Electrical Resistance Tomography: Mesh Grouping and Boundary Estimation Algorithms

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ABSTRACT

This paper presents the development and application of electrical resistance imaging techniques for the visualization of two-phase flow fields. Two algorithms, the so-called the mesh grouping and the boundary estimation, are described for potential applications of electrical resistance tomography (ERT) and results from extensive numerical simulations are also presented. In the electrical resistance imaging for two-phase flows, numerical meshes fairly belonging to each phase can be grouped to improve the reconstruction performance. In many cases, the detection of phase boundary is a key subject and a mathematical model to estimate phase boundary can be formulated in a different manner. Our results indicated that the mesh grouping algorithm is effective to enhance computational performance and image quality, and boundary estimation algorithm to determine the phase boundary directly.

Keywords: *Mesh Grouping, Boundary Estimation, Electrical Resistance Tomography, Electrical Impedance Tomography.*

1. INTRODUCTION

In many engineering fields, such as heat exchangers, oil or natural gas pumping system, and chemical processing, two-phase flow systems can be frequently encountered. The heterogeneous phase distribution affects the thermal hydraulic phenomena significantly and the determination of the distribution is a major concern for developing analytical methods to predict the phenomena. There have been many attempts to develop the techniques to measure two-phase flow fields, including X-ray imaging, computerized tomography (CT), gamma camera, magnetic resonance imaging (MRI), and ultrasonic imaging. The electrical impedance tomography (EIT) technique [1] has been developed for medical or industrial purposes as an alternative to conventional imaging techniques mentioned above, some of which are expensive and even cause adverse health impacts. Since EIT is characterized by good time resolution and low cost, it has obvious advantages in the application to the visualization of two-phase flow system. In most cases of EIT the resistance component of the impedance is used for the image reconstruction, so sometimes we use the term of Electrical Resistance Tomography (ERT) instead of EIT. The first extensive research relative to application of EIT to measurement of phase distribution in two-phase system was conducted by a Rensselaer research group [2]. In that approach, emphasis was placed on developing an iterative algorithm based on the finite element model. Results clearly indicated that approximate methods like back projection [3] or non-iterative methods like NOSER (Newton One Step Error Reconstruction) [4] are insufficiently accurate for two-phase visualization. Hence, they proposed the block decomposition exponential reconstruction method in the context of finite element, where each of the larger elements is decomposed into smaller elements and bilinear exponential variation of resistivity in each element is allowed to reduce the total number of elements required to describe the overall resistivity field. This paper describes two algorithms, the mesh grouping and the boundary estimation, that can be used to visualize two-phase flow fields for the appreciation of potential possibility of ERT. We review the mathematical models of these reconstruct ion algorithms for ERT, and some numerical simulations are also presented to show favorable features of each algorithm.

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2. MATHMATICAL MODELS

In the ERT, the internal resistivity distribution is reconstructed based on the known set of the injected currents and the measured voltages on the surface of the object. The schematic of the ERT is shown in Fig. 1.

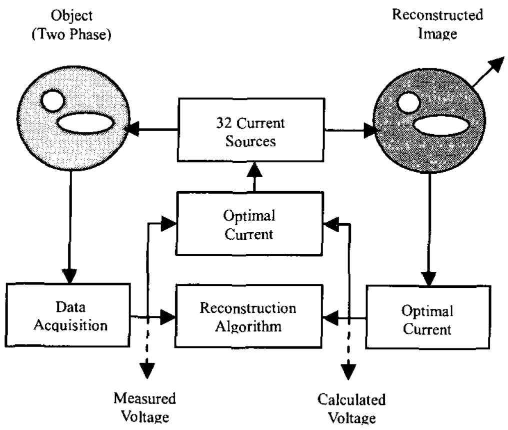


Fig. 1. Schematic diagram of the electrical resistance tomography (ERT).

The physics of relationship between the internal resistivity and the surface voltages is governed by a partial differential equation with the given injected currents. Mathematically, the ERT is composed of the *forward problem* to obtain the voltage distribution subject to the assumed resistivity distribution and the injected currents and the *inverse problem* to reconstruct the resistivity distribution with the measured boundary voltages in a suitable manner.

2.1 Mesh grouping algorithm

One of major drawbacks of EIT technique is the poor spatial resolution. Even the absolute value of resistivity cannot be reconstructed, but some useful information, such as the approximated outline of dispersed phase, can be extracted after a few iterations, especially for two-phase systems. Actually, in two-phase flows, there should be only two resistivity values. So, if we could group the elements whose resistivity values are similar, it would be expected to reduce the total number of unknowns and to increase the volume of effective meshes. By alternating conventional Newton-Raphson iterations and groupings repeatedly, we expect to improve the convergence as well as to reduce the computational load. If the mesh structure fits interfacial boundaries, the ideal reconstructed resistivities have two distinct values and only two groups will be enough to account for the resistivity distribution. However, the interfacial boundary cannot be known *a priori* and we cannot fit mesh structure to interfacial boundary. Moreover, due to the numerical and experimental errors, meshes with deviated resistivity values from the two exact ones will be inevitable. Experience shows that after a certain number of Newton-Raphson iterations, the sorted resistivity $\rho_j (j = 1,2,\Lambda\ ,M)$ is expected to have the idealized curve as shown in Fig. 2. In this figure, it is natural to assume that the elements in region I and III belong to the continuous phase (such as, water) and dispersed phase (such as, vapor), respectively. The elements in intermediate region II are assumed to be undetermined elements that do not belong to region I nor III. So, we classify each mesh into one of three mesh groups: ContGroup (or DispGroup) is the mesh group with the resistivity value of the continuous phase (or dispersed phase). TempGroup is the group of meshes neither in ContGroup nor in DispGroup. All meshes in ContGroup and in DispGroup are forced to have the same but unknown resistivity value (ρ_{Cont} and ρ_{Disp}), respectively. However, all meshes in TempGroup can have different resistivity values ($\rho_{Temp,i}$, $i = 1,2,\Lambda\ ,n$). However, since we cannot always expect to get such well-distinguished resistivity distribution curve as shown in Fig. 2.

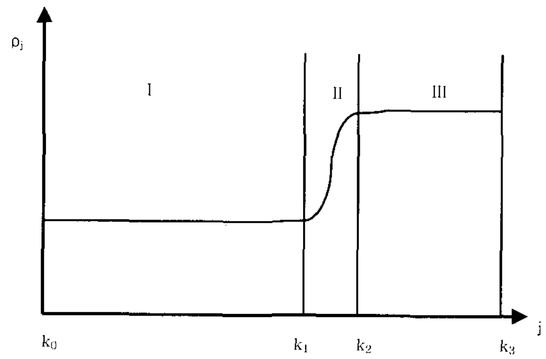


Fig. 2. Idealized resistivity distribution in an ascending order.

It is useful to divide the regions and determine a typical resistivity value of each region. Let $\bar{\rho}_i$ $(i = 1,2,3)$ be the representative values of each region and k_i $(i = 1,2)$ be the boundary location between regions. Then, we can formulate the following optimization problem to determine $\bar{\rho}_i$ and k_i: Find a solution vector **X**,

$$\mathbf{X} = \{\bar{\rho}_1, \bar{\rho}_2, \bar{\rho}_3, k_1, k_2\} \quad (1)$$

to maximize the fitness function f(**X**),

$$f(\mathbf{X}) = -\ln(D) \quad (2)$$

subject to the objective functional D,

$$D = \sum_{i=1}^{3} \sum_{j=k_{i-1}}^{k_i} (\rho_j - \bar{\rho}_i)^2, \quad k_0 = 1 \text{ and } k_3 = M \quad (3)$$

To solve the above optimization problem, several methods can be applicable. However, since the object funtion is not analytical and the derivatives cannot be calculated easily, the derivative-based method, such as Newton-Raphson method is unapplicable. We solved the above problem using the genetic algorithm (GA) [5]. In the simplest implementation of GA in the optimization problem, a set (population) of possible solution is generated, usually at random. Each individual consists in three resistivity values $(\bar{\rho}_{Cont}, \bar{\rho}_{Disp}, \bar{\rho}_{Temp})$ and two locations (k_1, k_2). A fitness value is computed for each individual. In the present case, the fitness function is the object function of Eq. (2). After several Newton-Raphson iterations, meshes are grouped according to the grouping algorithm. And then, with the grouped meshes, several Newton-Raphson iterations are conducted. In some cases, some meshes may be grouped in an improper manner and mis-grouped meshes can deteriorate the convergence. One of simple way to eliminate misgrouped meshes is to ungroup after a certain number of Newton-Raphson iterations with the grouped meshes. With this grouping, the sensitivity, that is the change of boundary voltages due to the change of resistivity value of a certain mesh, can be increased and the number of unknowns can be reduced as the grouping.

2.2 Boundary Estimation Algorithm

In many cases of two-phase visualization, the major concern is to determine the phase boundary rather than the resistivity distribution. Hence, some investigators have focused on the reconstruction of phase interface rather than phase distribution in the ERT image reconstruction. If the conductivity value of each component in mixtures could be given *a priori*, the unknown would be the interfacial boundary. Han and Prosperetti [6] considered a shape decomposition technique based on the boundary element method, where the boundary of each object was represented in terms of Fourier coefficients rather than a point-wise discretization. Kolehmainen *et al.* [7] developed an algorithm to recover the region boundaries of piecewise constant coefficients of an elliptic PDE from boundary data for the application to optical tomography, which is applicable to ERT problem. They also expressed boundaries in terms of Fourier coefficients:

$$C_\lambda(s) = \begin{pmatrix} x \\ y \end{pmatrix} = \sum_{n=1}^{N_\theta} \begin{pmatrix} \gamma_n^{x_\lambda} \theta_n^x(s) \\ \gamma_n^{y_\lambda} \theta_n^y(s) \end{pmatrix} \quad (4)$$

where $C_\lambda(s)$, is the boundary of the ℓ-th object ($\lambda = 1, \Lambda, m$), s is the coordinate for phase boundary, m is the number of the objects, $\theta_n(s)$ is a periodic and differentiable basis function with period 1, and N_θ is the number of basis functions. As the basis function, we used the form of

$$\theta_1^\beta(s) = 1$$

$$\theta_n^\beta(s) = \sin\left(2\pi \frac{n}{2} s\right), \qquad n = 2,4,6,\Lambda, even \quad (5)$$

$$\theta_n^\beta(s) = \cos\left(2\pi \frac{(n-1)}{2} s\right), \qquad n = 3,5,7,\Lambda, odd$$

where $s \in [0 \ \ 1]$, β denotes either x or y. The boundaries of the objects are identified with the vector γ of the shape coefficients,

$$\gamma = (\gamma_1^{x_1}, \Lambda, \gamma_{N_\theta}^{x_1}, \gamma_1^{y_1}, \Lambda, \gamma_{N_\theta}^{y_1}, \gamma_1^{x_m}, \Lambda, \gamma_{N_\theta}^{x_m}, \gamma_1^{y_m}, \Lambda, \gamma_{N_\theta}^{y_m})^T \quad (6)$$

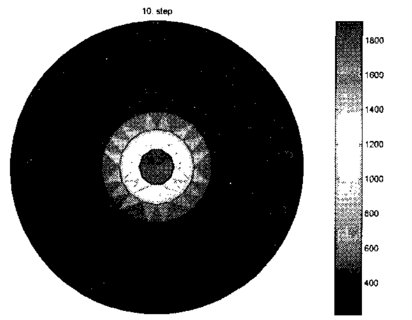
where $\gamma \in R^{2mN_\theta}$. Now our ERT problem is to determine the Fourier coefficients, not the resistivity value of each finite element mesh. For the optimal solution of the Fourier coefficients, Newton-type method can be employed [6,7]. It is known that Newton-type method is usually time consuming although it has shown good performance in many optimization problems. Such slow convergence would be an adverse effect in the application to mixture flows undergoing fast transient. For the computational efficiency in practical applications, as an alternative to Newton-type method, the neural network approach can be introduced. Adler and Guardo used the neural network method to reconstruct resistivity distribution and their concept could be applied to the present boundary estimation problem [8]. In the next section, our results from the algorithms mentioned above are presented.

3. RECONSTRUCTION RESULTS

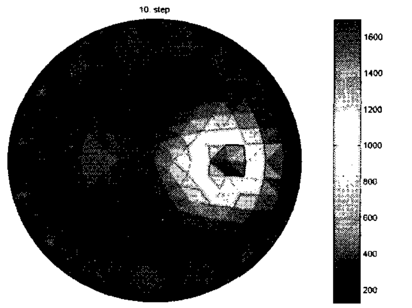
We have conducted extensive numerical and phantom experiments to verify the performance of each algorithm. Detailed description of phantom and experimental conditions can be found in [9]. It is worthwhile to mention so-called the *inverse crime* that means inadvertent cancellation of numerical error when the same mesh structure is used in the forward and the inverse solutions. In order to avoid the inverse crime, the mesh structure for the inverse solution should be different from that for the forward problem. The mesh structure used to generate the synthesized data of injected current patterns and corresponding boundary voltages for numerical experiments should also differ from that used for the inverse problem. A coarse mesh is considered for the inverse solution to reduce the number of unknowns, that is computational burden. This mesh structure corresponds to the spatial resolution of 1/16.

3.1 Mesh Grouping

Figs. 3 and 4 show the reconstructed images by the modified Newton-Raphson (mNR) and the mesh grouping based on the data generated from phantom experiments, respectively. It is estimated that the errors involved in the generation of injected currents and in the measurement of boundary voltages for homogeneous medium are maintained less than 2%. In Figs. 3 (a) and 4 (a), a cylindrical acryl rod is located at the center. This example seems to be very simple, but it is quite illustrative since the farther the target is located from the boundary electrodes, the less sensitive the boundary voltages are to the resistivity changes introduced during the inverse solution. Figs. 3 (b) and 4 (b) consider a target located off the center, toward the boundary electrodes. Figs. 3 and 4 clearly indicates that the mesh grouping can improve the image quality significantly although some blurs around the target are inevitable due to mismatch of mesh structure and phase boundary as well as measurement noise.

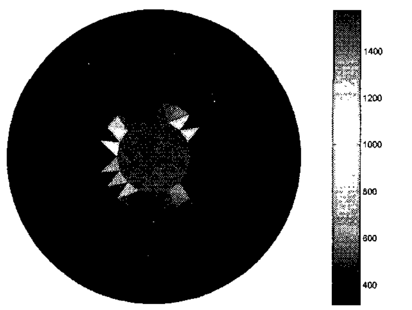


(a)

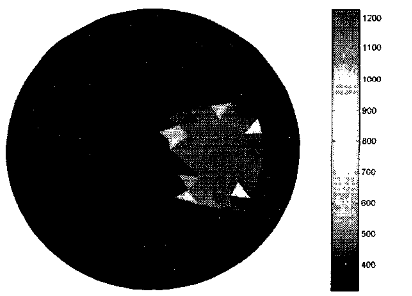


(b)

Fig. 3. Reconstructed images of a cylindrical acryl rod by using the mNR grouping algorithm: (a) at the center and (b) off the center.



(a)



(b)

Fig. 4. Reconstructed images of a cylindrical acryl rod by using the mesh grouping algorithm: (a) at the center and (b) off the center.

3.2 Boundary Estimation

For the verification of the algorithms developed for the boundary estimation, we conducted extensive numerical experiments although only a few results are presented in this paper. Phase boundaries are expressed in terms of truncated Fourier series. We assume targets to be circular or elliptic, so the dimension of Fourier series is set to $N_\theta = 3$. Two kinds of algorithms are used to determine the Fourier coefficients: mNR method and multilayer neural network (MNN). As can be seen in Fig. 5, both of the mNR and the MNN show good performance in the determination of the unknown Fourier coefficients even when measurement error of 1% is considered. However, numerical experiments show that the MNN has better performance in treating measurement error than the mNR.

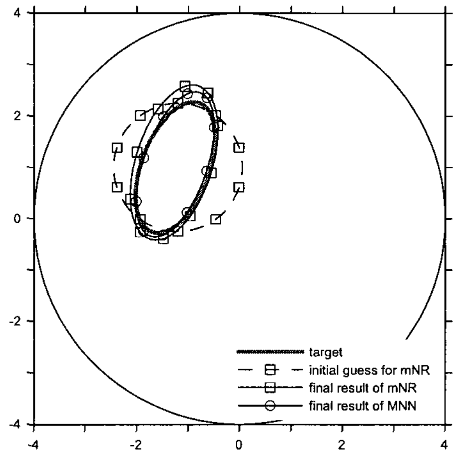


Fig. 5. Estimated boundary shape by MNN and mNR with 1% noise (γ = [-1.25, 0.75, 0.20, 1.00, 0.25, 1.25]).

4. CONCLUSION

In the paper, two promising approaches of electrical resistance tomography (ERT) technique to the visualization of two-phase flows were described [10]. Mesh grouping algorithm to enhance computational performance and image quality, and boundary estimation algorithm to determine the phase boundary directly were introduced and experimental results from numerical simulations and phantom experiments were presented. Even with up-to-date algorithms introduced in this paper, the spatial resolution of reconstructed images by the ERT should be improved further and the improvement will be quite challenging. As shown in the above, however, the ERT can give some information on the approximate location and size of target (e.g., bubble) and it seems to be promising at least in monitoring two-phase flows.

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