Path Planning of a Free Flying Object and its Application for Gymnastic Robots

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Abstract - The motion of animals and gymnasts in the air as well as free flying space robots without thrusters are subjected to nonholonomic constraints generated by the law of conservation of angular momentum. The purpose of this paper is to derive analytical posture control laws for free flying objects in the air. We propose the bang-bang control method for trajectory planning of a 3 link mechanical system with initial angular momentum. This technique is used to reduce the DOF (degrees of freedom) at first switching phase and to determine the control inputs to steer the reduced order system to the desired position. Computer simulations for motion planning of an athlete approximated by 3 link, namely platform diving, are provided to verify the effectiveness of the proposed control scheme.

Keywords: nonholonomic, path planning, flying object, bang-bang control

1. Introduction

Various motions carried out by an athlete in action while performing such feats as platform diving, horizontal bar, horse vault, and floor exercises are subject to nonholonomic constraints generated by the law of conservation of angular momentum. The nonholonomic system can be derived from the constraints that are not integral such as the law of conservation of angular momentum for the underactuated system. In particular, athlete's performances also include rotations in midair known as somersaults or flips. The initial angular momentum that athletes acquire in the pre-flight phase remains constant after he or she takes off from the ground and is transferred to their rotation axis. In other words, the gymnasts will automatically begin to rotate about their center of gravity. We plan to develop a planar gymnastic robot and to accomplish the motion of an athlete during the flight phase. To realize this, we first need to derive the necessary control laws. The purpose of this paper is to design a configuration control law for a free flying gymnast with an initial angular momentum.

Some related works of trajectory planning for nonholonomic systems with initial angular momentum are given in [1-3]. Sampei et al. [2] showed that errors in the system become locally controllable when the reference trajectory of the body angle is given by a certain first order function of time, and hence they proposed a linear feedback control method to stabilize the closed loop system. However, there is no guarantee that the error converges to zero when the control terminal time is finite. Kamon et al. [3] formulated the configuration control as a path planning problem and derived the minimum energy trajectory by the numerical optimization method to simulate the motion of a 3 dimensional somersault. Godhavn et al. [1] proposed motion planning for a planar diver using reaching control and manifold control based on numerical computation.

However, the solution is not unique since the control input is generated by a random process. An analytical solution for motion planning of a 2 DOF free flying object with drift by time optimal control was derived by Mita [6].

In this paper, we propose a configuration control law for 3 DOF free flying objects with initial angular momentum using bang-bang control (Sage [7], Mita [6]) with n-1 switching incorporated where n is the number of general coordinates. The method reduces the DOF at finite time $t_1, 0 < t_1 < T(\mathit{final\ time})$, and plans the trajectory from its initial states to its desired position for the reduced order system. The computer simulation for motion planning of an athlete approximated by 3 links, namely platform diving, is performed to verify the effectiveness of the proposed control algorithm.

2. Control object and Control problem

2.1 Control object

Here we deal with the configuration control problems of a 3 link planar gymnastic robot as shown in Fig.1.

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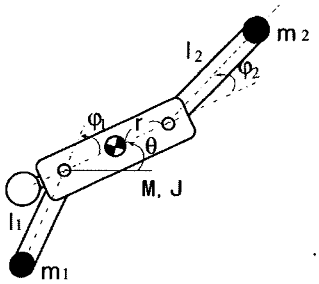


Fig. 1 Planar gymnastic robot composed of revolutive joints.

The robot is composed of an arm and a leg of lengths l_1, l_2, weights m_1 and m_2, and moments of inertia J_1, J_2. M and J denote a body of weight and moment of inertia, respectively. The limbs are attached to the body via revolute joints located in distance r from the center of CG (center of gravity) of the body. The configuration of the robot can be described by ψ_1, ψ_2, θ, where ψ_1 is the relative angle between the body and the arm, ψ_2 is the relative angle between the body and the leg, and θ implies the absolute angle of the body.

2.2 Control object

Suppose that the robot has a nonzero constant angular momentum P_0 that is provided by contact with the floor before taking off as an initial angular momentum. The initial angular momentum is given as

$$P_0 = (J_\alpha + \frac{a}{m_0})\dot{\theta} - (J_1 + \frac{b}{m_0})\dot{\psi}_1 - (J_2 + \frac{c}{m_0})\dot{\psi}_2 \quad (1)$$

where a, b, c are functions of ψ_1, ψ_2 (See Appendix I for details). Then the law of conservation of angular momentum can be derived

$$\begin{aligned} \dot{\theta} &= \frac{m_0 P_0}{m_0 J_\alpha + a} + \frac{J_1 + b}{m_0 J_\alpha + a}\dot{\psi}_1 + \frac{J_2 + c}{m_0 J_\alpha + a}\dot{\psi}_2 \\ &:= \gamma_1(\psi_1, \psi_2) + \gamma_2(\psi_1, \psi_2)\dot{\psi}_1 + \gamma_3(\psi_1, \psi_2)\dot{\psi}_2 \end{aligned} \quad (2)$$

Defining the generalized coordinates as $x = (\psi_1, \psi_2, \theta)^T$ and the control inputs as $u_1 = \dot{\psi}_1, u_2 = \dot{\psi}_2$, then we have

$$\dot{x} = \begin{bmatrix} 0 \\ 0 \\ \gamma_1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \gamma_2 & \gamma_3 \end{bmatrix} u \quad (3)$$

The control dilemma is how to derive a control input that can drive the system from its initial state x_0 to its desired state x_r at fixed final time T. Defining the error between the current state value and the desired value $q = x - x_r$, we have

$$\begin{aligned} \dot{q} &= \begin{bmatrix} 0 \\ 0 \\ \alpha_1(q_1, q_2) \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \alpha_2(q_1, q_2) & \alpha_3(q_1, q_2) \end{bmatrix} u \\ &:= f(q) + G(q)u \end{aligned} \quad (4)$$

where $\alpha_1, \alpha_2, \alpha_3$ are functions of q (See Appendix. II). We can see that the control problem became the derivation of control input to make all error variables to zero.
Godhavn [1] generalized the STLC (Small Time Locally Controllability) for an affine nonlinear control system with drift. Applying the control input $u = \lambda u, \lambda > 0$ to (4), and scaling the time with $\tau = \lambda t$, we then have

$$\frac{dq}{d\tau} = \frac{1}{\lambda} f(q) + G(q)u \quad (5)$$

Since the drift term is to be compensated by a large control input, i.e, $\lambda / f \cong 0$, the system will be equivalent to a driftless system, therefore the controllability is guaranteed. This STLC property is guaranteed for 3 DOF nonholonomic systems that have 2 control inputs, but is not generally guaranteed for a 2 DOF nonholonomic system with one control input [6].

3. Trajectory Control by Reducing Dof

We will design control laws that steer the system (4) from its initial configuration to the origin. As the first step in deriving the control law, we consider the control input that is needed to reduce the order of the system. If an initial state ($q_2(0) > q_1(0)$) lies in Reg.1, i.e. $q_2(0) > |q_1(0)|$ as shown in Fig.2, we determine the control input

$$u(t) = (u_m \; -u_m)^T \quad (6)$$

where u_m denotes the maximum control input.
Applying control inputs (6) to (4), trajectories of q_1, q_2 become

$$\begin{aligned} q_1(t) &= u_m t + q_1(0) \\ q_2(t) &= -u_m t + q_2(0) \end{aligned} \quad (7)$$

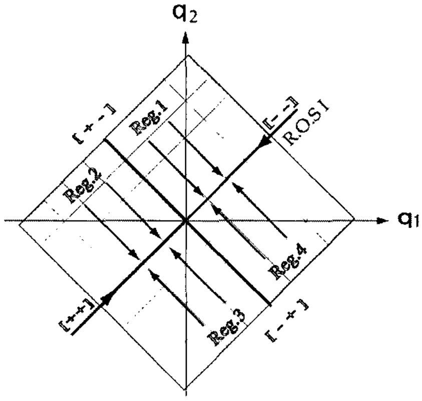


Fig. 2 Control input to reduce the system order

Here, we define the reducing order surface (ROS I)

$$ROS\ I = \{q_1, q_2 \in R \mid q_1(t) = q_2(t)\} \tag{8}$$

The initial states in Reg.1 or Reg.2 will move to the ROS I by the control input (6). When the states q_1 *and* q_2 arrive at the ROS I, $q_1(t)$ becomes equal to $q_2(t)$, reducing the system to a second order system. We can also consider control input

$$u(t) = (-u_m \;\; u_m)^T \tag{9}$$

for initial states in Reg.3 ($-q_2(0) \le |q_1(0)|$) or Reg.4 ($q_1(0) > q_2(0)$). Applying control inputs (9) to (4), we have

$$q_1(t) = -u_m t + q_1(0),\ q_2(t) = u_m t + q_2(0) \tag{10}$$

We see that the states q_1, q_2 move to ROS I and the system becomes a second order system. We will define t_1, the time at which the states satisfy $q_1(t) = q_2(t)$. Since the reduction of DOF, we now consider the problem of how to move states q_1, q_3 from their initial states of reduced order system to the point of origin. The states and control inputs of the reduced order system, i.e. the system on the ROS I ($t > t_1$) are defined as

$$\begin{aligned} q_1 &= q_2 = \eta, \\ u_1 &= u_2 = u \end{aligned} \tag{11}$$

Now the reduced order system equation can be described as

$$\begin{aligned} \dot{\eta} &= u \\ \dot{q}_3 &= \alpha_1(q) + (\alpha_2(q) + \alpha_3(q))u \\ &= \alpha_1(\eta) + \beta_1(\eta)u \end{aligned} \tag{12}$$

In order to steer the states η, q_3 to the origin, we will apply the bang-bang control input. We have to analyze the trajectories of system (12) subjected to bang-bang input, e.g. $u = u_m$ or $u = -u_m$. Considering control input

$$u = -u_m \tag{13}$$

Substituting (13) into (12) gives us

$$\begin{aligned} \dot{\eta} &= -u_m \\ \dot{q}_3 &= \alpha_1(\eta) - \beta_1(\eta)u_m \end{aligned} \tag{14}$$

The body angle q_3, which is the solution of (14) is given as

$$q_3(t) = h_1(\eta) + C_1 \tag{15}$$

where $h_1(\eta) = -\frac{1}{u_m}\int_0^{\eta}(\alpha_1(p) - \beta_1(p)u_m)dp$ and C_1 denotes the integral constant. Equation (14) can be derived from a differentiation of (15). From (4) we have

$$\begin{aligned} h_1(\eta) &= \int_0^{\eta} \frac{W_1/2\sin p + W_2/2\cos p + W_4}{W_1 \sin p + W_2 \cos p + W_3} dp \\ &= \frac{\eta}{2} + \frac{2W_4 - W_3}{\sqrt{W_3^2 - W_1^2 - W_2^2}} \tan^{-1}\left(\frac{(W_3 - W_2)\tan(\eta/2) + W_1}{\sqrt{W_3^2 - W_1^2 - W_2^2}}\right) \end{aligned} \tag{16}$$

where

$$\begin{aligned} W_1 &= -2(k_6 \sin\psi_{1r} + k_7 \sin\psi_{2r}), \\ W_2 &= 2(k_6 \cos\psi_{1r} + k_7 \cos\psi_{2r}), \\ W_3 &= m_0 J_\alpha + k_1 + k_2 + k_3 + 2k_8 \cos(\psi_{1r} - \psi_{2r}), \\ W_4 &= -\frac{m_0 P_0}{u_m} + J_1 + J_2 + k_4 + k_5 + 2k_8 \cos(\psi_{1r} - \psi_{2r}) \end{aligned}$$

The function $\tan^{-1}(\cdot)$ in (16) has discontinuity at $\eta = \pm\pi$. Therefore, we need to modify the calculation of $\tan^{-1}(\cdot)$ as [6]

$$k\pi + \tan^{-1}\left(\frac{(W_3 - W_2)\tan(\eta/2 - k\pi) + W_1}{\sqrt{W_3^2 - W_1^2 - W_2^2}}\right) \tag{17}$$

The trajectory of $h_1(\eta)$ is shown in Fig. 3.

Next, we consider control input $u = u_m$, then we have

$$\begin{aligned}\dot{\eta} &= u_m \\ \dot{q}_3 &= \alpha_1(\eta) + \beta_1(\eta) u_m\end{aligned} \tag{18}$$

from (12). The solution of q_3 is

$$q_3(t) = h_2(\eta) + C_2 \tag{19}$$

where

$$\begin{aligned} h_2(\eta) &= \frac{1}{u_m}\int_0^{\eta}(\alpha_1(p) + \beta_1(p)u_m)dp \\ &= \frac{\eta}{2} + \frac{2W_5 - W_3}{\sqrt{W_3^2 - W_1^2 - W_2^2}}\tan^{-1}\left(\frac{(W_3 - W_2)\tan(\eta/2) + W_1}{\sqrt{W_3^2 - W_1^2 - W_2^2}}\right)\end{aligned}$$

and $W_5 = \dfrac{m_0 P_0}{u_m} + J_1 + J_2 + k_4 + k_5 + 2k_8\cos(\psi_{1r} - \psi_{2r})$

Trajectory $h_2(\eta)$, which is compensated by the discontinuity is illustrated in Fig. 3.

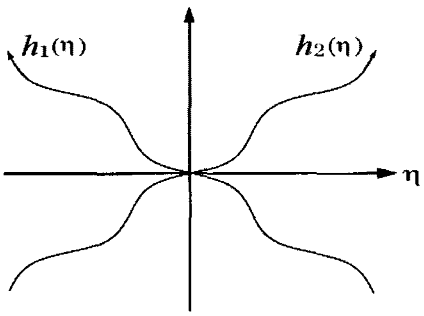


Fig. 3 Trajectories of $h_1(\eta)$ and $h_2(\eta)$.

The trajectories of q_3 calculated from (15) and (19) are indicated in Fig. 4. The trajectories I leading to the left and trajectories II leading to the right are obtained by changing C1 in (15) and C2 in (19), respectively.

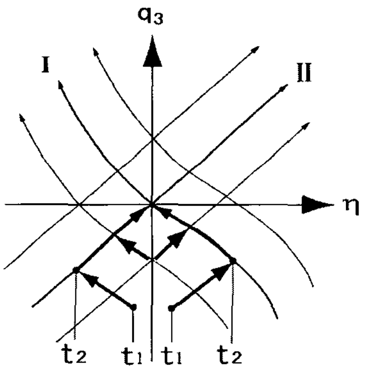


Fig. 4 Trajectory of q_3 to η.

From Fig. 4 we can determine a suitable trajectory from the initial states to the origin. Suppose that the initial states $\eta(t_1), q_3(t_1)$ were located in the fourth quadrant and under the invariant manifold I, we then have $\eta(t_1) \geq 0$. In this case, we apply the control input as $u = u_m$ until the trajectory meets a path that leads towards the left. Then the trajectory leading towards the right direction will intersect at $t = t_2$ with another path that moves from the fourth quadrant to the origin. The final approach is to switch the control input to $u = -u_m, t_2 \leq t \leq T$. Therefore, we can steer the states of the reduced system from their initial positions to the origin.

Repeating this procedure, initial states $\eta(t_1), q_3(t_1)$ do not exist under invariant manifolds I or II, and so we introduce new initial states as $\eta(t_1), q_3(t_1) - 2\pi$. This provides a solution for the robot body with 2π rotation.

It is shown that the initial states of the reduced order system can be steered to the origin by one time control switching, i.e. switching from $u = u_m$ to $u = -u_m$ or from $u = -u_m$ to $u = u_m$. Now we can compile configuration control for the gymnastic robot from its initial states to the origin. If an initial state is located in either of the following

$$\begin{aligned}&\text{Reg.1} \quad q_2(0) > |q_1(0)|, q_3(0) < 0 \\ &\text{Reg.2} \quad q_1(0) < |q_2(0)|, q_3(0) < 0\end{aligned} \tag{20}$$

then the control input becomes

$$u(t) = \begin{cases} (u_m \quad -u_m)^T, (t_0 \leq t \leq t_1) \\ (-u_m \quad -u_m)^T, (t_1 < t \leq t_2) \\ (u_m \quad u_m)^T, (t_2 < t \leq T) \end{cases} \tag{21}$$

For initial states in

$$\begin{aligned}&\text{Reg.3} \quad q_2(0) < |q_1(0)|, q_3(0) < 0 \\ &\text{Reg.4} \quad q_1(0) > |q_2(0)|, q_3(0) < 0\end{aligned} \tag{22}$$

the control input becomes

$$u(t) = \begin{cases} (-u_m \quad u_m)^T, (t_0 \leq t \leq t_1) \\ (-u_m \quad -u_m)^T, (t_1 < t \leq t_2) \\ (u_m \quad u_m)^T, (t_2 < t \leq T) \end{cases} \tag{23}$$

Consequently, the gymnastic robot represented by (3) can be controlled from its initial configuration to its desired one by the sequence of control input as in (21) and (23) with twofold control switching.

4. Switching Time Calculation

In this section we will calculate the switching time required to steer all states to their origin with twofold control switching. Let us assume the initial states lie in Reg. 2. From (4) and (21), we can derive

$$q_1(t) = u_m t + q_1(0), \quad q_2(t) = -u_m t + q_2(0) \tag{24}$$

Since the states in ROS I satisfy $q_1(t_1) = q_2(t_1)$, we have

$$t_1 = \frac{q_2(0) - q_1(0)}{2u_m}, \; q_1(0) > q_2(0) \tag{25}$$

Applying the second control input $u = (-u_m \quad u_m)^T$, $t_1 < t \le t_2$, we then get

$$q_1(t_2) = -u_m(t_2 - t_1)q_1(t_1) \tag{26}$$

From (26), we obtain

$$t_2 = \frac{u_m t_1 + q_1(t_1) - q_1(t_2)}{u_m} \tag{27}$$

Finally, applying $u = (u_m \quad u_m)^T$, $t_2 < t \le T$, we have

$$q_1(T) = u_m(T - t_2) + q_1(t_2) \tag{28}$$

using $q_1(T) = 0$ gives us the final time

$$T = \frac{u_m t_2 - q_1(t_2)}{u_m} \tag{29}$$

$q_1(t_2)$ must be known before calculating the switching time t_2, T.

In order to obtain information for $q_1(t_2)$, consider the state trajectory $q_3(t)$, $t_1 \le t \le T$. From (15) and (19) we get

$$\begin{aligned} q_3(t) &= h_1(\eta) + C_1, t_1 \le t \le t_2 \\ q_3(t) &= h_2(\eta) + C_2, t_2 < t \le T \end{aligned} \tag{30}$$

We can calculate an integral constant C_1 from (30).

$$C_1 = q_3(t_1) - h_1(\eta(t_1)) \tag{31}$$

Considering $q_3(T) = h_2(\eta(T)) = 0$ and (31), we have

$$\begin{aligned} q_3(t) &= h_1(\eta) + q_3(t_1) - h_1(\eta(t_1)), t_1 \le t \le t_2 \\ q_3(t) &= h_2(\eta), t_2 < t \le T \end{aligned} \tag{32}$$

Both trajectories of (32) should be equal at $t = t_2$, i.e.

$$h_1(\eta(t_2)) - h_2(\eta(t_2)) = -q_3(t_1) + h_1(\eta(t_1)) \tag{33}$$

It is difficult to calculate $q_3(t_1)$, which satisfies (33), by the analytical method. Here, we used the numerical integration method to calculate it. Finally, we can derive $\eta(t_2)$, *i.e.* $q_1(t_2)$ from (33)

$$\eta(t_2) = -2W_1 + 2\tan^{-1}(C_3 \tan(-q_3(t_1) + h_1(\eta(t_1)))) \tag{34}$$

where C_3 is constant calculated from (16) ~ (19).

$$C_3 = \frac{W_3{}^2 - W_1^2 - W_2^2}{2(W_3 - W_2)(W_4 - W_5)} \tag{35}$$

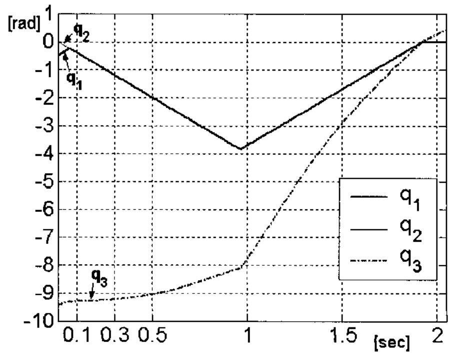
Using the information of $q_1(t_2)$, we can acquire switching time t_2, and final time T from (27) and (29), respectively.

5. Application to Planar Diving

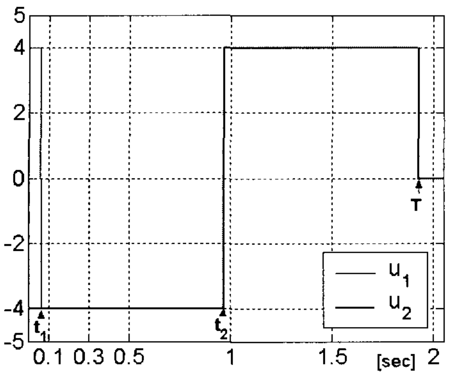
We applied the proposed control scheme to the configuration control of a planar diver. Simulation results for a 3 link gymnastic robot with parameters

$$P_0 = 70[kg.m^2/s], M = 40[kg], m_1 = 10[kg], m_2 = 15[kg],$$
$$l_1 = 0.6[m], l_2 = 0.9[m], r = 0.15[m] \text{ with } u_m = 4[/s]$$

are shown in Fig. 5.



(a)



(b)

Fig. 5 Simulation Results

The initial and desired states were $x_0 = (-0.5\ \ 0\ \ -0.5\pi)^T$ and $x_r = (0\ \ 0\ \ 2.5\pi)^T$, i.e. $q_0 = (-0.5\ \ 0\ \ -3\pi)^T$ and $q_r = (0\ \ 0\ \ 0)^T$, respectively. We can see that the control performance will be one and a half rotations in midair. The switching times were determined as $t_1 = 0.0625[s], t_2 = 0.9628[s], T = 1.9256[s]$. Fig. 5 (a) and Fig. 5 (b) depict the time evolution of the state and the control input, respectively. We can observe that all states were zero at the final time T, i.e. the control purpose was achieved by the proposed control scheme. An animation of the simulation result is given in Fig. 6.

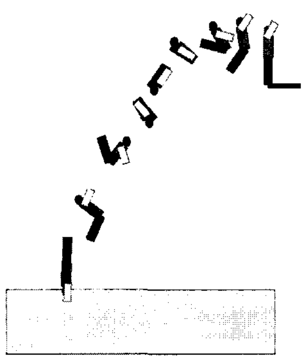


Fig. 6 Animation of simulation result.

6. Conclusions

We addressed the control problem of 3 DOF free flying objects from their initial configurations to the desired configurations using bang-bang control inputs. We reduced the DOF of the original system in the first control phase and determined the control input to steer the reduced order system to the desired configuration by bang-bang control. The computer simulation for a motion planning of the planar diver approximated by 3 links was carried out to verify the effectiveness of the proposed control scheme. We consider this bang-bang control input that has switching time information useful in actual experiments because of its simplicity.

Appendix I

The angular momentum of 3 link flying objects is described as

$$P_0 = (J_\alpha + \frac{a}{m_0})\dot{\theta} - (J_1 + \frac{b}{m_0})\dot{\psi}_1 - (J_2 + \frac{c}{m_0})\dot{\psi}_2 \quad \text{(A.1)}$$

where

$$a_0 = m_1 M(l_1^2 + r^2) + m_1 m_2 (l_1^2 + l_2^2 + 4r^2) + m_2 M(l_1^2 + r^2) + 2m_1 l_1 r(M + 2m_2)\cos\psi_1 + 2m_2 l_2 r(M + 2m_1)\cos\psi_2 +$$

$$2m_1 m_2 l_1 l_2 \cos(\psi_1 - \psi_2) := k_1 + k_2 + k_3 + 2k_6 \cos\psi_1 + 2k_7 \cos\psi_2 + 2k_8 \cos(\psi_1 - \psi_2)$$

$$b_0 = m_1(M + m_2)l_1^2 + m_1 l_1 r(M + 2m_2)\cos\psi_1 + m_1 m_2 l_1 l_2 \cos(\psi_1 - \psi_2) := k_4 + k_6 \cos\psi_1 + k_8 \cos(\psi_1 - \psi_2)$$

$$c_0 = m_2(M + m_1)l_2^2 + m_2 l_2 r(M + 2m_1)\cos\psi_2 + m_1 m_2 l_1 l_2 \cos(\psi_1 - \psi_2) := k_5 + k_7 \cos\psi_1 + k_8 \cos(\psi_1 - \psi_2)$$

$$m_0 = m_1 + m_2 + M, \quad J_\alpha = J_1 + J_2 + J$$

Appendix II

$$\alpha_1(q) = \frac{m_0 P_0}{m_0 J_\alpha + a(q)}, \quad \alpha_2(q) = \frac{J_1 + b(q)}{m_0 J_\alpha + a(q)}, \quad \alpha_3(q) = \frac{J_2 + c(q)}{m_0 J_\alpha + a(q)} \quad \text{(A.2)}$$

where

$$a(q) = k_1 + k_2 + k_3 + 2k_6(\cos(\psi_{1r})\cos(q_1) - \sin(\psi_{1r})\sin(q_1)) + 2k_7(\cos\psi_{1r}\cos(q_2) - \sin(\psi_{1r})\sin(q_2)) + 2k_8(\cos(\psi_{1r} - \psi_{2r})\cos(q_1 - q_2) - \sin(\psi_{1r} - \psi_{2r})\sin(q_1 - q_2))$$

$$b(q) = k_4 + 2k_6(\cos(\psi_{1r})\cos(q_1) - \sin(\psi_{1r})\sin(q_1)) + k_8(\cos(\psi_{1r} - \psi_{2r})\cos(q_1 - q_2) - \sin(\psi_{1r} - \psi_{2r})\sin(q_1 - q_2))$$

$$c(q) = k_5 + k_7(\cos(\psi_{2r})\cos(q_2) - \sin(\psi_{2r})\sin(q2)) + k_8(\cos(\psi_{1r} - \psi_{2r})\cos(q_1 - q_2) - \sin(\psi_{1r} - \psi_{2r})\sin(q_1 - q_2))$$

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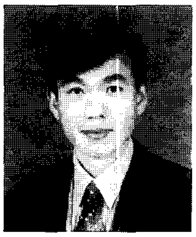

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