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속성 가중치에 대한 서수 정보가 주어질 때 다요소 의사결정 방법의 비교분석에 관한 연구*

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Comparative Analysis of Multiattribute Decision Aids with Ordinal Preferences on Attribute Weights*

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Abstract

In a situation that ordinal preferences on multiattribute weights are captured, we present two solution approaches: an exact approach and an approximate method. The former, an exact solution approach via interaction with a decision-maker, pursues the progressive reduction of a set of non-dominated alternatives by narrowing down the feasible attribute weights region. Subsequent interactive questions and responses, however, sometimes may not guarantee the best alternative or a complete rank order of a set of alternatives that the decision-maker desires to have. Approximate solution approaches, on the other hand, can be divided into three categories including surrogate weights methods, dominance value-based decision rules, and three classical decision rules. Their efficacies are evaluated in terms of choice accuracy via a simulation analysis. The simulation results indicate that a proposed hybrid approach, intended to combine an exact solution approach through interaction and a dominance value-based approach, is recommendable for aiding a decision making in a case that a final choice is seldom made at single step under attribute weights that are imprecisely specified beyond ordinal descriptions.

Keyword : Multiattribute Decision, Rank Order, Approximate Weights, Dominance Values

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1. Introduction

Decision making involves choosing some course of action among various alternatives. In almost all decision making problems, there are several conflicting criteria for judging alternatives. A multiple attribute decision making (MADM) method largely consists of two phases : 1) construction and information input, and 2) aggregation and exploitation. With regard to the parameter inputs in the first phase, the types of preference information allowed in the research models vary from exact information to imprecise information. Earlier studies on multiple criteria decision making (MCDM) under imprecise information can be found in a range of literatures and some authors refer to it as incomplete information, partial information, linear partial information (LPI) or incomplete knowledge. It is said that the need for handling imprecise data occurs in situations such as time pressure or lack of data, intrinsically intangible or non-monetary nature of criteria, decision maker's limited attention and information processing capability, and the like [11, 25]. In some cases, imprecise information can be utilized for the method when many alternatives are available in a decision problem, and evaluating all of them in detail is not practical and thus allow an efficient preliminary evaluation for the identification of the more promising alternatives to be studied in detail [13]. Some inferior alternatives, although they are non-dominated alternatives as they are, can be dropped off from further consideration by screening them with imprecise preference information.

In an MADM problem, one usually considers a finite discrete set of alternatives, $A = \{x, y, z, \cdots\}$, which is valued by a finite discrete set of attributes, $I = \{1, 2, \cdots, n\}$. A classical evaluation of alternatives leads to the aggregation of all criteria into a unique criterion called a value function under certainty and a utility function under uncertainty. In this paper, we assume that there exists an additive value function under preferential independence [12, 27] and thus the underlying model is a multiattribute value (MAV) model of the form :

$$V(x) = \sum_{i \in I} w_i v_i(x) \quad (1)$$

where V is the overall multiattribute value, $0 \leq V \leq 1$; $v_i(x)$ is a single attribute value of alternative x with respect to ith attribute, $0 \leq v_i(x) \leq 1$; $w_i \geq 0$ are weights reflecting the relative importance of the range of the attribute values, and $\sum_{i \in I} w_i = 1$.

When we assume that the underlying evaluation model is an MAV model, an aggregation method under imprecise attribute weights requires using linear programs for the identification of the most preferred alternative since the attribute weights lie in a pre-specified weights region. In a case that the attribute weights are assessed *ordinally* by the decision maker, an alternative with the highest multiattribute value when evaluated on the basis of extreme points of a feasible weight region is the best alternative to implement. The problem is, however, the number of final decision results available. Kirkwood and Corner [14] show in their simulation study that imprecise information about the ordinal preference information of weights is often not sufficient to determine the most preferred alternative for realistic decision problems. An interactive decision aid has features to make the decision maker specify additional preference information or modify existent preference information for the purpose of restricting the feasible weights region. Further interactions with the decision maker may proceed to the extent to compensate for the initial comforts of preference specifications. These interactions may not, however, guarantee to result in the best alternative or a complete ranking of a set of alternatives. With regard to this, surrogate weights that satisfy the ordinal rank order of attribute weights can be considered as an option for aiding a multi-attribute decision analysis and several well-established rank-based surrogate weights have been proposed [4, 10, 24]. Two other categories of approximate solution methods such as dominance value-based decision rules and three classical decision rules are considered and their efficacies with the rank-based surrogate weights are evaluated in terms of choice accuracy via a simulation analysis. The results show that the dominance value-based method, which is characterized by the aggregation of solutions from the paired dominance checks, reveals outstanding performance in all cases studied except the rank order centroid weights. It is believed that this has occurred since the weights in the method are disseminated into the alternatives the way in which they can take under the ordinal weights and that the dominance results are then combined for identifying a preferred alternative based on a reasonable way of aggregation. Further, in a case that linear partial information is obtained, it is not easy to obtain surrogate weights by the use of formulae and hence their usage is somewhat limited.

For the purpose of preview, we outline the methods in <Table 1> that were already developed in the literature of decision analysis. As was previously mentioned, this paper focuses, via a simulation analysis, on the efficacies of these methods in leading to a right decision when insufficient information on attribute weights is provided.

The paper is organized as follows : in Section 2, we present an exact solution method via interaction with the decision-maker ; in Section 3, we deal with three categories of approximate solution methods and their performances are tested through a simulation analysis, followed by some discussions in Section 4.

〈Table 1〉 A Summary of the Methods Considered

Method	1st Classification	2nd Classification	Characteristics
Exact Method			Several Rounds of Interactions
Approximate Method	Rank-based Decision	Rank Sum (RS)	Flatter than ROC Weights
		Rank Reciprocal (RR)	
		Rank Order Centroid (ROC)	Average of Extreme Points
		Expected Weights (EW)	Insufficient Reason
	Dominance Value-based Decision	Dominating Value (OUT I)	Outranking Concept
		Net Dominating Value (OUT II)	Outranking Concept
		Central Value (CENT)	Midpoint of Value Interval
	Classical Decision Rules	Optimistic (OPT)	Maxi-max
		Pessimistic (PESS)	Maxi-min
		Minimize Maximum Regret (REG)	Min-max Regret

2. Exact solution approach via interaction with decision-maker

Consider a car selection problem with four alternatives and four attributes shown in <Table 2>, which was reproduced from Edwards and Barron [7].

Given the decision matrix, the decision-maker thinks of alternatives that maximize her/his value of expense for buying a car. When applying a swing weighting method to determine the weights for the linear additive model in (1), this operation establishes a rank order of criteria, thus yielding, e.g., $w_{Power} \geq w_{ShopTtrips} \geq w_{Crushability} \geq w_{Styling}$ (refer [7] for more detailed derivation of the rank order). Simple multiattribute rating technique by swing (SMARTS) proceeds one step further to elicit the exact attribute weights via direct magnitude estimates or indifference judgments. The elicited weights, however, are controversial. Barron and Barrett [4] state that various methods for eliciting exact weights from the decision maker may suffer on several counts because the weights are highly dependent on the elicitation method [8, 9, 22] and there is no agreement as to which method produces more accurate results.

Let alternatives x and y represent two alternatives. It is said that alternative x strictly dominates alternative y if and only if $\xi_{min}(x) > \xi_{min}(y)$, in which

$$\xi_{min}(x) = \min\{\textstyle\sum_{i\in I} w_i v_i(x) \mid w_i \in W\} \text{ and}$$
$$\xi_{max}(y) = \max\{\textstyle\sum_{i\in I} w_i v_i(y) \mid w_i \in W\},$$

where a set of ordinal weights is given as

$$W = \{(w_1, w_2, \cdots, w_n) \in R^n \mid w_1 \geq w_2 \geq \cdots \geq w_n, \; w_i \geq 0, \text{ for } i = 1, \cdots, n, \; \textstyle\sum_{i\in I} w_i = 1\}.$$

In other words, the strict dominance concept implies that the worst weighted value of alternative x exceeds the best weighted value of alternative y within the feasible region of attribute weights. In our example, the rank-ordered weight set consists of four extreme points such as (1, 0, 0, 0), (1/2, 1/2, 0, 0), (1/3, 1/3, 1/3, 0), (1/4, 1/4, 1/4, 1/4). Since the additive value model is linear, we only have to calculate the extreme points in order to determine dominance relations. The value intervals becomes [47.5, 100] for *Anapest*, [0, 65.0] for *Dactyl*, [55.0, 70.0] for *Iamb*, and [25.0 50.0] for *Trochee*. The value intervals overlap each other except the value intervals for *Iamb* and *Trochee*. Thus, it can be concluded that alternative *Iamb* strictly dominates alternative *Trochee*, that is *Iamb* ≻ *Trochee* since a minimum MAV for Iamb (= 55.0) is larger than one for Trochee.

It is said that alternative x strictly pairwise dominates alternative y if and only if $\xi_{min}(x, y) > 0$, in which

<Table 2> Single Dimensional Utility for the Car Purchase Example

	Value Dimension			
Cars	Power	Shop Trips	Crushable Steel	Styling
Anapest	100	90	0	0
Dactyl	0	100	90	70
Iamb	70	40	100	40
Trochee	50	0	30	100

Note. The entries in the <Table 2> are utilities, not physical measures. For all value dimensions, 100 is best and 0 is worst.

$$\xi_{min}(x,y) = \min\{\sum_{i\in I} w_i[v_i(x)-v_i(y)] \mid w_i \in W\}. \quad (2)$$

The pairwise dominance results reveal that the alternatives *Anapest* and *Iamb* dominate alternative *Trochee*, that is *Anapset* > *Trochee* and *Iamb* > *Trochee*, since each of their pairwise dominance values is positive. It can be verified that a set of preference orders by the strict dominance rule, Ω_{SD} = {(*Iamb*, *Trochee*)} is included in a set of preference orders by the pairwise dominance rule, Ω_{PSD} = {(*Anapset*, *Trochee*), (*Iamb*, *Trochee*)}.

Under ordinal preferences on attribute weights, some difficulties that the decision-maker has in selecting the best alternative often occur since the number of non-dominated alternatives obtained from binary dominance checks is more than she/he wants. If the decision purpose is on choosing the most preferred alternative or on a rank order of a set of alternatives, the binary dominance rules may not be enough to reach such goals. One of the ways to cope with this problem is to direct our attention to an interactive decision procedure with the decision-maker. This approach basically assumes that it is possible to obtain more restrictive (i.e., precise) preference information about the decision parameters, thus reducing the feasible decision region and therefore results in less number of non-dominated alternatives. In our example, suppose that the decision-maker who is a speed maniac decides to put more emphasis on the speed attributes, thus resulting in a modified attribute weights set

$$W' = \{(w_1, w_2, w_3, w_4)\in R^4 \mid w_1 \geq 2w_2, w_2 \geq w_3 \geq w_4, w_i \geq 0, \text{ for } i=1,\cdots,4, \sum_{i=1,2,\cdots,4} w_i = 1\}.$$

The extreme points of the modified weights set consist of four extreme points such as (1, 0, 0, 0), (2/3, 1/3, 0, 0), (2/4, 1/4, 1/4, 0) and (2/5, 1/5, 1/5, 1/5). In general, if attribute weights are assessed in the type of ratio scale inequalities among the attributes such as $W = \{(w_1, w_2, \cdots, w_n)\in R^n \mid w_1 \geq (m_1/m_2)w_2, w_2 \geq (m_2/m_3)w_3, \cdots, w_n \geq 0, w_i \geq 0, i = 1, \cdots, n, \sum_{i\in I} w_i = 1\}$, the extreme points can be derived from the following formula [16, 18] : $w_i = (1/m^i)(m_1, m_2, \cdots, m_i, 0, \cdots, 0)$, $i = 1, \cdots, n$, where $m^i = \sum_{j=1,\cdots,i} m_j$, $i = 1, \cdots, n$. The pairwise dominance checks are performed to make these changes effective and the results are Ω_{PSD} = {(*Anapset*, *Dactyl*), (*Anapset*, *Trochee*), (*Iamb*, *Dactyl*), (*Iamb*, *Trochee*)}. A dominance relation between *Anapset* and *Iamb* still remains to be resolved and hence further interactive dialogues about attribute weights are required to be consistently specified in more precise ways.

Though such interactive approaches make sense and provide us with a useful decision principle, there exist, on the other hand, somewhat complicated problems in which we have to address how to assess more specific information from the decision-maker until the final decision is made. Further, we have to consider a situation in which the decision-maker is not willing to provide more restrictive information as situations advance but she/he expects to have a more specific recommendation for decision making. To deal with this situation, we now direct our attention to the approximate solution approach.

3. Approximate solution approach

3.1 Decision aid with surrogate weights method

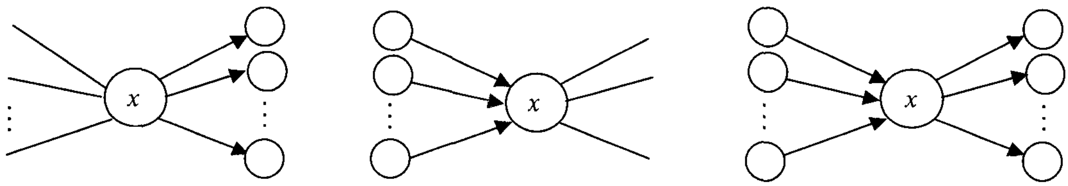
In a case where a rank order about attribute weights is assessed from the decision maker as in SMARTS [7, 27], several methods for determining approximate attribute weights have been presented. Stillwell et al. [24] present three surrogate weighting methods for determining the attribute weights that preserve the rank order of weights : rank sum weights, rank reciprocal weights, and rank exponent weights. Assuming that the significance of attribute weights is arranged in a descending order from the most important attribute to the least important attribute such as $w_1 \geq w_2 \geq \cdots \geq w_n$, they present approximate weights having the following forms :

(a) Rank sum (RS) weights :
$w_i = (n+1-i)/\sum_{j=1,\cdots,n} j = 2(n+1-i)/n(n+1)$,
$i = 1, \cdots, n$,
(b) Rank reciprocal (RR) weights :
$w_i = (1/i)/\sum_{j=1,\cdots,n} 1/j, \quad i = 1,\cdots,n$,
(c) Rank order centroid (ROC) weights [4, 27] :
$w_i = (1/n)/\sum_{j=i,\cdots,n} 1/j, \quad i = 1,\cdots,n$.

Barron and Barrett [4] suggest that ROC weights among other approximate weights are more adequate to accurately rank alternatives. In their simulation study, the superiority of ROC weights is evaluated and verified in terms of the degree of identification of the best alternative and value losses with respect to the various combinations of the number of alternatives, the number of attributes, and four different distributions from which attribute values are generated. We note, however, that in all these approximate weights methods including ROC weights method, not much is done to incorporate all information pertinent to the imprecise attribute weights, in other words, to pervade imprecise weights as they are prescribed in determining dominance relations among alternatives. Thus, the strength of preference between pairs of alternatives, which is the result of being taken effect through using possibly different weights that satisfy the rank order, can be considered a decision rule for ranking alternatives. Kmietowcz and Pearman's [15] concept of paired weak dominance basically falls into the category of reflecting the strength of preference between paired alternatives. A paired weak dominance relation of alternative x over y does ensure that the weight and value score most favorable to alternative x yield a higher expected difference than does the weight and value score most favorable to alternative y. Ahn [1, 2] extends this concept to encompass aggregated dominance values which can be derived from pairwise strict and weak dominance values. The procedure for deriving the aggregated dominance values is outlined below.

3.2 Decision aid with aggregated dominance values

It is said that a pairwise weak dominance relation holds between alternative x and y if and only if it holds that $\xi_{min}(x, y) > \xi_{min}(y, x)$. We now direct our attention to the pairwise dominance value, $\xi_{min}(x, y)$ obtained from solving (2) as helpful decision-relevant information for the use in establishing preference relations among the alternatives. We can then compute the *dominating* value of alternative x by means of adding paired dominance values between x and all the other alternatives [Figure 1a]. In other words, the leaving flow is the sum of the values of the arcs leaving node x (i.e., alternative x) and therefore provides a measure of the outranking character of x. Similarly we can obtain the *dominated* value



(a) Dominating (b) Dominated (c) Net Preference Value

[Figure 1] Aggregated Dominance Values

of x which is the addition of paired dominance values between other alternatives and x (i.e., in the reversely ordered pairs of alternatives) [Figure 1b]. The entering flow measures the outranked character of x. Then the difference between the dominating and dominated values can be regarded as a *net* dominance value that x has purely over all the other alternatives [Figure 1c].

We can thus use the net dominance value as a measure of strength of preference in a manner that an alternative to have larger is better. Note that this idea is consistent with the sense of outranking dominance relationship [5, 19]. This consideration can be set forth by the following five steps.

Step 1 : Solve the problem (2) to obtain the pairwise strict or weak dominance values. The pairwise dominance value between alternative x and y, that is $\xi_{min}(x, y)$, for instance, can be computed as $\min\{[v_1(x) - v_1(y)],\ 1/2[v_1(x) - v_1(y)] + 1/2[v_2(x) - v_2(y)], \cdots, 1/n[v_1(x) - v_1(y)] + \cdots + 1/n[v_n(x) - v_n(y)]\}$ if the ordinal preference on attribute weights is elicited from the decision-maker.

Step 2 : Using the paired dominance values in Step 1, compute the aggregated dominating preference intensities for each of alternatives.

$$\Phi^+(x) = \sum_{y \subset A \setminus \{x\}} \xi_{min}(x, y),\ \forall x \in A.$$

Step 3 : Using the paired dominance values in Step 1, compute the aggregated dominated preference intensities for each of alternatives.

$$\Phi^-(x) = \sum_{y \in A \setminus \{x\}} \xi_{min}(y, x),\ \forall x \in A.$$

$\Phi^+(x)$ is a more-is-better measure, i.e., the intensity by which alternative x is preferred to all the others. Conversely, $\Phi^-(x)$ becomes a less-is-better measure for alternative x. The ranks of the alternatives in two complete preorders from the Step 2 and Step 3 are not necessarily the same as they are based on different computations. In order to compromise the disagreements between two indexes, we introduce a net dominance value that is difference between dominating and dominated values and it can be regarded as pure strength of preference of an alternative over all others. We thus use the net dominance value as a measure of strength of preference in a manner that an alternative to have larger is better.

Step 4 : Compute the net preference intensity for each of alternatives by means of the difference between the dominating and the dominated values.

$$\Phi^N(x) = \Phi^+(x) - \Phi^-(x),\ \forall x \in A.$$

Step 5 : Establish the preference relations among the alternatives according to the following rules :

$$xPy \text{ if } \Phi^N(x) > \Phi^N(y)$$
$$xIy \text{ if } \Phi^N(x) = \Phi^N(y)$$
$$yPx \text{ if } \Phi^N(x) < \Phi^N(y)$$

where xPy means that alternative x is preferred to alternative y if the net preference strength of x is greater than that of y and xIy represents indifferent preference between x and y. We denote a decision method by the magnitude of aggregated dominating values as the OUT I (i.e., Steps 1-2) and a decision method by the magnitude of the net dominance values as the OUT II (i.e., Steps 1-5).

3.3. Other decision rules

In the course of decision analysis, different decision rules can be applied to help the decision-maker make a decision. Three classical decision rules which are considered to reflect the decision-maker's propensity under noncompensatory decision context are modified to encompass an imprecise decision context considered. Alternative decision rules among others include (i) the choice of an alternative with the largest possible overall value (i.e., maximax rule), (ii) the choice of an alternative for which the smallest possible value is largest (i.e., maximin rule), and (iii) the choice of an alternative such that the maximum value difference to some other alternative is minimized (i.e., minimax regret) [18, 21].

- maximax (OPTimistic) : $\max_{x \in A}[\xi_{max}(x)]$
- maximin (PESSimistic) : $\max_{x \in A}[\xi_{min}(x)]$
- minimax regret (REG) :

$$\min_x[\max_{y \neq x} \max_W [V(y) - V(x)]]$$

In addition to the three classical decision rules, we consider a choice of an alternative which is the greatest in the midpoint of the value intervals, that is

- central values (CENT) :

$$\max_{x \in A}[\xi_{min}(x) + \xi_{max}(x)].$$

3.4 Comparative analysis via simulation[1)]

In the simulation study, we demonstrate the performance of approximate weights methods (i.e., ROC, RR, RS, and EW) in terms of selection of the best alternative and overall rank ordering of alternatives, compared with aggregated dominance values (OUT I, OUT II, and CENT) and three classical decision rules (i.e., OPT, PESS, and REG) as a function of decision problem size. The quality of decisions is assessed by comparing decisions resulting from the use of each of approximate methods with those arising from knowledge of "true" weights, which are generated from random numbers which satisfy the rank order of attribute weights. The simulation study can be outlined by the following five steps:

Step 1 : Create the simulated decision problems. Each sequentially generated random number from independent uniform distribution ranging in (0, 1) constitutes the m x n matrix of attribute values. By convention, the single-attribute values in each column are normalized so that the smallest value is zero and the largest is one.

Step 2 : Perform the simple dominance checks. Without consideration of relative magnitude of attribute weights, the simple dominance checks are performed for each of generated decision problems by checking if the value differences between paired alternatives are all positive or negative. If dominating or dominated al-

1) The simulation procedure was implemented in the Excel's programming language, Visual Basic for Applications (VBA) on an IBM compatible personal computer.

ternatives exist, go to Step 1. The simple dominance checks render the decision matrix to some extent negatively correlated and thus more realistic decision problem since attributes for alternatives in the non-dominated set are usually negatively correlated.

Step 3 : Compute the attribute weights. The five different sets of attributes weights need to be generated ; one is the randomly generated weights constrained so as to satisfy the rank order (hereafter we call them TRUE weights and the decision made by TRUE weights is called TRUE method), and ROC, RR, RS, and EW weights are generated according to the formulae shown in Section 3.1. To generate the TRUE weights for the n attributes, we first select $n-1$ independent random numbers from a uniform distribution on (0, 1), then rank these numbers. Suppose the ranked numbers are $1 > r_{n-1} \geq \cdots \geq r_2 \geq r_1 > 0$. The differences of these consecutively ranked numbers can be obtained as the weights of the n-attributes, that is $w_n = 1 - r_{n-1}$, $w_{n-1} = r_{n-1} - r_{n-2}, \cdots, w_1 = r_1$ Then, the set of weights will sum to 1 and be uniformly distributed on the possible domain of weights [6, 10].

Step 4 : Determine the final ranking of a set of alternatives, applying the weights derived in Step 3, for each of generated decision problems. We calculate an MAV for each of alternatives only by substituting each of approximate weights, thus an alternative with the highest MAV is positioned in the first rank, one with the second-highest in the second rank and so on. In case of using the aggregated dominance values and the classical decision rules, determine the final ranking of a set of alternatives according to the decision criteria in Section 3.2 and Section 3.3 respectively.

Step 5 : Compare the decision results by each of proposed methods with those by TRUE method in terms of efficacy measures.

Two measures for the performance evaluation include hit ratio and rank order correlation (Kendall's τ). The hit ratio evaluates how frequently the coincidence of best alternative occurs between methods under consideration and TRUE method throughout simulation runs. Thus, the best alternative resulted from each of proposed methods is compared with the best alternative chosen from TRUE method. As another indicator representing the accuracy of considered methods, we use Kendall's τ for calculating rank order correlations between the methods under consideration and TRUE method, and Kendall's τ is defined as follows [26] :

$$\tau = 1 - \frac{2(\text{Number of Pairwise Preference Violations})}{\text{Tolal Number of Pairs of Preferences}}$$

The value +1 in Kendall's τ, which ranges from -1 to +1, means perfect correspondence between the two rank orders. In addition to two efficacy measures, a measure is motivated by the question, "When the TRUE weights and the methods select different alternatives as best, what is the level of disagreement in their MAVs?" We designate this quantity as the Value Loss. It is the difference in MAV for the alternative selected by TRUE weights and that selected by the competing methods. Of course, in those trials where the decision rule weights and the TRUE weights select the same alternative, the Value Loss is zero.

Kirkwood and Sarin [13] algorithm (shortly K-S algorithm) provides a criterion about whether the final ranking of a set of alternatives derived by each of methods preserves the (partial) ranking of a set of alternatives derived by the K-S algorithm, which can be stated as follows: if an additive value function exists and $w_1 \geq w_2 \geq \cdots \geq w_n$, then x is guaranteed to be preferred to y if and only if $\sum_{m=1,\cdots,i}[v_m(x) - v_m(y)] \geq 0$, $i = 1, \cdots, n$. It is trivial to show that the final rankings by approximate weights methods except EW method preserve the (partial) ranking by the K-S algorithm. It is, however, meaningful to see the final ranking by the OUT (OUT I and OUT II) method preserves the ranking by the K-S algorithm since the final ranking by the OUT method is derived by using various extreme points of weights and then performing numerical manipulations. If the K-S algorithm identifies alternative x is preferred to y, then the OUT method identifies the same dominance result as well (i.e., $\phi^N(x) > \phi^N(y)$) since $\phi^N(x) - \phi^N(y) = \sum_{z \in A-\{x\}}[\xi_{min}(x, z) + \xi_{max}(x, z)] - \sum_{z \in A-\{y\}}[\xi_{min}(y, z) + \xi_{max}(y, z)]$, and $\xi_{min}(x, z) \geq \xi_{min}(y, z)$, $\xi_{max}(x, z) \geq \xi_{max}(y, z)$ for $z \in A$, $z \neq x, y$ and $\xi_{min}(x, y) > 0$. Thus the final ranking by the OUT method perfectly coincides with one by the K-S algorithm.

We design the simulation with four different levels of alternatives (m = 3, 5, 7, 10) and with five different levels of attributes (n = 3, 5, 7, 10, 15). For each of 20 design elements (alternatives × attributes), the process of generating and analyzing decision problems repeated until 10 replications of 10,000 trials had been obtained (see Appendix A for the illustration of a simulation analysis for a specific value matrix). The simulation results for each of 20 design elements are arranged with respect to two efficacy measures in <Table 3> and <Table 4>. Throughout the different combinations of alternatives and attributes numbers, ROC method shows the highest degree of coincidences of best alternative with TRUE method in terms of hit ratio criterion whereas the OUT I method is consistently superior to the OUT II, CENT and three classical decision rules. With relatively less number of alternatives and attributes (m = 3, 5 and n = 3, 5, 7), the OUT I maintains correspondence of more than 80% with TRUE method and shows peculiar declines in hit ratio as the number of both alternatives and attributes increases (m =10 and n = 10, 15) whereas ROC and RR show stable rates of prediction. Therefore, it can be said that ROC > RR > RS > EW on rank-based weights methods, OUT I > OUT II > CENT on dominance value-based rules, and no regular trends on classical decision rules. As was expected, EW method is the worst method to adopt in resolving a decision problem with rank-ordered attribute weights.

With regard to Kendall's τ, ROC method shows predominantly higher correlations with the ranking by TRUE method than other methods, irrespective of the various alterations of simulation parameters. From the smallest to the largest number of alternatives and attributes, ROC method, which ranges from 88% to 86%, shows only 2% decreases in rank order correlations with TRUE method whereas RR and RS method show rapid decreases of 7%~8%. On the other hand, the OUT I method, which is in between 84% and 70%, consistently shows better performance than the OUT II, CENT and three classical decision rules. The results in terms of rank order correla-

〈Table 3〉 Simulation Results in Terms of Average Hit Ratio Criterion

		Rank-based Weights				Dominance Value-based Rules			Classical Decision Rules		
Alternative	Attribute	ROC	RR	RS	EW	OUT I	OUT II	CENT	OPT	PESS	REG
3	3	0.891	0.881	0.873	0.721	0.884	0.883	0.841	0.737	0.812	0.832
	5	0.898	0.887	0.864	0.705	0.878	0.844	0.833	0.723	0.848	0.836
	7	0.889	0.866	0.857	0.686	0.848	0.830	0.815	0.692	0.836	0.826
	10	0.898	0.860	0.835	0.654	0.824	0.792	0.776	0.665	0.823	0.787
	15	0.875	0.835	0.829	0.648	0.762	0.720	0.719	0.620	0.759	0.723
5	3	0.846	0.830	0.816	0.654	0.816	0.833	0.782	0.590	0.680	0.759
	5	0.864	0.845	0.826	0.618	0.835	0.811	0.780	0.573	0.775	0.784
	7	0.855	0.830	0.805	0.593	0.809	0.786	0.753	0.558	0.773	0.756
	10	0.859	0.808	0.787	0.578	0.783	0.725	0.699	0.518	0.757	0.708
	15	0.876	0.799	0.773	0.565	0.730	0.680	0.652	0.486	0.719	0.658
7	3	0.815	0.793	0.792	0.622	0.785	0.815	0.695	0.576	0.605	0.701
	5	0.836	0.819	0.800	0.577	0.802	0.793	0.753	0.505	0.719	0.769
	7	0.825	0.799	0.773	0.551	0.782	0.767	0.717	0.483	0.730	0.714
	10	0.842	0.790	0.762	0.541	0.768	0.712	0.668	0.433	0.721	0.674
	15	0.831	0.781	0.757	0.525	0.713	0.652	0.620	0.411	0.704	0.618
10	3	0.760	0.735	0.734	0.578	0.729	0.737	0.719	0.477	0.718	0.711
	5	0.827	0.803	0.783	0.537	0.778	0.781	0.742	0.430	0.713	0.744
	7	0.806	0.779	0.748	0.507	0.727	0.723	0.678	0.402	0.696	0.692
	10	0.829	0.783	0.745	0.504	0.715	0.691	0.642	0.354	0.693	0.664
	15	0.849	0.774	0.740	0.489	0.702	0.645	0.611	0.350	0.681	0.608

〈Table 4〉 Simulation Results in Terms of Average Rank Order Correlation

		Rank-based Weights				Dominance Value-based Rules			Classical Decision Rules*		
Alternative	Attribute	ROC	RR	RS	EW	OUT I	OUT II	CENT	OPT	PESS	REG
3	3	0.864	0.853	0.843	0.611	0.843	0.852	0.809			
	5	0.857	0.845	0.805	0.589	0.805	0.795	0.784			
	7	0.848	0.821	0.790	0.558	0.789	0.772	0.748	–	–	–
	10	0.862	0.812	0.782	0.531	0.753	0.704	0.695			
	15	0.870	0.775	0.773	0.525	0.701	0.650	0.619			
5	3	0.861	0.847	0.836	0.637	0.829	0.846	0.795			
	5	0.858	0.843	0.814	0.590	0.809	0.812	0.788			
	7	0.855	0.832	0.801	0.570	0.784	0.771	0.747	–	–	–
	10	0.865	0.819	0.786	0.546	0.751	0.720	0.692			
	15	0.875	0.793	0.775	0.529	0.724	0.671	0.630			
7	3	0.857	0.842	0.835	0.647	0.825	0.846	0.770			
	5	0.857	0.843	0.816	0.593	0.810	0.812	0.792			
	7	0.856	0.832	0.797	0.562	0.775	0.774	0.743	–	–	–
	10	0.864	0.820	0.784	0.543	0.746	0.720	0.702			
	15	0.877	0.797	0.778	0.534	0.695	0.668	0.640			
10	3	0.836	0.822	0.813	0.613	0.823	0.826	0.817			
	5	0.860	0.842	0.816	0.592	0.814	0.813	0.793			
	7	0.858	0.833	0.798	0.566	0.787	0.771	0.749	–	–	–
	10	0.866	0.822	0.790	0.545	0.743	0.725	0.706			
	15	0.880	0.802	0.782	0.542	0.716	0.694	0.645			

* : In the classical decision rules, the calculation of a complete rank is excluded.

tions are analogous to the results in terms of hit ratio criterion due to the high correlation between two efficacy measures. The simulation results in terms of the value loss measure show the similar ones with two efficacy measures (the detailed data are not included in this paper). Therefore, it can be concluded that, to summarize, the overall simulation results based on these two efficacy measures reveal ROC > RR > RS > EW on rank-based weights methods, OUT I > OUT II > CENT on dominance value-based rules, and no regular trends on classical decision rules.

A couple of things with regard to the results can be mentioned. First, in 10 replications of 10,000 trials, any instance where ROC method was surpassed by other methods in two efficacy measures was never occurred (the simulation results in <Table 3> shows the average values of 10 replications). Second, the superiority of ROC weights under rank-ordered preference on attribute weights was found in several research studies and thus ROC weights can be utilized to support decision making in case that it is difficult to elicit further precise preference information from the decision-maker. Our simulation results show similar results in favor of ROC weights to those studies.

Our final remarks concern the issue such that other decision rules, then, except ROC weights are useless. With several decision rules suggested, it may be advisable not to follow them early on if there are quite a few non-dominated alternatives. This is because one cannot, in advance, exclude the possibility that the decision rule will support an alternative which could turn out to be inferior to another, if the analysis were to be continued [21]. Further if the preference information on attribute weights is not just ordinal but possibly general partial information (the various types of partial information are shown in Section 4 in detail), as interaction with the decision-maker progresses to precisely specify the attribute weights, it is rather difficult to obtain the centroid weights which are central points of extreme points of feasible attribute weights. Such circumstances, which can occur in realistic decision problems, render the OUT method among others to be an alternative decision instrument.

4. Discussions

If we do not force the decision maker to specify parameters as input data to the extent that this becomes overly stressful or behaviorally and physically irrelevant in view of the inherent imprecision associated with domain knowledge of parameters characterizing the decision situation, the decision-maker provides his/her knowledge or preference information on the weights $\{w_i\}_{i \in I}$ of which the precise values are not known possibly on some of attributes in such way that information is to satisfy any combinations of linear constraints [1, 15, 17] : (a) $w_i \geq w_j$ or $w_i - w_j \geq \varepsilon$, where ε is a small positive number, (b) $w_i \geq a_{ij}w_j$, (c) $l_i \leq w_i \leq u_i$, (d) $w_i - w_j \geq w_k - w_l$, for $i \neq j \neq k \neq l$.

The ordinal ranking on attribute weights is widely-used imprecise information which can be found in e.g., SMARTS and so many others. The ratio judgments such as preference with ratio comparisons are the scales found in the analytic hierarchy process (AHP) and the parameter a_{ij} represents well-established verbal description [20]. The interval judgments impose the linear constraints on the weights in a natural way [3]. The preference differences present two levels of strength of attribute importance and can be constructed, for example, in case attribute w_i is believed to be strongly important to attribute w_j and attribute w_k is believed to be weakly important to attribute w_l. In a case that a mixture of imprecise preferences is provided, it is not easy to obtain approximate weights by the use of formulae and hence their usage is somewhat limited. In the OUT method, however, we only have to solve small linear programs for checking paired dominance relations between alternatives, which are, in turn, utilized for determining the strength of preference of each of alternatives. In the simulation study, the OUT method (especially the OUT I method), which is characterized by both exact solutions from the paired dominance checks and their aggregation, shows outstanding performance in all cases studied except the rank-based attribute weights. It is believed that this has occurred due to the fact that the weights in the OUT method are disseminated into the alternatives the way in which they can take under imprecisely specified weights and that the dominance results (i.e., strict or weak) are then combined for identifying a preferred alternative based on a reasonable way of aggregation. In other words, the OUT method utilizes as pertinent weights as possible depending on the value scores which alternatives in paired comparison take rather than using point estimates which only satisfy the ordinal relation of weights.

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Appendix A

〈Table 5〉 Rank-based Decision Results for a Specific Value Matrix

	Attributes ($w_1 \geq w_2 \geq w_3 \geq w_4 \geq w_5$)					MAV				
Alternatives	1	2	3	4	5	TRUE	ROC	RR	RS	EW
1	1	0.86	0.59	0.51	0	0.789	0.817	0.768	0.748	0.592
2	0.58	0.05	1	0.46	0.76	0.565	0.507	0.528	0.519	0.570
3	0	0.62	0.31	1	0.06	0.246	0.300	0.295	0.365	0.398
4	0.82	1	0	0	1	0.655	0.672	0.666	0.607	0.564
5	0.34	0	0.54	0.17	0.9	0.346	0.291	0.425	0.304	0.390
Methods	Approximate Weights					Final Rank Order				
TRUE	0.539	0.132	0.129	0.119	0.081	1, 3, 5, 2, 4[a]				
ROC	0.457	0.257	0.157	0.090	0.040	1, 3, 4, 2, 5				
RR	0.438	0.219	0.146	0.109	0.088	1, 3, 5, 2, 4				
RS	0.333	0.267	0.200	0.133	0.067	1, 3, 4, 2, 5				
EW	0.200	0.200	0.200	0.200	0.200	1, 2, 4, 3, 5				

주) [a] Rank order : $x_1 > x_4 > x_2 > x_5 > x_3$.

An example of simulation analysis is presented for a specific value matrix of five alternatives and five attributes in <Table 5>.

The attribute values for five non-dominated alternatives are generated from repeated random value generations and simple dominance checks. The random TRUE weights are generated so as to satisfy the order of importance magnitude of attribute weights, $w_1 \geq w_2 \geq w_3 \geq w_4 \geq w_5$ and four approximate weights in bottom rows in <Table 5> are derived from the formulae. The final ranks of alternatives are determined by the magnitude of MAV and shown in the right-hand column. The best alternative chosen by approximate weights methods shows a perfect coincidence with that of TRUE method whereas only RR weights method show the perfect correspondence of a rank order with TRUE method in this case. For determining a final ranking of a set of alternatives by the OUT method, 20 dominance checks for paired alternatives are required, and the results are shown in <Table 6>.

〈Table 6〉 Dominance Value-based Decision Results for a Specific Value Matrix

	Alternatives					OUT I		OUT II
Alternatives	1	2	3	4	5	Φ^+	Φ^-	Φ^N
1	–	0.022[a]	0.194	0.020	0.202	0.438	-2.660	3.098
2	-0.615	–	0.005	-0.595	0.145	-1.060	-0.886	-0.175
3	-1	-0.580	–	-0.820	-0.340	-2.740	-0.049	-2.692
4	-0.285	-0.068	-0.028	–	0.174	-0.206	-2.135	1.929
5	-0.760	-0.260	-0.220	-0.740	–	-1.998	0.181	-2.161
Rank Order						1,3,5,2,4		1,3,5,2,4

주) [a] This entry denotes a pairwise dominance value between x_1 and x_2, $\xi_{\min}(x_1, x_2)$.

The aggregated dominating and dominated values of each of alternatives are shown in the right-hand columns in <Table 6>. The OUT I produces a rank order $x_1 > x_4 > x_2 > x_5 > x_3$, arranging the aggregated values in a descending order. Based on OUT II, the net strength value of alternative x_1 is the largest one of all the net values and we thus come to a conclusion that alternative x_1 is the most preferred alternative among others. The rank order by the OUT II method also preserves partial rankings, $x_1 > x_2 > x_3$, $x_2 > x_5$, and $x_1 > x_4 > x_5$ by the K-S algorithm. The value intervals of each of alternatives become [0.59, 1.00] for alternative x_1, [0.32, 0.58] for alternative x_2, [0.00, 0.48] for alternative x_3, [0.46, 0.91] for alternative x_4, and [0.17, 0.39] for alternative x_5. Thus, the rank order by CENT is $x_1 > x_4 > x_2 > x_3 > x_5$, arranging the midpoints of the value intervals in a descending order. The best alternatives are identified as alternative x_1 (1.00) by OPT, x_1 (0.59) by PESS and x_3 (2.355) by REG.