韓國經營科學會誌
第30권 第1號
2005年 3月

(Max, +)-대수를 이용한 2-노드 유한 버퍼 일렬대기행렬에서의 대기시간 분석*

서 동 원**

Application of (Max, +)-algebra to the Waiting Times in Deterministic 2-node Tandem Queues with Blocking*

Dong-Won Seo**

Abstract

In this study, we consider characteristics of stationary waiting times in single-server 2-node tandem queues with a finite buffer, a Poisson arrival process and deterministic service times. The system has two buffers: one at the first node is infinite and the other one at the second node is finite. We show that the sojourn time or departure process does not depend on the capacity of the finite buffer and on the order of nodes (service times), which are the same as the previous results. Furthermore, the explicit expressions of waiting times at the first node are given as a function of the capacity of the finite buffer and we are able to disclose a relationship of waiting times between under communication blocking and under manufacturing blocking. Some numerical examples are also given.

Keyword : Tandem Queues, Finite Buffer, Waiting Times, (Max, +)-Linear Systems

1. Introduction

Stochastic networks with finite buffers have been widely studied. In particular, as common models of communication and manufacturing systems, tandem queues with finite buffers have been investigated. Many researchers have interests in characteristics in stochastic net-

논문접수일 : 2004년 7월 5일 논문게재확정일 : 2005년 1월 26일
* 본 연구는 한국과학재단 목적기초연구(R01-2004-000-10948-0) 지원으로 수행되었음.
** 경희대학교 국제경영학부

works such as mean waiting times, sojourn times, departure processes, steady-state probabilities, blocking probabilities and cycle times and so on. Since the computational complexity and difficulty in the analysis of stochastic networks, most studies are focused on very restrictive and small size of tandem queues over the past decades such as the systems with constant or exponential or phase-type service times and only two or three nodes. Much of literature on tandem queues with finite buffers has proposed several approximation methods (see for example Brandwajn and Jow [9] and references therein). Lee and Zipkin [14] studied on approximation of performances in a make-to-order system which corresponds to tandem queues with Poisson demand process and exponential service times.

In Poisson driven 2-node tandem queues with exponential service times Grassmann and Drekic [10] determined the joint distribution of both lines in equilibrium by using generalized eigenvalues. Onvural and Perros [16] studies equivalencies on joint queue length distribution in queueing networks with exponentially distributed service times and interarrival time under three types of configurations : tandem, split and merge. By comparing the state space of systems they showed that for a 2-node tandem queues with exponential service times all types are equivalent to each other if the first queue is infinite. For nonoverlapping service times, Whitt [19] (see also references therein) studied the optimal order of nodes, which minimizes the expected sojourn times in tandem queues with infinite buffers and an arbitrary arrival process. Nakade [15] derived bounds for expected cycle times in tandem queues with general service times under communication and manufacturing blocking. He gave upper bounds from the synchronous systems and lower bounds by solving the simultaneous equations using distribution functions of processing times.

Wan and Wolff [18] showed that the departure processes in finite capacity tandem queues with an infinite buffer at the first node and nonoverlapping service times with respect to tasks are independent of the size of finite buffers when it is greater than 2 under communication blocking or when it is greater than 1 under manufacturing blocking. They have assumed that the capacities of buffers include the space for a customer in service and that arrival process is arbitrary. Labetoulle and Pujolle [13] gave the same results for the mean response time in deterministic finite tandem queues, and derived the mean waiting times of each node in tandem queues with infinite buffers. In our best knowledge, however, there is no result on the waiting times in the subarea of tandem queueing networks with finite buffers.

Recently, more generous stochastic queueing networks which are called (max, +)-linear systems are studied. (Max, +)-linear systems cover various types of queueing networks which are prevalent in telecommunication, transportation, manufacturing and production systems. Various instances of (max, +)-linear systems can be represented by stochastic event graphs, a special type of stochastic Petri net. Petri nets allow one to analyze and model (max, +)-linear systems which are nonconcurrent (choice-free) and nonovertaking nets and consist of single server queues under FIFO service discipline. Complex discrete event systems (DESs) can be properly modeled by this method involving only 'max' and '+' operations, but unfortunately it is very hard to obtain closed form expressions for performance measures of these complex systems except for some restricted models.

Baccelli and Schmidt [8] derived a Taylor series expansion for the expected value of stationary waiting times with respect to the arrival rate in Poisson driven (max, +)-linear systems. This expansion approach was generalized to other characteristics (such as higher order moments, Laplace transform, tail probability) of stationary waiting times and transient waiting times by Baccelli, Hasenfuss and Schmidt [6, 7], Hasenfuss [12], Ayhan and Seo [2-4] and Seo [17] and to joint characteristics of stationary waiting times by Ayhan and Baccelli [1]. In [2-4, 17] they derived the explicit expressions on characteristics for stationary and transient of waiting times in (max, +)-linear systems with a Poisson input process.

As an application of these results, we are able to investigate certain properties of deterministic tandem queueing networks with a Poisson arrival process and blocking. The object of this research is to disclose the effect of finite buffers to stationary waiting times in all areas of the systems under various kinds of blocking. In addition, a relationship on stationary waiting times in the deterministic systems between with communication blocking and with manufacturing blocking is also studied. We focus on 2-node deterministic tandem queues with a Poisson arrival process and a finite buffer in this study. The analysis on the deterministic tandem queues with more than 2 nodes will be studied later on.

The paper is organized as follows. In Section 2 some preliminaries on waiting times in (max, +)-linear systems are given. Section 3 contains our main results and Section 4 shows examples to illustrate this result. Conclusion and some future research topics are mentioned in Sections 5.

2. Waiting Times in (Max, +)-Linear Systems

Under the notion of an open (max, +)-linear stochastic system, one understands a sequence $\{X_n\}$ of random vectors satisfying α-dimensional vectorial recurrence equations

$$X_{n+1} = A_n \otimes X_n \oplus B_{n+1} \otimes T_{n+1} \quad (2.1)$$

with an initial condition X_0 where

- $\{T_n\}$ is a nondecreasing sequence of real-valued random numbers (the epochs of the Poisson arrival process),
- $\{A_n\}$ is a stationary and ergodic sequence of $\alpha \times \alpha$ matrices with real-valued random entries,
- $\{B_n\}$ is a stationary and ergodic sequence of $\alpha \times 1$ matrices with real-valued random entries,
- $\{X_n\}$ is a sequence of α-dimensional state vectors.

Here, the addition $\oplus$(o-plus) means coordinatewise maximization and the multiplication $\otimes$ (o-times) means addition for scalars and (max, +) algebra product for matrices (see Baccelli et al. [5]). Such systems allow one to represent the dynamics of stochastic Petri nets belonging to the class of event graphs ([5, 8]). In most cases, and particularly for systems with α greater than 2, it is very difficult to determine the characteristics of the random vector $W=(W^1, W^2, \cdots, W^\alpha)$ in closed form.

In particular, this class contains various instances of queueing networks like acyclic and cyclic fork-join queueing networks, finite or infinite capacity tandem queueing networks with various kinds of blocking rules (manufacturing and communication), synchronized queueing networks etc. It also contains some basic manufacturing systems like Kanban networks, assembly systems and so forth.

In all these models, T_n is the arrival epoch of the n-th customer to the network and the coordinates X_n^i of the state vector $\{X_n\} = (X_n^1, X_n^2, \cdots, X_n^\alpha)$ represent absolute times (like beginning of the n-th service in the i-th queue) which grow to ∞ when n increases unboundedly. For this reason one is actually more interested in the differences $W_n^i = X_n^i - T_n$ (like the waiting time of the n-th customer until the beginning of his service in queue i), which are expected to admit a certain stationary state $W^i = \lim_{n\to\infty} W_n^i$ (in distribution) under certain rate conditions. Let $\tau_n = T_{n+1} - T_n$ (the interarrival time) with $T_0 = 0$ and let $C(x)$ be the $\alpha\times\alpha$ matrix with all diagonal entries equal to $-x$ and all nondiagonal entries equal to $-\infty$. By subtracting T_{n+1} from both sides of (2.1), the new state vector W_{n+1} can be written as

$$W_{n+1} = A_n \otimes C(\tau_n) \otimes W_n \oplus B_{n+1}$$

for $n \geq 0$ and with some initial condition W_0.

Under certain conditions, it is shown in [5] that for all $\lambda < a^{-1}$ where a is the maximal (max, +) Lyapunov exponent of the sequence $\{A_n\}$, W is unique and determined by the matrix-series

$$W = D_0 \oplus \bigoplus_{k\geq 1} C(T_{-k}) \otimes D_k \tag{2.2}$$

with $D_0 = B_0$ and

$$D_k = \left(\bigotimes_{n=1}^{k} A_{-n} \right) \otimes B_{-k} \tag{2.3}$$

for all $k\geq 1$. For a general (max, +)-linear stochastic system the i-th component of the random vector D_k can be interpreted as the longest path from the initial node to node i in the corresponding task graph. Note that the components of D_k can be written in terms of service times.

We assume that each entry of the random matrix A_n is either almost surely nonnegative or equal to $-\infty$, and that all entries on the diagonal of A_n are nonnegative. We also assume that there is an integer $0 \leq \alpha' \leq \alpha$ such that the first α' coordinates of the α-dimensional random vectors B_n are nonnegative, i.e. $B_n^i \geq 0$ for all $1 \leq i \leq \alpha'$. Let D_n be defined as in (2.3) with $D_0 = B_0$. First α' coordinates of D_n are assumed to be nondecreasing in n i.e.

$$0 \leq D_0^i \leq D_1^i \leq \cdots \text{ for all } i = 1, 2, \cdots, \alpha'$$

Under the assumption that $\{T_n\}$ is a stationary Poisson process with intensity λ and that the sequences $\{A_n\}$ and $\{B_n\}$ have certain independence properties, Baccelli, Hasenfuss and Schmidt [6, 12] derived a Taylor series expansion for the expected value of functional of stationary waiting times, $E[G(W^i)]$ for $i=1,\cdots,a'$. For an integrable, bounded and nonnegative function $G(\cdot)$, if $E[G(W^i)]$ is $(m+1)$-times differentiable in the arrival intensity $\lambda\in[0,a^{-1})$, then

$$E[G(W^i)]=\sum_{k=0}^{m}\lambda^k E[q_{k+1}(D_0^i,D_1^i,\cdots,D_k^i)] + O(\lambda^{m+1}) \qquad (2.4)$$

for all $k=0,1,\cdots,m$. Here the polynomials $q_k(\cdots)$ is defined as

$$\begin{aligned} q_{k+1}(x_0,x_1,\cdots,x_k) &= \sum_{n=0}^{k}\binom{k}{n}(-1)^{k-n}H^{[k]}(x_n) \\ &- \sum_{n=0}^{k-1}\sum_{j=n}^{k-1}\binom{j}{n}(-1)^{j-n}H^{[j]}(x_n) \\ &\{p_{k-j}(x_{n+1},\cdots,x_{k-j+n}) \\ &- p_{k-j}(x_n,\cdots,x_{k-j+n-1})\}, \end{aligned}$$

with $H^{[0]}(x)=G(x)$ and $H^{[n]}(x)$ is recursively defined by a suitably chosen version of the indefinite Reimann-integral $\int H^{[n-1]}(x)\,dx$. And the polynomial $p_k(\cdots)$ is defined as

$$p_k(x_0,\cdots,x_{k-1})=\sum_{(i_0,i_1,\cdots,i_{k-1})\in N_k}(-1)^{\gamma_k(i_0,i_1,\cdots,i_{k-1})}\frac{x_0^{i_0}}{i_0!}\frac{x_1^{i_1}}{i_1!}\cdots\frac{x_{k-1}^{i_{k-1}}}{i_{k-1}!}$$

where

$$\begin{aligned} N_k=\{(i_0,i_1,\cdots,i_{k-1})\in\{0,1,\cdots\}^k: \\ i_0+i_1+\cdots+i_{k-1}=k, \\ \text{if } i_s=l>1, i_{s-1 \bmod k}=\cdots=i_{s-l+1 \bmod k}=0\} \end{aligned}$$

and

$$\gamma_k(i_0,i_1,\cdots,i_{k-1})=1+\sum_{n=0}^{k-1}I(i_n>0)$$

for all $k\geq 1$, with $I(x)=1$ whenever x is true and $I(x)=0$ otherwise.

For instance when $G(x)=x^v$ for $v\in N$ their expression (2.4) leads to expansions for moments of W^i, for $i=1,\cdots,a'$. In particular, $v=1$ gives the expansion result for the first moment of W which was first derived by Baccelli and Schmidt [8]. In [12] they showed that q_k is the same expression as the polynomial p_k when $G(x)=x$.

From now on, we consider a class of deterministic (max, +)-linear systems where the net topology such that for each $i\in\{1,\cdots,a'\}$ the elements of the sequence $\{D_m\}$ is given by

$$D_m^i=\begin{cases}\eta_m^i & for\ m=0,\cdots,\xi_i-1\\ \eta_{\xi_i}^i+(m-\xi_i)a_i & for\ m\geq\xi_i\end{cases} \qquad (2.5)$$

for deterministic real numbers $0\leq\eta_0^i\leq\eta_1^i\leq\cdots\leq\eta_{\xi_i}^i$ and a_i and some nonnegative integer ξ_i. The Lyapunov exponent of the entire system is hence given by $a=\max_{i=1,\cdots,a'}\{a_i\}$. Even though not all deterministic (max,+)-linear systems fall into this category, this class covers many queueing systems with deterministic service times such as tandem queues with various types of blocking, fork-and-join type queueing networks, queuing networks operating under Kanban, CONWIP control strategies etc. Ayhan and Seo [2] (see also [17]) derived the following closed form expression on the stationary waiting times in a class of (max, +)-linear systems with deterministic service times.

Theorem 1 : Let $\{T_n\}$ be a stationary Poisson process with rate λ such that $\lambda\in[0,a_i^{-1})$. Suppose that $i\in\{1,\cdots,a'\}$ and D_m^i has the structure given in (2.5). Then for $\xi_i\geq 1$

$$E[(W^i)^r]=\sum_{j=0}^{r}\binom{r}{j}\sum_{k=0}^{j}\frac{(-1)^k(\eta^i_{\xi_i})^{j-k}k!}{\lambda^k}\binom{j}{k}\binom{\xi_i-1+k}{k}E[(W)^{r-j}]$$
$$+(1-\rho_i)\sum_{m=0}^{\xi_i-1}e^{-\lambda(\eta^i_{\xi}+(m-\xi_i)a_i-\eta^i_m)}\sum_{k=0}^{r-1}\frac{(-1)^k rk!}{\lambda^{k+1}}\binom{r-1}{k}\binom{m+k}{k}(\eta^i_m)^{r-k-1}$$
$$+(1-\rho_i)\sum_{m=0}^{\xi_i-2}e^{-\lambda(\eta^i_{\xi}+(m-\xi_i)a_i-\eta^i_m)}\sum_{k=0}^{r-1}\frac{(-1)^k rk!}{\lambda^{k+1}}\binom{r-1}{k}\binom{m+k}{k}(\eta^i_m)^{r-k-1}$$
$$\times\sum_{l=1}^{\xi_i-1-m}e^{-\rho_i l}\{p_l(\lambda(\eta^i_{\xi_i}+(m+l-\xi_i)a_i),\lambda\eta^i_{m+1},\cdots,\lambda\eta^i_{m+l-1})-p_l(\lambda\eta^i_{m+1},\cdots,\lambda\eta^i_{m+l})\}$$

with the convention that summation over an empty set is 0 and for $\xi_i = 0$

$$E[(W^i)^r]=\sum_{j=0}^{r}\binom{r}{j}(\eta^i_0)^j E[(W)^{r-j}]$$

where $\rho_i=\lambda a_i$ and W is the stationary waiting time in an M/D/1 queue with service time equal to a_i and arrival rate λ.

The moments of stationary waiting times in an M/D/1 queue can be computed easily using the recursive formula of Takacs. It is well known that (see for example Gross and Harris [11]) for $r\geq 1$,

$$E[W^r]=\frac{\lambda}{1-\rho_i}\sum_{j=1}^{r}\binom{r}{j}E[W^{r-j}]\frac{(a_i)^{j+1}}{j+1}.$$

3. Waiting Times in 2-node Tandem Queues with Deterministic Service

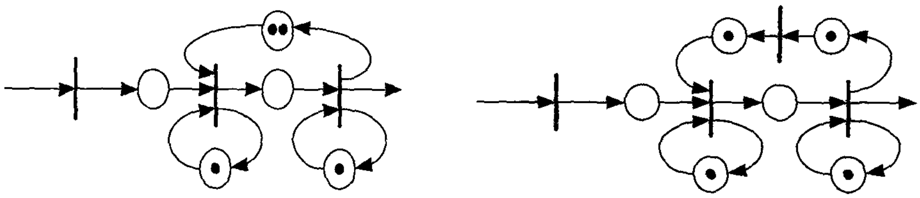
In this study, we investigate on waiting times in single-server 2-node tandem queues with a finite buffer, a Poisson arrival process and deterministic service times. The system has two buffers : one at the first node is infinite and another one at the second node is finite.

Let σ^i and K_i be the deterministic service time and the size of buffer at node i ($i=1,2$). The buffer size includes a room for a customer in service. We first mention about the waiting times in 2-node tandem queues with infinite buffers ($K_1=K_2=\infty$). From the definition of random vector D_n, one can obtain the expressions on the components of D_n as

$$D^1_n=n\sigma^1 \quad \text{for } n\geq 0,$$
$$D^2_n=\sigma^1+n\max\{\sigma^1,\sigma^2\} \quad \text{for } n\geq 0.$$

As we mentioned earlier, the components of D_n can be written in terms of service times and depend on the structure of stochastic networks and service times. Therefore, if the components of D_n for all $n\geq 0$ in one system are the same as those in another, then we can see that two systems are equivalent.

For 2-node tandem queues with a finite buffer, we consider waiting times under two blocking policies : communication blocking and manufacturing blocking. Under communication blocking a customer at node j cannot begin his service unless there is a vacant space in the buffer at node $j+1$. For manufacturing blocking, a customer served at node j moves to node $j+1$ only if the buffer of node $j+1$ is not full ; otherwise the blocked customer stays in node j until a vacancy is available. During that time, node j is blocked from serving other customers.



[Figure 1] 2-node tandem queues with a finite buffer of size 2 and infinite buffers

To derive recursive equations in (max, +)-linear systems with finite buffers one depicts a corresponding event graph and then convert it to an event graph with infinite buffers by inserting dummy nodes with zero service times. For example, 2-node tandem queues with $K_1 = \infty, K_2 = 2$ depicted in [Figure 1] (left) can be converted to the event graph with infinite buffers depicted in [Figure 1] (right) by inserting a dummy node with zero service time.

We first derive expressions of the random vector D_n under communication blocking. The finite buffer systems with communication blocking we consider here have infinite buffer at the first node and finite buffer at the second node, i.e. $K_1 = \infty, K_2 < \infty$. Similarly as done in the infinite buffer case, one is able to obtain the expressions for the components of the random vector D_n as follows :

if $K_2 = 1$,

$$D_n^1 = n(\sigma^1 + \sigma^2) \quad \text{for} \quad n \geq 0,$$

$$D_n^2 = \sigma^1 + n(\sigma^1 + \sigma^2) \quad \text{for} \quad n \geq 0,$$

if $K_2 = 2$,

$$D_0^1 = 0$$

$$D_n^1 = \sigma^1 + (n-1)\max\{\sigma^1, \sigma^2\} \quad \text{for} \quad n \geq 1,$$

$$D_n^2 = \sigma^1 + n \max\{\sigma^1, \sigma^2\} \quad \text{for} \quad n \geq 0,$$

if $K_2 \geq 3$,

$$D_n^1 = n\sigma^1 \quad \text{for} \quad n = 0, 1, \cdots, K_2 - 1, \tag{3.1}$$

$$D_n^1 = \sigma^1 + \max\{(n-1)\sigma^1, (n - K_2 + 1)\sigma^2\} \quad \text{for} \quad n \geq K_2, \tag{3.2}$$

$$D_n^2 = \sigma^1 + n \max\{\sigma^1, \sigma^2\} \quad \text{for} \quad n \geq 0.$$

From the above expressions, we know that the expression of D_n^2 for all $n \geq 0$ and $K_2 \geq 2$ is the same as that of D_n^2 for all $n \geq 0$ and $K_2 = \infty$. It shows the same result in [17] that when $K_2 \geq 2$, the sojourn time (here $W^2 + \sigma^2$) or departure process in a tandem queue with deterministic or nonoverlapping service times under communication blocking are the same as that in the system with infinite buffers. In other words, a customer's sojourn time is not dependent of the size of buffer at node 2. Furthermore, the order of nodes (service times) does not affect the sojourn time of a customer (see for example Whitt [19]).

The analysis on waiting times at the first node W^1 (the time interval from the arrival until the beginning of the service at node 1) is much difficult. One can obtain the following Lemma.

Lemma : In a deterministic two-node tandem queue with buffers of capacities $K_1 = \infty$ and $3 \leq K_2 < \infty$, D_n^1 has the following structure. If $\sigma^1 \geq \sigma^2$, then

$$D_n^1 = n\sigma^1 \quad \text{for} \quad n \geq 0$$

or if $\sigma^1 < \sigma^2$, then

$$D_n^1 = n\sigma^1 \qquad for\ n=0,1,\cdots,\xi-1$$
$$D_n^1 = \sigma^1 + (n-K_2+1)\sigma^2 \quad for\ n=\xi$$
$$D_n^1 = D_\xi^1 + (n-\xi)\sigma^2 \qquad for\ n\geq\xi+1 \qquad (3.3)$$

where an integer $\xi = \lceil \kappa \rceil$, $\lceil x \rceil$ is the smallest integer greater than or equal to x and a real number $\kappa = 1+(K_2-2)\frac{\sigma^2}{\sigma^2-\sigma^1}$.

Proof In order to derive the expression of D_n^1, one considers two exclusive cases on deterministic service times. When $\sigma^1 \geq \sigma^2$ (no blocking occurs at the first node), one can see easily that since $(n-1)\sigma^2$ is always bigger than $(n-K_2+1)\sigma^2$ in (3.2), $D_n^1 = n\sigma^1$ for all $n \geq 0$ and $\xi=0$ in (2.5).

When $\sigma^1 < \sigma^2$ (blocking occurs at the first node), then one can find an inequality $n \geq 1+(K_2-2)\frac{\sigma^2}{\sigma^2-\sigma^1}$ for $n \geq K_2$ such that $(n-1)\sigma^1 \leq (n-K_2+1)\sigma^2$. So, one can obtain a unique value $\kappa = 1+(K_2-2)\frac{\sigma^2}{\sigma^2-\sigma^1}$. Letting $\xi = \lceil \kappa \rceil$ shows the expressions of D_n^1 given in (3.3). Also, we know $\kappa \geq K_2$ because $\kappa = 1+(K_2-2)\frac{\sigma^2}{\sigma^2-\sigma^1} > 1+(K_2-2) = K_2-1$.

Now we are able to use Theorem 1 introduced in the previous section to compute moments of waiting times in a Poisson driven 2-node tandem queues with a finite buffer even though the simple explicit form expressions of stationary waiting times at the first node are not available. Note that if $\kappa = \infty$ (as $K_2 \to \infty$, $\kappa \to \infty$, i.e. a finite buffer system becomes an infinite buffer one), then $\xi=0$ in (2.5). When $\xi=0$, it becomes a simple M/D/1 queue with arrival rate λ and constant service time σ^1.

As done in communication blocking, for manufacturing blocking one can obtain the expressions on the components of the random vector D_n as follows :

if $K_2=1$,

$$D_0^1 = 0,$$
$$D_n^1 = \sigma^1 + (n-1)\max\{\sigma^1,\sigma^2\} \quad for\ n\geq 1,$$
$$D_n^2 = \sigma^1 + n\max\{\sigma^1,\sigma^2\} \qquad for\ n\geq 0$$

if $K_2 \geq 2$,

$$D_n^1 = n\sigma^1, \qquad for\ n=0,1,\cdots,K_2$$
$$D_n^1 = \sigma^1 + \max\{(n-1)\sigma^1,(n-K_2)\sigma^2\} \qquad for\ n\geq K_2+1$$
$$D_n^2 = \sigma^1 + n\max\{\sigma^1,\sigma^2\} \quad for\ n\geq 0$$

For systems with manufacturing blocking we can see the same results as the system with communication blocking, except for one difference in the value of the finite capacity buffer at node 2. That is, one can obtain the same expressions by substituting K_2 in (3.1) and (3.2) for K_2+1. It means that waiting times at the first node in deterministic 2-node tandem queues under manufacturing blocking with K_2 buffer at the second node are the same as those in the system under communication blocking with K_2+1 buffer. This also says that when two systems have equal size of buffer capacity the stationary waiting times in all areas under manufacturing blocking are always smaller than or equal to those under communication blocking. Therefore, we can immediately conclude the following Proposition since all components of random vector D_n are nonincreasing in K_2, and thus W^i is also stochastically nonincreasing in K_2 (see (2.2)).

Proposition : In a Poisson driven deterministic 2-node tandem queue with D_m^i satisfying the structure given in (2.5), when $K_1=\infty$ and $K_2 \geq 1$, then for $i=1,2$

$$E[G(W^i_{Communication\ Blocking\ with\ K_2+1})] = E[G(W^i_{Manufacturing\ Blocking\ with\ K_2})],$$

and

$$E[G(W^i_{Communication\ Blocking\ with\ K_2})] \geq E[G(W^i_{Manufacturing\ Blocking\ with\ K_2})]$$

where $G(\cdot)$ is the same as is defined above in (2.4).

4. Examples

To illustrate the results we consider a deterministic 2-node tandem queue with a finite buffer. The service times at each node are given as $\sigma^1=1$, $\sigma^2=4$. In this particular example, the Lyapunov maximum value a is 4. Using the explicit expressions of the random vector D_n^i together with Theorem 1 we are able to compute the exact value of mean waiting times. <Table 1> shows exact and simulation values for the expected waiting time (time interval between the arrival and just before the beginning of service) at node 1 under communication blocking with $K_2=5$. In this case, the values are given as $\kappa=\xi=5$.

Under manufacturing blocking policy the same system has $\kappa=6.3333$ and $\xi=7$. For various traffic intensities the values of mean stationary waiting times at the first node are shown in <Table 2>.

From the numerical results, we can see that our expressions of D_n^i as a function of buffer capacity are accurate and that the mean values of waiting times under manufacturing blocking are smaller than or equal to those under communication blocking. Besides, we can ob-

<Table 1> Waiting Times under Communication Blocking with $K_1=\infty, K_2=5$

Arrival Intensity λ	$E(W^1_{Communication\ Blocking\ with\ 5})$	Simulation
0.025($\rho=0.1$)	0.01282	0.01247 ∓0.00147
0.05($\rho=0.2$)	0.02634	0.02537 ∓0.00156
0.125($\rho=0.5$)	0.08851	0.08624 ∓0.00641
0.20($\rho=0.8$)	1.68781	1.6658 ∓0.15152
0.225($\rho=0.9$)	8.35323	8.1551 ∓0.69208

<Table 2> Waiting Times under Manufacturing Blocking with $K_1=\infty, K_2=5$

Arrival Intensity λ	$E(W^1_{Manufacturing\ Blocking\ with\ 5})$	Simulation
0.025($\rho=0.1$)	0.01282	0.01247 ∓0.00147
0.05($\rho=0.2$)	0.02632	0.02535 ∓0.00154
0.125($\rho=0.5$)	0.07629	0.07473 ∓0.00344
0.20($\rho=0.8$)	1.14075	1.1470 ∓0.13059
0.225($\rho=0.9$)	6.81748	6.6547 ∓0.64847

tain the exactly same values of waiting times at node 1 for one difference buffer capacity under manufacturing and communication blocking. In particular, we are able to obtain the exactly same values of mean stationary waiting times for all various traffic intensities, i.e. $E(W^{i}_{Communication\ Blocking\ with\ 6}) = E(W^{i}_{Manufacturing\ Blocking\ with\ 5})$ for $i=1,2$.

5. Conclusion

In this paper, we studied stationary waiting times in Poisson driven 2-node tandem queues with deterministic service times. There are two nodes. One at the first node is infinite, and the other at the second node is finite. Recursive expressions for waiting times in the stochastic system with a finite buffer under communication or manufacturing blocking can be obtained in (max, +)-algebra notation. From these explicit expressions we can show the following fact, which are the same results as the previous studies. When the capacity of buffer at the first node is infinite, the system sojourn time is independent of the capacity of buffer at the second node and does not depend on the order of nodes (service times). Moreover, we are able to disclose a relationship on stationary waiting times in the system with a finite buffer between under communication blocking and under manufacturing blocking.

This simple version of result can be extended to more complex (max, +)-linear systems with finite buffers such as m-node tandem queues, fork-and-join type queues and so on. Even though it is much difficult to derive an explicit expression on waiting times in all areas of the deterministic system with $K_1=\infty$, $K_j<\infty$ $(j=2,\cdots,m)$, one may be able to find certain common patterns of the expressions of the random vector D^{i}_{n} under various types of blocking, which allow one to compute characteristics of stationary waiting times in stochastic systems.

References

[1] Ayhan, H. and F. Baccelli, "Expansions for Joint Laplace Transform of Stationary Waiting Times in (Max, +)-Linear Systems with Poisson Input," Queueing Systems, Vol.37, No.3(2001), pp.291-328

[2] Ayhanm, H. and D.W. Seo, "Laplace Transform and Moments of Waiting Times in Poisson Driven (Max,+)-Linear Systems," *Queueing Systems*, Vol.37, No.4(2001), pp. 405-438.

[3] Ayhan H. and D.W. Seo, "Characteristics of Transient and Stationary Waiting Times in Poisson Driven (Max, +) Linear Systems," *Proceedings of the IFAC Symposium on System Structure and Control*, (2001), pp. 227—234.

[4] Ayhan H. and D.W. Seo, "Tail Probability of Transient and Stationary Waiting Times in (Max,+)-Linear Systems," *IEEE Transactions on Automatic Control*, Vol.47, No.1 (2002), pp.151-157.

[5] Baccelli, F., G. Cohen, G.J. Olsder, and J-P. Quadrat, *Synchronization and Linearity : An Algebra for Discrete Event Systems*, John Wiley & Sons, 1992.

[6] Baccelli, F., S. Hasenfuss and V. Schmidt, "Transient and Stationary Waiting Times in (Max, +) Linear Systems with Poisson Input," *Queueing Systems*, Vol.26(1997),

pp.301-342.

[7] Baccelli, F., S. Hasenfuss and V. Schmidt, "Expansions for Steady State Characteristics in (Max, +) Linear Systems," *Stochastic Models*, Vol.14(1998), pp.1-24.

[8] Baccelli, F. and V. Schmidt, "Taylor Series Expansions for Poisson Driven (Max,+) Linear Systems," *Annals of Applied Probability*, Vol.6, No.1(1996), pp.138-185.

[9] Brandwajn, A. and Y.-L.L. Jow, "An Approximation method for Tandem Queues with Blocking," *Operations Research*, Vol. 36, No.1(1988), pp.73-83.

[10] Grassmann, W.K. and S. Drekic, "An Analytical Solution for a Tandem Queue with Blocking," *Queueing Systems*, Vol.36(2000), pp.221-235.

[11] Gross, D. and C.M. Harris, *Fundamentals of Queueing Theory*. John Wiley & Sons, New York, 2nd Edition, 1985.

[12] Hasenfuss, S., *Performance Analysis of (Max,+)-Linear Systems via Taylor Series Expansions*, PhD thesis, University of Ulm, 1998.

[13] Labetoulle, J. and G. Pujolle, "A Study of Queueing Networks with Deterministic Service and Application to Computer Networks," *Acta Informatica*, Vol.7(1976), pp. 183-195.

[14] Lee, Y.-J. and P. Zipkin, "Tandem Queues with Planned Inventories," *Operations Research*, Vol.40, No.5(1992), pp.936-947.

[15] Nakade, K., "New Bounds for Expected Cycle Times in Tandem Queues with Blocking," *European Journal of Operations Research*, Vol.125(2000), pp.84-92.

[16] Onvural, R.O. and H.G. Perros, "On Equivalencies of Blocking Mechanisms in Queueing Networks with Blocking," *Operations Research Letters*, Vol.5, No.6 (1986), pp.293-297.

[17] Seo, D.-W., Performance Analysis of Queueing Networks via Taylor Series Expansions, PhD thesis, Georgia Institute of Technology, 2002.

[18] Wan, Y.-W. and R.W. Wolff, "Bounds for Different Arrangements of Tandem Queues with Nonoverlapping Service Times," *Management Science*, Vol.39, No.9(1993), pp.1173-1178.

[19] Whitt, W., "The Best Order for Queues in Series," *Management Science*, Vol.31, No. 4(1985), pp.475-487.