Optimal Design of Piezoelectric Cantilever Fan by Three-Dimensional Finite Element Analysis

Byoung-Jai Kim†, Jong-Seok Rho* and Hyung-Kyo Jung*

Abstract - As the structure of the piezoelectric bimorph cantilever becomes increasingly more complicated, a more accurate and efficient analysis of piezoelectric media is needed. In this paper, the piezoelectric transducer is analyzed by using the three-dimensional finite element method. The validity of the three-dimensional finite element routine is confirmed by comparing the experimental result. The resonance characteristics, such as resonance frequency and anti-resonance frequency, of the piezoelectric cantilever are calculated by the experimentally verified three dimensional finite element method. Subsequently, the characteristics, such as mechanical displacement and impedance, are calculated at the resonance frequency. Besides, to design the piezoelectric bimorph cantilever shape that maximizes displacement at the tip, the ES (Evolution Strategy) algorithm is applied. Finally, optimal design for the fan of the piezoelectric cantilever is fulfilled to obtain maximum displacement at the tip. From these results, the application potentiality of the piezoelectric bimorph cantilever fan is identified.

Keywords: Displacement analysis, Evolution Strategy, Impedance analysis

1. Introduction

As piezoelectric materials are widely used in various industrial fields, a more accurate and efficient analysis for electromechanical devices is required. There is the Mason Model established as a standard model for piezoelectric transducers, which represents a piezoelectric device by using electromechanical three-port. However, this model treats the piezoelectric devices one-dimensionally [1]. For two-dimensional or three-dimensional simulation of piezoelectric media, the complete set of fundamental equations governing piezoelectric media should be solved. Recently, the finite element analysis has been extensively employed for vibration model analysis. As the electrodes and structures of piezoelectric actuators become more complicated, a simpler and more efficient analysis method for optimal design is demanded. As such, numerical analysis techniques of the Finite Element Method are investigated. The piezoelectric media, which is characterized by its simple shape, low power, very low noise generation and small size, is applied to integrated industrial parts such as the ultrasonic motor, the piezoelectric transformer and the piezoelectric cantilever. Particularly, the piezoelectric cantilever, which stands out for its very low noise generation, is used as an alternative actuator to the piezoelectric fan, micro pump and data storage in the various vibration systems. In these actuators, piezoelectric fans are resonant devices that use piezoelectric excitation to drive a thin blade into resonance to create a fluid flow for electronics cooling. [2] Since very minimal strain can be expected from piezoelectric effects, small wobbling angle, typically in the range of 0.01-0.5 degrees, can be achieved directly from the bending piezoelectric actuator [3]. Also, little seems to be known about an optimal actuator-beam configuration including patch location, actuator patch-to-beam length ratio, and patch-to-beam thickness ratio for the piezoelectric cantilever. [2] Therefore, to maximize the displacement of the cantilever, optimal design is executed by using the ES (Evolution Strategy) algorithm with four variables; the length and thickness of the piezoelectric material and the elastic beam. From these results, an optimal scheme is designed for the larger deflection angle of the piezoelectric cantilever fan. The focus of the present paper is optimization of the piezoelectric cantilever fan. Validity of the optimized piezoelectric cantilever fan model is inspected through displacement analysis.

2. The Operating Principle of the Piezoelectric Cantilever

Fig. 1 represents the structure and operating principle of the piezoelectric cantilever. The piezoelectric cantilever is composed of one elastic body and two piezoelectric

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materials. The representation in Fig. 1 is generally called the 'parallel cantilever'. If the two piezoelectric materials are anti-parallel in polarization, then it is referred to as the 'series cantilever'. The series shape is wired by only two wires but the parallel shape is wired by three wires.

However, parallel operation requires only half voltage for series operation. Since the piezoelectric materials are excited by alternative voltage with the same phase, each piezoelectric material is repeatedly contracted and expanded. As such, the elastic body and two piezoelectric bars, which are attached to each other, are bent as shown in Fig. 1 As the operating characteristics of piezoelectric media are determined by the applied voltage frequency, the applied voltage frequency is decided through resonance characteristic analysis of the piezoelectric cantilever.

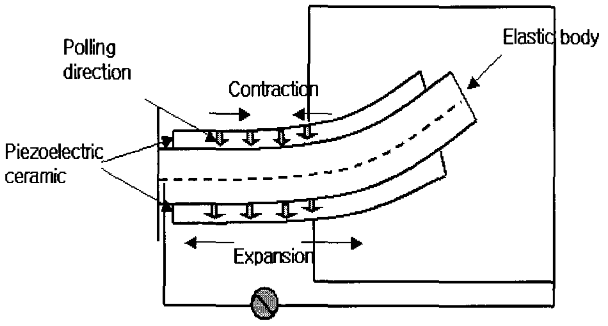


Fig. 1 Structure and operating principle of piezoelectric cantilever fan

3. Formulation for Finite Element Analysis

The matrix equations of (1) are the bases for the derivation of the finite element formulation, which relate mechanical and electrical quantities in piezoelectric media.

$$\begin{aligned} T &= c^E S - e^t E \\ D &= eS + \varepsilon^{t} E \end{aligned} \tag{1}$$

T: vector of mechanical stresses
S: vector of mechanical strains
E: vector of electric field
D: vector of dielectric displacement
C_E: mechanical stiffness matrix for constant electric field E
ε_S: permittivity matrix for constant mechanical strain S
e: piezoelectric matrix

From Hamilton's variational, the matrix equation of (2) and (3) can be obtained [1].

$$-\omega^2 Mu + j\omega D_{uu}u + K_{uu}u + K_{u\Phi}\Phi = F_{total} \tag{2}$$

$$K^t_{u\Phi}u + K_{\Phi\Phi}\Phi = Q_S + Q_p \tag{3}$$

K_{uu} : mechanical stiffness matrix
D_{uu}: mechanical damping matrix
$K_{u\Phi}$: piezoelectric coupling matrix
$K_{\Phi\Phi}$: dielectric stiffness matrix
M: mass matrix
F_B: mechanical body forces
F_S: mechanical surface forces
F_P: mechanical point forces
Q_S: electrical surface charges
Q_P: electrical point charges

The damping matrix, $\boldsymbol{D}_{uu}$ can be assembled from the damping properties of the structure. In this paper, for confirmation of the finite element analysis, the damping coefficients are selected as the typical values of the Rayleigh ones at 1 MHz. For analysis of the piezoelectric cantilever, the damping coefficients are also selected as the same value.

4. Finite Element Analysis

4.1 Impedance Analysis for Piezoelectric Transducer

The electrical impedance is important, since it can confirm the characteristic quantities such as the resonance and anti-resonance frequencies of piezoelectric devices and can also be verified from simple experiments with a network analyzer. The input impedance can be calculated by using the external electrical charge and the potential on the electrode. The current can be obtained from the external electric charge of $Q_o \sin(wt)$. Then the electrical impedance is given by (4).

$$Z(w) = \frac{\Phi(w)}{jwQ_o} \tag{4}$$

Fig. 2 indicates the impedances from three-dimensional calculation and experimentation for the test model of the piezoelectric transducer, whose experimental impedance is referred from[1]. Also, Fig. 2 shows the resonance and anti-resonance frequencies for the test model in the finite element analysis. Since the simulation result using the finite element method and experimental results have the same trend, the finite element formulation used in this paper is verified.

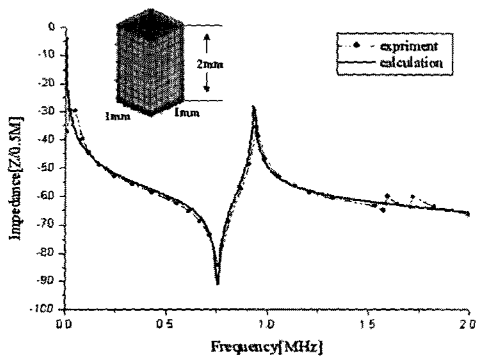
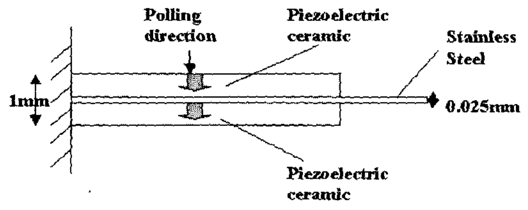


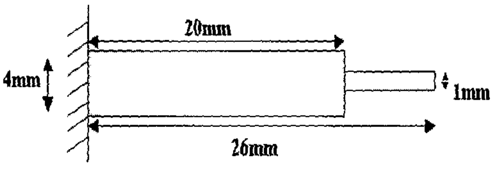
Fig. 2 Comparison between experimental and calculated impedance of transducer

4.2 Impedance Analysis for Piezoelectric Cantilever

Fig. 3 presents an analysis model to verify the analysis routine for the piezoelectric cantilever fan. Fig. 4 shows the impedance analysis of the piezoelectric cantilever using the verified finite element method. Experimental result and simulation result are identical. The resonance frequency is almost 526Hz and 1.2KHz. [2] Through this result, the validity of the three dimensional finite element method for the piezoelectric cantilever is verified.



(a) side view



(b) top view

Fig. 3 Analysis model of piezoelectric cantilever fan to verify FEM

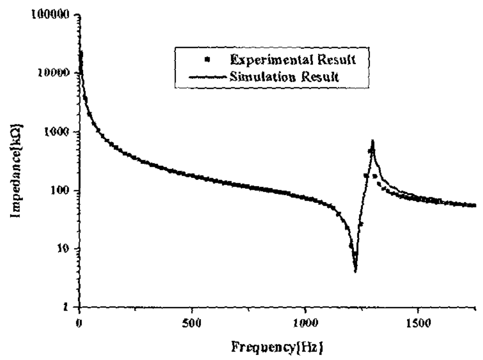


Fig. 4 Comparison between experimental and impedance results of analysis model

5. Optimal Design for Maximum Displacement

5.1 Evolution Strategy (ES)

Among the optimization methods a powerful stochastic method that offers probably the most universal compromise between reliability and performance is the evolution strategy. Evolution strategy copies the natural principle of mutation and selection ('survival of the fittest') into the technical optimization problem [4]. Beginning with an initial parent generation in each iteration, a child generation is generated by randomly modifying the parent parameters.

After the mutation step the objective function is evaluated for all children and the best of them are selected to form the new parent generation. In this study, the evolution strategy is used to determine optimal dimensions of the piezoelectric bimorph cantilever for maximum displacement. The design parameters are the length and thickness of the piezoelectric material and elastic body. The objective functions are maximum displacement at the tip. An example of meshed shape by using the auto-mesh generator and the flow chart of the evolution strategy are shown respectively in Fig. 5 and Fig. 6. The dimensions of Fig. 5 are totally 26×4×1.1mm, which is composed of the piezoelectric material (16×4×0.5mm) and stainless steel (26×4×0.1mm). Fig. 7 and Table 1 represent tested sinc

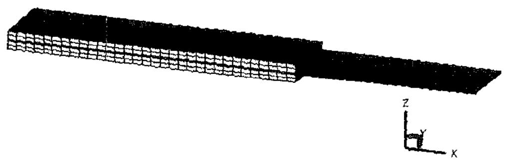


Fig. 5 Meshed piezoelectric cantilever shape generated from auto-mesh generator (26 ×4 ×1.1*mm*)

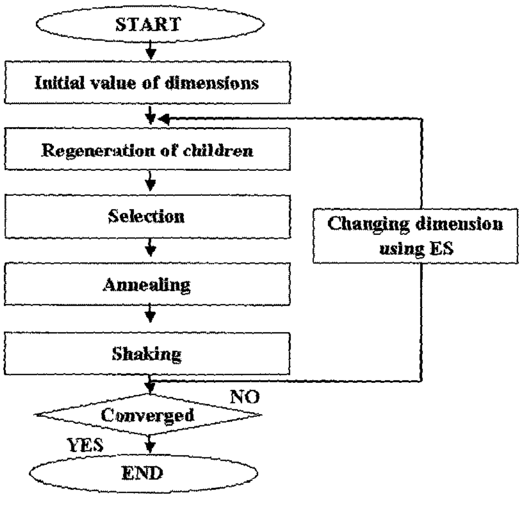
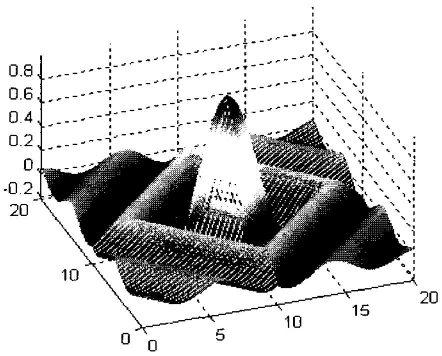


Fig. 6 Flow chart of evolution strategy

function and convergent results of the sinc function by applying the evolution strategy algorithm. According to the results from Table 1, the evolution strategy algorithm used in this study is reasonable.



$$f=\frac{\sin(|x_1-10|+|x_2-10|)}{|x_1-10|+|x_2-10|}$$

Fig. 7 Test function

Table 1 Convergent Process of test function through ES

Initial value of x_1	Initial value of x_2	Converged value of x_1	Converged value of x_2	Iteration number
9	11	10.001360	10.024390	4
16	13	9.990341	9.978443	5
4	6	9.990341	9.978443	5
13	18	9.990341	9.978443	5
2	19	9.990341	9.978443	5

5.2 Optimal Design for Maximum Displacement

As mentioned previously, the applications of the piezoelectric cantilever are extended. So, the optimal design of the piezoelectric cantilever according to various applications is needed. In this paper, optimal design considering maximum displacement for cooling is advantageous to users as well as manufacturers. The dimension optimization of the piezoelectric cantilever can be performed by using analytic calculations or finite element solutions, however, analytic calculations are not suited for complicated shapes and finite element methods take a great deal of time to model their shapes corresponding to dimension changes. In this study, dimension optimizations for maximum displacement are accomplished very quickly and simply by using the evolutions strategy. Table 2 shows the initial value of the ES. With four variables, length and thickness of piezoelectric material and elastic body, the optimization is executed and initial values, ranges and a part of the results are shown. Table 3 presents the results of dimension optimizations by using the evolution strategy up to iteration 200 with an applied voltage of just 1V. The optimal design model is totally 29.9×4×0.407. Finally, Fig. 8 indicates the simulation result pertaining to the optimal model. The resonance frequency is 459Hz, which is first mode frequency. The maximum displacement is 7.36E-04 when the applied voltage is 1V.

Table 2 Initial Value And Ranges of Dimensions

	Initial value	Minimum value	Maximum value
Length of Piezo-electric media (mm)	16	12	20
Thickness of Piezo-electric media (mm)	0.5	0.1	1.3
Length of Elastic body (mm)	26	22	30
Thickness of Elastic body (mm)	0.6	0.1	1

Table 3 Displacement with respect to Dimensions (Applied 1V)

Length of Piezoelectric media (mm)	Length of Elastic body (mm)	Thickness of Piezoelectric media (mm)	Thickness of Elastic body (mm)	Displacement (m)
16	26	0.5	0.6	**1.37E-7**
14.1	27.95	0.207	0.604	**6.5151E-07**
14.15	28.05	0.173	0.344	**7.4811E-06**
19.39	29.92	0.229	0.221	**1.0604E-05**
19.75	29.99	0.163	0.133	**5.7458E-04**
19.85	29.97	0.167	0.147	**5.8481E-04**
⋮	⋮	⋮		⋮
19.8	29.8	0.169	0.123	**6.1327E-04**
18.99	29.72	0.142	0.144	**6.358E-04**
19.69	29.84	0.369	0.204	**6.908E-04**
19.91	**29.9**	**0.152**	**0.103**	**7.36E-04**

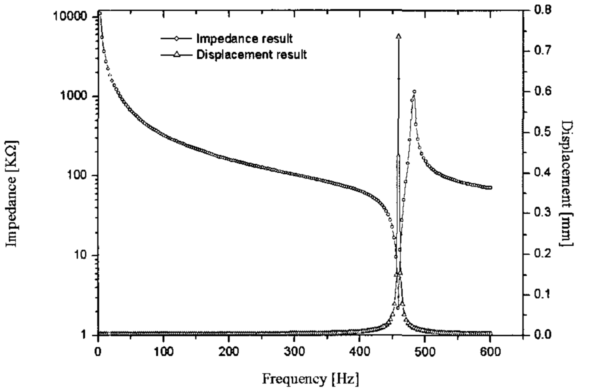


Fig. 8 The simulation result of the optimal design model

6. Conclusion

Impedance analysis of the piezoelectric transducer is calculated by the three-dimensional finite element method in this paper, the simulation result is verified through the experiment. Impedance of the piezoelectric cantilever is calculated by the verified three-dimensional finite element method, the impedance result is verified through the experimental result [2]. Also, to design the piezoelectric cantilever model for maximizing the displacement on the edge, Evolution Strategy Algorithm is imported into the analysis code. From this attempt, optimal design of the piezoelectric cantilever fan is done. Through all results, the possibility to the piezoelectric bimorph cantilever fan is verified and the optimal design method is established.

Further studies should be continued to consider the more variables; the location of piezoelectric material and coupling factor.

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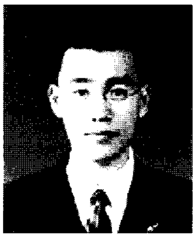

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