Analysis of the Transient State of the Squirrel Cage Induction Motor by Means of the Magnetic Equivalent Circuit Method

Jong-Ho Jeong† , Eun-Woong Lee* and Hyun-Kil Cho**

Abstract - The finite element method is very flexible for new shapes and provides flux distribution, magnetomotive force, eddy currents, and torques. However, it requires lengthy computational time in order to achieve desired accuracy. The magnetic equivalent circuit method takes less computation time than the finite element method. Therefore, the finite element method is mainly used to confirm the completed design. The magnetic equivalent circuit method is convenient for complicated analysis of the transient state of the induction motor. The magnetic equivalent circuit method is restricted to only one direction of magnetic flux. In this paper, the construction elements (that is, stator iron, rotor iron, yoke, air gap, etc.) of the squirrel cage induction motor were represented by a flux tube and the air gap magnetomotive force was calculated by the magnetic equivalent circuit method. Starting transient torque and phase current of the squirrel cage induction motor were verified by the theoretical calculation and the experiment.

Keywords: air gap permeance, flux tube, magnetic equivalent circuit, magnetomotive force distribution, starting transient torque

1. Introduction

The transient states of the induction motor can flow currents several times larger than its rated current. Also, magnetic flux in the induction motor can be much higher than its rated value. The developed torque during the transients is much higher than its rated value, causing excessive mechanical strain on the shaft and coupling [1].

Therefore, the analysis of the transient state of the induction motor needs a design parameter for stable drive and long life of the induction motor.

The electrical equivalent circuit method is composed of self inductance and mutual inductance, etc. However, numbers of stator and rotor slots, skewing, and magnetic saturation, etc. are spatial values. Therefore, self and mutual inductance cannot be described by their many electromagnetic phenomena. Permeance of the magnetic equivalent circuit is a quantity that describes the electromagnetic property of the space occupied by the flux. Therefore, the magnetic equivalent circuit is more useful than the electrical equivalent circuit.

The Finite Element Method has been used for the analysis of an induction motor. Analysis of the induction motor using the Finite Element Method provides flux distribution, flux density, magnetomotive force, eddy currents, and torques, etc. [2, 3]. The Finite Element Method is very flexible for new shapes, but required computational times of the method can be excessive and the availability of commercial program packages have financial limitations [4].

In the early 1980's, the Magnetic Equivalent Circuit Method using the concept of the flux tube, which constrains to only two directions of flux path, was proposed by Vlado Ostovic. The Magnetic Equivalent Circuit Method can overcome which has long computation times required by the Finite Element Method and can be applied to the analysis of both steady and transient states. Also, the Magnetic Equivalent Circuit Method can be considered as; the number of stator and rotor slots, the state of magnetic saturation of the iron core, and the connection of windings etc. in the electrical machine [1].

The Magnetic Equivalent Circuit Method can include various modeling of the induction motor such as the distribution of windings, stator slotting, and the iron saturation, etc.

In this paper, it is assumed that each element of the induction motor has no wave phenomenon, which is a quasi-stationary state. Also, the concept of a flux tube was introduced to three-phase squirrel cage induction motor. Each element of the squirrel cage induction motor was represented by the flux tube. In addition, the air gap magnetomotive force was computed by the Magnetic Equivalent Circuit Method. At no load, starting transient

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torque and phase current of the squirrel cage induction motor were confirmed by the theoretical calculation and the experiment.

2. Magnetic equivalent circuit

A space including a quasi-stationary state electric or magnetic field is represented by a flux tube and all flux lines are perpendicular to the cross section. The flux through the flux tube is a function of the tube geometry such as shown by Eq. (1):

$$R = \int_0^l \frac{dx}{A(x)} \quad (1)$$

Where

R : reluctance, l : flux tube length,

$A(x)$: cross-sectional area

Magnetic Equivalent Circuits consist of an active element of magnetomotive force and a passive element of permeance. It is possible to reduce the number of elements without jeopardizing accuracy. In case of symmetry, only one part of the magnetic equivalent circuit needs to be calculated rather than simultaneous analysis of the complete circuit. Fig. 1 shows the magnetic equivalent circuit of the stator.

Fig. 2 shows the magnetic equivalent circuit of the rotor. The rotor slot and yoke between the j th teeth and j+1 th teeth are the magnetomotive force and magnetic resistance of the j+1 th element. When the rotor slot is closed, the magnetic equivalent circuits of iron core have a nonlinear property.

Fig. 3 shows a complete magnetic equivalent circuit that is composed of a stator magnetic equivalent circuit and a rotor magnetic equivalent circuit of an induction motor.

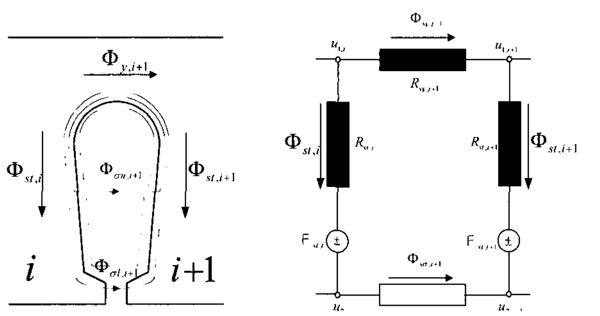

Fig. 1 Flux path and magnetic equivalent circuit of stator

It is necessary that the air gap permeance connects between the stator magnetic equivalent circuit and the rotor magnetic equivalent circuit. The air gap permeance depends on the rotor angle and constructs a flux path between the stator teeth and rotor teeth.

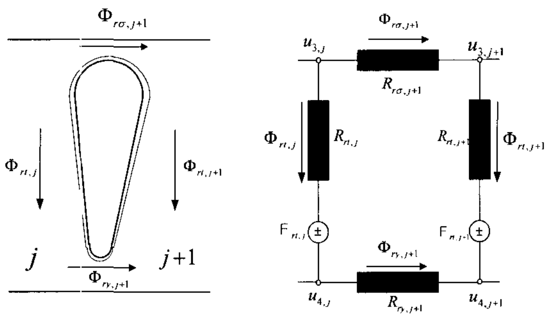

Fig. 2 Flux path and magnetic equivalent circuit of rotor

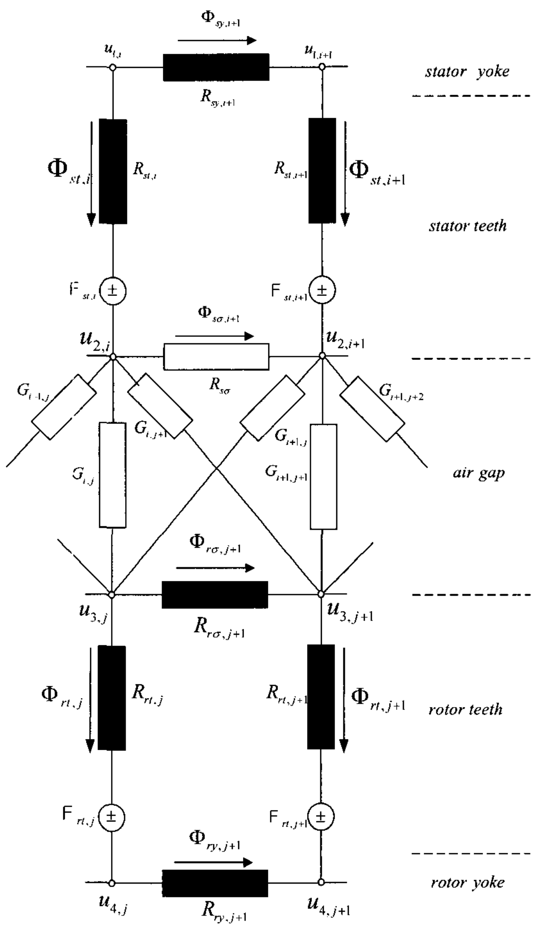

Fig. 3 Complete magnetic equivalent circuit of induction motor

3. Magnetomotive force

Flux tubes of the Magnetic Equivalent Circuit allow only radial direction of fluxes in the teeth and only tangential direction of fluxes in the yokes. Therefore, when the magnetomotive force of the stator teeth does not vary in the radial direction, the corresponding equation can be written as: [5]

$$\mathbb{F}_{i+1} = \mathbb{F}_i + (NI)_i \qquad i = 1,2,3,\cdots,z \tag{2}$$

$\mathbb{F}_i$ is the $i-th$ magnetomotive force that acts in the radial direction. $(NI)_i$ is the $i-th$ ampare-turn. Z is the total conductor number in the stator slot.

The induction motor has a structure that repeats the $i-th$ tooth next to the first tooth. Therefore, a necessary condition is that the magnetomotive force varies relatively as in Eq. (3):

$$\sum_{i=1}^{z} \mathbb{F}_i = 0 \tag{3}$$

Where the magnetomotive force position vector matrix of Eq. (2) is C, the number of conductor matrices is N, current vector matrix is I, and magnetomotive force matrix is $\mathbb{F}$. Eq. (2) can be written as Eq. (4):

$$\mathbb{F} = C^{-1}\, N\, I \tag{4}$$

Phase current vector I_{ph} multiplied by winding direction vector S is current vector I as shown in Eq. (5):

$$I = S\, I_{ph} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{5}$$

In the case of three phases, winding direction vector S has six rows and three columns in the matrix.

Table 1 contains specifications of the analysis model and Fig. 4 shows the magnetomotive force distribution of the stator teeth applied by Eq. (4).

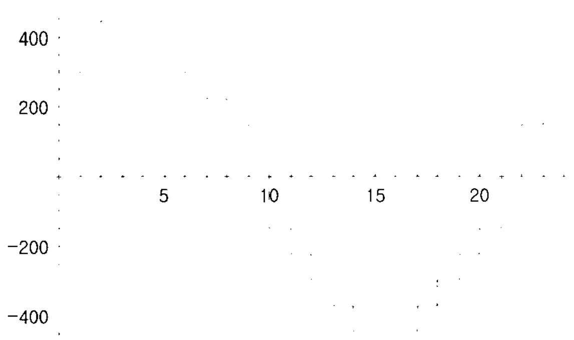

Fig. 4 Magnetomotive force distribution of the stator teeth

Table 1 Spec. of Analysis Model

3 ϕ, 4-poles, 220V, 2.3A, Δ-connection, double layer winding, 60Hz, 0.4kW		
Air gap length		0.3[mm]
Active axis length		55[mm]
Stack factor		0.96
Stator	Slot number	24
	Outer diameter	139[mm]
	Inner diameter	70[mm]
	Slot depth	14.4[mm]
	Slot opening	2.3[mm]
	Slot bottom width	9.152[mm]
	Slot upper width	5.382[mm]
	Winding number	76 turns
Rotor	Slot number	34
	Outer diameter	69.4[mm]
	Inner diameter	20[mm]
	Slot depth	7.85[mm]
	Inertia	0.00269[$kg{\cdot}m^2$]

When phase current is applied in time-varying conditions, magnetomotive force $\mathbb{F}(x,t)$ in the distributed air gap varies with respect to time and its spatial distribution can be formulated as:

$$\mathbb{F}_{k.n}(x,t) = \mathbb{F}_{k.n} \sin k\omega t \sin \frac{n\pi x}{\tau_p} \tag{6}$$

where

$$\mathbb{F}_{k.n} = \frac{4NI_k}{n\pi} \sin\frac{n\pi}{2} \sin\frac{n\pi y}{2\tau_p}$$

The change of magnetomotive force on the time varying current is indicated in Fig. 5.

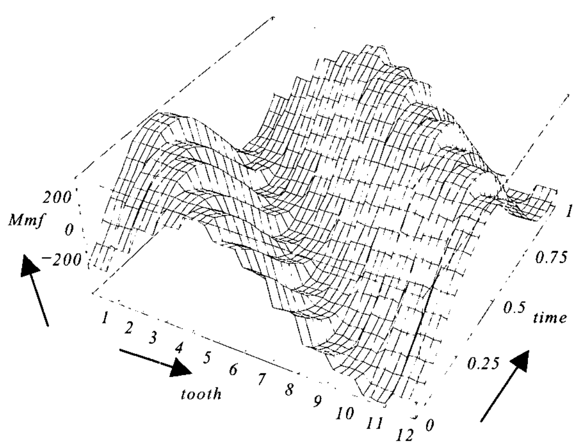

Fig. 5 Magnetomotive force distribution on the time varying current

4. Torque

The energy balance of electrical energy and mechanical energy consists of magnetic energy $\left(dW_{mag}\right)$, which is accumulated in the iron core and air gap, rotor kinetic energy $\left(dW_{mech}\right)$ and electrical loss $\left(dW_{loss}\right)$ shown as:

$$T d\gamma + \sum_{k=1}^{N} i_k v_k dt = dW_{mag} + dW_{mech} + dW_{loss} \qquad (7)$$

Where

T : torque, i : phase current, v : phase voltage,

N : number of coils

Neglecting electrical loss, the energy balance for the rotating angle γ can be expressed as:

$$\left(T - J\frac{d^2\gamma}{dt^2} - \sum \mathbb{F}_k^2 \frac{dG_k}{d\gamma}\right) d\gamma = dF - \sum_{k=1}^{N} i_k d\lambda_k \qquad (8)$$

Where

G : permeance, dF : differential magnetic energy,

λ : linkage flux

When the rotor is locked $\left(\gamma = 0\right)$, electromagnetic torque can be given as:

$$T = J\frac{d^2\gamma}{dt^2} + \sum \mathbb{F}_k^2 \frac{dG_k}{d\gamma} \qquad (9)$$

The decrease of magnetomotive forces in the iron core has an effect on the energy conversion efficiency. However, electromagnetic torque is dependent only on the air gap magnetomotive force amplitude and rotor shift.

5. Experiments

Electromagnetic torque is developed by air gap permeance and the angle between stator and rotor. Fig. 6 shows starting torque that is calculated by Eq. (9) at no load and Fig. 7 shows starting phase current on a time scale. Starting torque reaches the maximum value of approximately 3.5Nm and starting phase current reaches about 9.7A. Starting phase current is 4.2 times higher than the rated current.

Fig. 8 shows the experimental results in which the upper wave is starting torque and the lower wave is starting phase current. It can be seen that starting torque reaches a maximum at 5.8Nm and starting phase current reaches a maximum at about 12.3A.

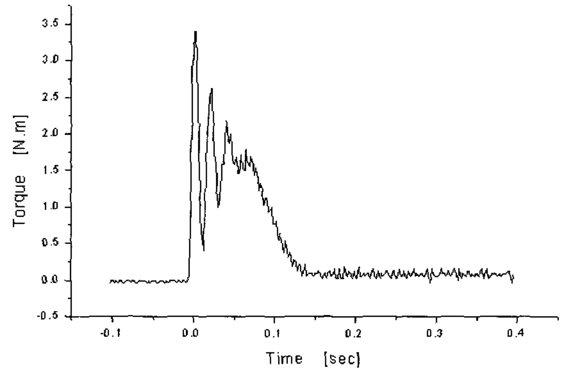

Fig. 6 Starting torque at no load

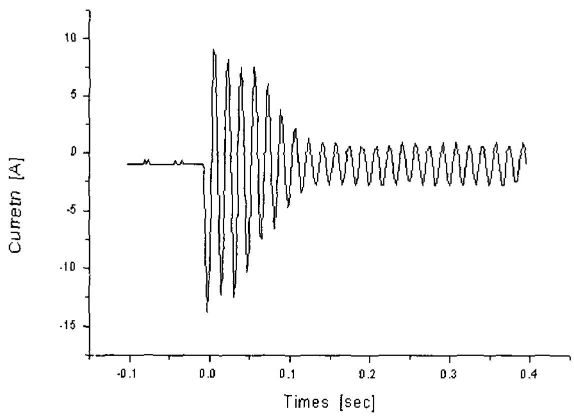

Fig. 7 Starting phase current at no load

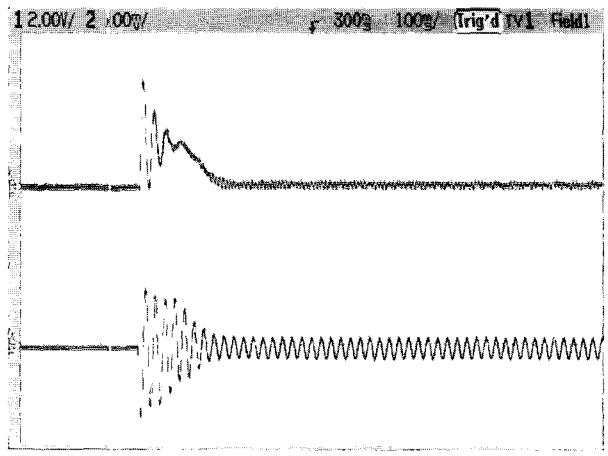

Fig. 8 Experiment results at no load

6. Conclusion

The authors calculated air gap magnetomotive force by using the magnetic equivalent circuit method. Furthermore, electromagnetic torque was calculated by means of the air gap permeance, air gap magnetomotive force and rotor shift angle.

Starting torque and starting phase current of the squirrel cage induction motor were both verified by the theoretical calculation and the experiment.

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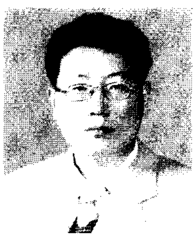

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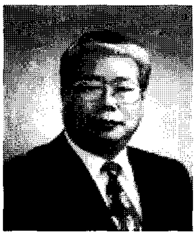

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