a verification of the Modified B-spline approximation with the points measured by manual operation. Three curves are shown and compared with each other in Fig. 4. They are 1) a B-spline interpolating curve, 2) a B-spline approximating curve which uses the measured points for control vertices, 3) a modified B-spline approximating curve. They are programmed with Visual C++ and OpenGL library. The figure shows that the Modified B-spline is the smoothest curve among them.

The maximum curvature of the Modified B-spline is the smallest as shown in Table 1. From the curvature analysis using the CATIA (Dassalt Systems, France), the Modified B-spline approximation generates the smoothest curve.

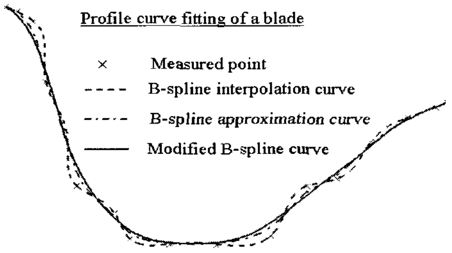


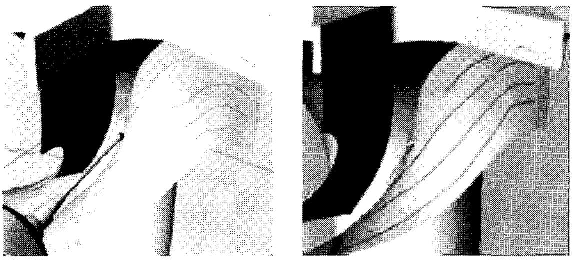
Fig. 4 Comparison of curves according to fitting methods

Table 1 Comparison of max. curvatures according to fitting methods

Division	B-spline interpolating curve	B-spline approximating curve	Modified B-spline curve
Maximum curvature	0.159	0.0951	0.0514

2.4 Programmable Measurement using Point Data of Modified B-spline Approximating Curve

The curve constructed with the Modified B-spline is an approximating curve that passes in between the measured points that are unevenly scattered on the surface. But it is inaccurate since the curve does not pass the measured points on the surface. Programmable measurement that uses the points approximated from the curve fitting is done to compensate for the errors. The section curve of a blade obtained from the programmable measurement that uses the points from the Modified B-spline curve is smoother as shown in Fig. 5(b) than the one from without approximation as shown in Fig. 5(a).



(a) Measured curves by manual measurement

(b) Measured curves by programmable measurement with proper process

Fig. 5 Effect of proper process

This curve is used for a section curve to get point data for the inputs to the construction algorithm of a NURBS surface.

3. Surface Construction and Verification of Proper Process

3.1 Surface Construction with NURBS

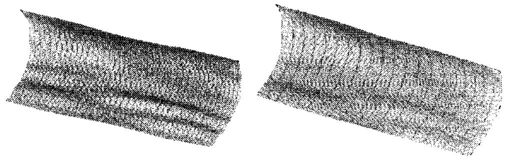
The point data obtained from the proper process previously described are used for fitting in surface modeling with a NURBS11 algorithm. The expanding knot spans of the end parts of constructed surface are set to all '0's and the end tangents are derived from the free end condition.

Fig. 6(a) and Fig. 7(a) are the results with a NURBS fitting algorithm without a proper process after manual measurement on the surfaces of the bucket blade and the impeller blade. Fig. 6(b) and Fig. 7(b) are the results with the proper process using both Visual C++ and OpenGL library.

3.2 Verification of Effectivity in Proper Process

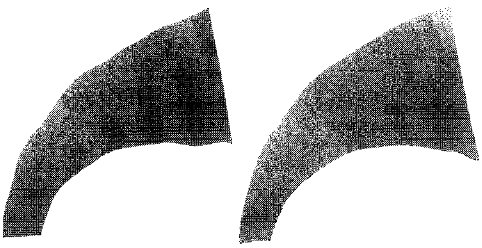
Figs. 6 and 7 show that the NURBS surface properly processed by the B-spline Approximating algorithm and the programmable measuring process proposed in this paper unfolds the wrinkles of the surface on the whole. They become smoother in the unevenly spaced section curve than the previous surface improperly processed.

Fig. 8 and Fig. 9 show error comparison between the standard model and the reversed models. The standard model is the surface that is constructed with the fitting criterion of 0.02 mm deviation against measured points by using the CATIA. Figs. 8(a) and 9(a) present the errors in between the standard surface and the improper surface. Figs. 8(b) and 9(b) show the errors between the standard surface and the improper surface. As the result, the properly processed surface of the bucket blade fits within the allowing tolerance of 0.1 mm such as thin lines represented less than 0.1mm. This is same to the impeller blade except the end parts of the surface. On the other hand, the most area of the improperly processed surfaces do not fit within the allowing tolerance of 0.1 mm such as thick lines represented more than 0.1mm. The maximum deviation computed with the CATIA against the standard surface is presented in Table 2. The maximum deviation of the proper surface is decreased as shown in the table.



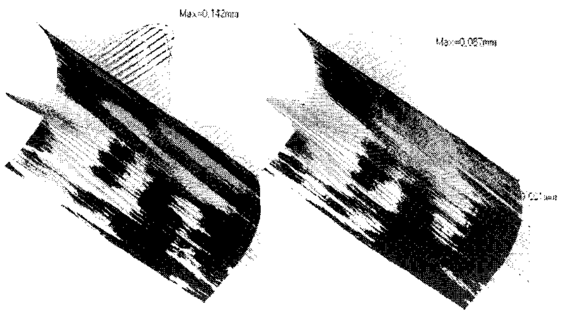
(a) Improper NURBS surface (b) Proper NURBS surface

Fig. 6 Comparison between improperly fitted surface and properly fitted surface with NURBS algorithm for the bucket blade



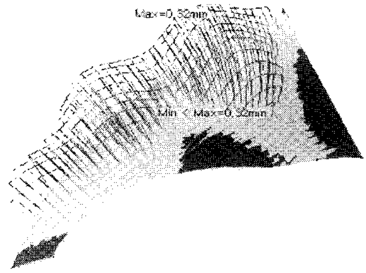
(a) Improper NURBS surface (b) Proper NURBS surface

Fig. 7 Comparison between the improperly fitted surface and properly fitted surface with NURBS algorithm for the impeller blade

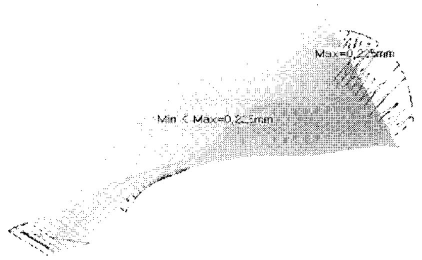


(a) Standard surface versus improper NURBS surface (b) Standard surface versus proper NURBS surface

Fig. 8 Comparison between the standard surface with CATIA and NURBS surfaces for the bucket blade



(a) Standard surface versus improper NURBS surface



(b) Standard surface versus proper NURBS surface

Fig. 9 Comparison between the standard surface with CATIA and NURBS surfaces for the impeller blade

Table 2 Comparison of max. deviation between the standard surface using CATIA and NURBS surfaces

Division	Bucket blade		Impeller blade	
	The standard surface versus		The standard surface versus	
	Improper NURBS surface	Proper NURBS surface	Improper NURBS surface	Proper NURBS surface
Maximum deviation	0.142(mm)	0.087(mm)	0.32(mm)	0.225(mm)

4. Solid Modeling and CNC Machining

4.1 Solid Modeling

The output data from the proposed algorithm in this paper use an IGES format to interface other types of software and use CC (Cutter Contact) data to generate tool path. A solid modeling is constructed by power fit command in the CATIA accepting point data. This solid model is input data in generating cutter-paths for CNC machining.

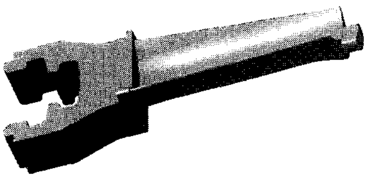


Fig. 10 Solid model of the bucket blade

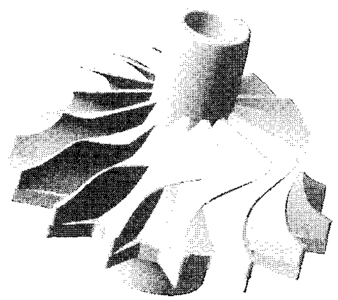
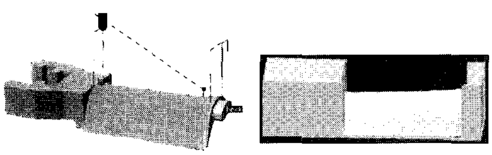


Fig. 11 Solid model of the impeller

4.2 CNC Machining

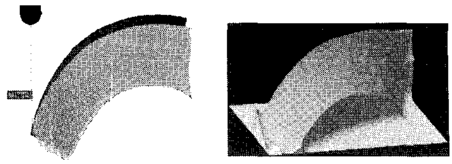
Surface machining is achieved after the generation of cutter paths and verifying the paths with the CATIA as shown in Figs. 12 and 13. On the purpose of comparing with the master model, two reversed models are manufactured. One is a properly reversed model and the other is an improperly reversed model.



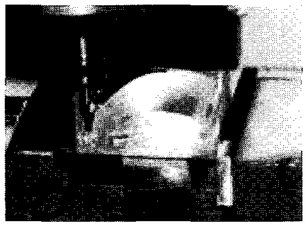
(a) Generation and verification of cutter paths

(b) CNC machining with a machining center

Fig. 12 CNC machining of the bucket blade



(a) Generation and verification of cutter paths



(b) CNC machining with a machining center

Fig. 13 CNC machining of the impeller blade

5. Comparative Measurement between Master Model and Reversed Models

On the purpose of identifying errors between the master model and the machined model, we measured them with a CMM as shown in Fig. 14. A program is prepared to measure points on the surface of the reversed models obtained from sampling points on the surface of the master model. The graph of measurement results is shown in Figs. 15 and 16. The deviation of the properly reversed model is smaller than that of the improperly reversed model and the average deviation of the properly reversed model is also in a same situation.

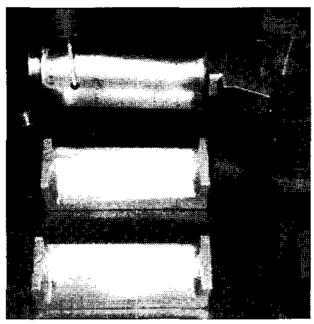


Fig. 14 Comparative measurement between master model and reversed models

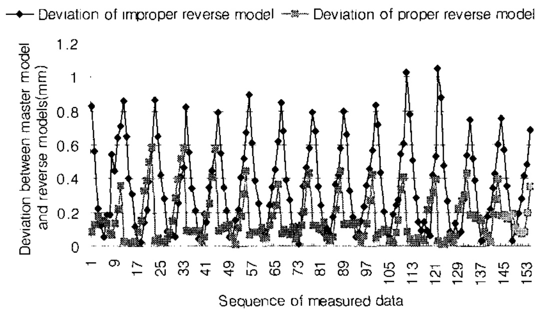


Fig. 15 The comparative graph between the master model and reversed models for the bucket blade

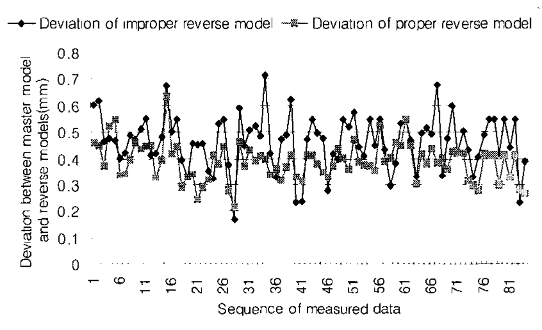


Fig. 16 The comparative graph between the master model and reversed models for the impeller blade

Table 3 Comparison of average deviation between master model and reversed models

Division	Bucket blade		Impeller blade	
	Master model versus		Master model versus	
	Improperly reversed model	Properly reversed model	Improperly reversed model	Properly reversed model
Average deviation	0.357(mm)	0.157(mm)	0.463(mm)	0.389(mm)

6. Conclusion

We draw the following conclusions from this research.

1. In developing the Modified B-spline approximation algorithm to minimize the errors of functional surfaces, we have proper section curves utilizing the programmable measurement that uses the measured points.
2. The deviation of the properly reversed model from the master model is smaller than that of the improperly reversed model. This is identified from the surface fitting program coded in this research with Visual C++ program and OpenGL library along with the NURBS algorithm.
3. The deviation of the NURBS surface from the properly modified algorithm is smaller than that of the improperly reversed model. This is known through comparative measurement of machined surfaces that are achieved from the CNC machining. The cutter paths for the CNC machining are generated from the solid models constructed with the CATIA.
4. The curve constructed by the Modified B-spline algorithm proposed in this paper has an advantage that the algorithm is an easier fitting method to have a smoother curve than other previously introduced fairing methods. This can reduce the deviation caused from the designers' subjectivity for modifying the measured data. When the surface constructed by the manual measurement is continuously convex or concave, the surface is possible to skew to one side. This should be studied for correction in the further research.

Acknowledgement

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