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Are the Primary School Teachers of the Future Ready to Solve the Word Problems without Algebra?

Tatar, Enver
Department of Primary Teacher Education, Agri Education Faculty,
Ataturk University, 04100-Agrı, Turkey; Email: entatar@atauni.edu.tr

Isleyen, Tevfik
Department of Secondary Science and Mathematics Education, Kazim Karabekir Education Faculty,
Ataturk University, 25240-Erzurum, Turkey; Email: tisleyen@atauni.edu.tr

Okur, Muzaffer
Department of Primary Mathematics Education, Kazim Karabekir Education Faculty,
Ataturk University, 25240-Erzurum, Turkey; Email: mokur@atauni.edu.tr

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The aim of this study is to investigate future teachers' skills that can make problem solving methods concrete for 7–11 year old students. For the students in the concrete operations level, solutions of word problems should also be taught by concreting. But most of teacher candidates can not solve the problems without algebra because they got used to solve the word problems with algebra during their high school and university education. In this study, whether the teacher candidates have the skills of solving the primary school level problems without using algebra or not are being observed. At the end of this observation it is determinated that primary level teacher candidates generally prefer using algebra operations because of their former habits. The results show that in the education of the primary level teacher candidates, there is the need of developing the solving skills using figures and diagrams without algebra rather than algebraic solutions in word problems.

Keywords: problem solving, word problem, primary teacher education, problem solving skills.
ZDM Classification: B59, D59
MSC2000 Classification: 97B50, 97D50

INTRODUCTION

What is the mathematics? This question can be answered in many ways by the people based upon their knowledge and major areas. It is known that many students were scared of mathematics classes and they were not interested in doing anything related to mathematics. The factors such as teachers and educational systems of the schools create mathematical knowledge gap on the students.

Especially, mathematics for students just started to primary school needs to be taught with interesting real world example problems. Teachers should motivate the students who may not have many mathematical experiences and show them a sense of what lies beyond simple arithmetical operations such as addition, subtraction, multiplication, and division. For instance, "180 students want to take a trip by a bus. The bus can carry only 50 students. How many buses do they need for this trip?" If division is applied to solve this problem, answer will be 180 / 50 = 3.6. On the other hand, the answer cannot be 3.6. Not only do teachers teach the simple division operation to the students, but also they need to show the sense of what lies beyond this arithmetical operation. It is important to apply the sense of mathematical thinking to those types of questions. Understanding the problem is main point to start for solving it (Barb & Quinn 1977). Mathematics instruction has been characterized to a large extent by its emphasis on rote memorization of facts and computational skills, rather than on developing important concepts and applying mathematics to real-world problem situations (Baroody & Hume 1991; Bottge 1999; Parmar, Cawley & Miller 1994; Woodward & Montague 2000).

Problem solving is one of the pieces of mathematical education. In one of Polya's studies the skill of problem solving was assumed to be a center of mathematical education Polya's book, named "How to Solve it", is one of main sources for problem solving techniques in mathematical education. One of main duties of mathematics teacher in this book is to help the students improve their abilities of problem solving (Polya 1957). Later on, most of the mathematical educators also emphasized the importance of problem solving skills in the mathematical education (*e.g.*, De Corte, Greer & Verschaffel 1996; Goldman, Hasselbring & the Cognition and Technology Group at Vanderbilt, 1997; Patton, Cronin, Bassett & Koppel 1997). The teachers should help the students in order to improve their problem solving skills and problem solving ability should be the main goal of mathematical education (Charles & Lester 1982). Belge (1979) also agreed with the mathematical educators on the problem solving skills that is one of the important compartment of the mathematical education. The main advices for mathematical educators are given as follows (NCTM, An Agenda for Action 1980).

1) Problem solving ability or skills should be main objective of mathematical educations
2) Many example problems should be in the mathematical textbooks. Each subject needs to be well explained by example problems.
3) The example problems in the textbooks need to be represented by interesting real world problems.

Problem solving is not answering the questions given to the students. Problem solving is one of the thinking methods, knowledge comparisons, using all mathematical activities, and applying procedural and conceptual knowledge on the problems (Barb & Quinn 1997; İsleyen & Isik 2003). One of the study showed that there was a relationship between problem solving skills and applying the skills to overcome their problems faced with every day in their lives (Saleh 1999).

Mathematics is known as an abstract lecture. Students do not easily understand abstruse problems and ideas. The difficulties can be reduced by real world example problems during Mathematical lecture (Baykul 1999). Petit & Zowojewski (1999) showed that an appropriate method applied for mathematics classes by the teachers could decrease the difficulty levels of problems given to the students. The appropriate method must be selected based upon the students' mental development and ages. The students whose ages are between 7 and 11 are accepted in concreted operational periods by Jean Piaget studied children's mental developments (Albayrak 2000). The solution of mathematical problems needs to be explained to the students (7–11 year olds) using an appropriate method. One of the most common methods is to draw an appropriate picture and diagram related to the question. Especially in Turkey, teachers use only equations/formulas in order to solve the problems during their high school and undergraduate educations. Therefore, the teacher candidates prefer to use only equations to solve the problems since they are not good at solving the same problems by drawing pictures and diagrams. The reasons why the teachers use only equations for answering the questions are:

Time is so important during their examinations. The main purposes of the students are to find the correct answer in a short time. Students will not have enough time to draw a picture for the question. For example, 180 questions are given to students in the National University Exam (Selection and Placement of Students in Higher Education Institutions in Turkey). The students have only 60 second to answer each question. During this example, the students cannot use calculators. There is no break allowed for the students during this examination.

Examinations are graded based on correct answers. If students do not find the correct answers for the questions, the grade for the questions will be zero even if the students make simple mistakes at the end of the solutions. Their efforts during the examinations are not graded unless there is a correct answer.

For the problems given in a mathematics class, the teachers need to solve them not only using equations but also drawing pictures. Applying only straightforward equations to solve the problem may not be enough for the students to understand the solutions. For example, if the problem consists of some unknowns as *x, y, a, b* etc. while the teacher is solving the problem using only equations, it will not be enough for primary school students to understand the problem. The solution method using straightforward equation is not appropriate method for primary students who have not finished their mental developments. Thus, the teacher must use a reasonable approach to explain the solution of the problems to the students and their ages must be considered to determine the approach. Sometimes, the teachers prefer to use only equations/formulas in order to solve mathematics problems in primary schools and the teachers do not draw any picture to explain the problems to the students. Conversely, solutions of the problems with pictures (drawing picture or photographs) and diagrams are well understood by the students in primary school (Altun 2001). One of the studies showed that the students were trying to solve the questions by using simple arithmetic's operations and figures and diagrams instead of using equations even if solutions using equations were taught to them during their educations (Greer 1997). Therefore, primary school teacher candidates during their undergraduate education are accustomed to solving mathematics problem with simple mathematical (arithmetic's) operations instead of equations.

Mandatory education in Turkey is 8 years. Children whose ages are 7 years old have to start to go to primary schools. The first five years and the following 3 years of the mandatory education are called as "Primary School Education-Step I" (PSES-I) and "Primary School Education-Step II" (PSES-II), respectively. Only one teacher teaches all of the classes in PSES-I. To be a teacher for PSES-I is required to have an undergraduate degree from school of education (Education Faculty). After having 4 years undergraduate degree from school of education (if they passed all the classes), students from school of educations in Universities can be a teacher for PSES-I. Each class in PSES-II is given by a teacher majored in the specific field. For instance, a teacher who graduated from mathematical education from the universities teaches mathematics classes. In this study, the arithmetic and algebra word problem solving skills of future teachers for PSES-I has been investigated.

METHODOLOGY

Objective:

The aim of this study was to investigate solving skills in word problems of teacher candidates of Primary School Education Step I. The teacher candidates were asked to solve the questions given during this research both with and without equations and formulas.

Problem:

World problems need to be taught to the students whose ages are 7 to 11 by a concretized manner. Therefore, the teacher for PSES-I should apply mathematical operations in order to solve word problems without using equation in the mathematics class. In this study, we investigated whether teacher candidates use equation when they are solving word problems.

Universe and Sample

86 PSES-I teacher candidates studying at the School of Education at Ataturk University in Agri, in Turkey were used for the study. All teacher candidates were studying as fourth-year students would be a PSES-I teachers in the following year.

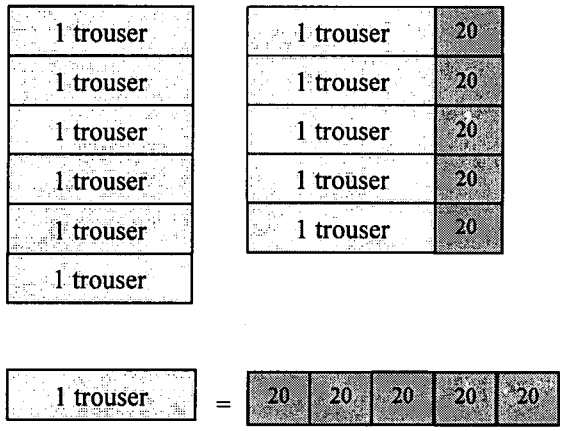
Data Collection and Analysis:

Teacher candidates must perform Teaching Practice Lesson when they are studying at the last year of their undergraduate education in Turkey. In the first three years of undergraduate education, these students are educated in how they will solve word problems with using no equation when they practice. One author of the present study (Enver Tatar) observed whether candidates solved word problems in usual manner such as using diagram or schema.

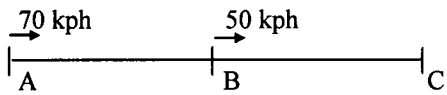
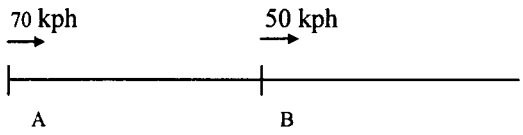
Investigators prepared 5 word problems and asked to candidates. All questions were so simple and they could also be solved by using no equation. It is asked to the teacher candidates to solve each question both with and without using algebra. Questions and their solutions are as follows:

Test Items:

Question 1: A tailor had received an order to make 6 pants supposed to be same size and leg-length. The tailor used certain amount of cloth to make the pants. If the tailor increased the leg-lengths of the pants 20 cm, He would have made 5 pants using the clothes used for 6 pants. Then, what is the length of a trouser?

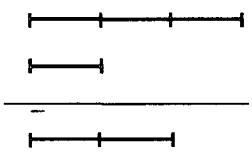
Algebraic Solution	Solution without Algebra
Let length of a trouser be x cm. $6x = 5(x+20)$ $6x = 5x+100$ $6x-5x = 100$ $x = 100$ cm.	 1 length of a trouser is 5 × 20 = 100cm.

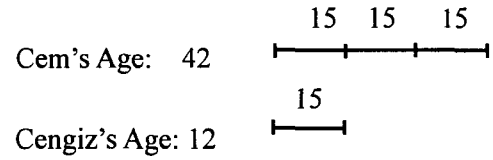
Question 2: Suppose a truck leaves Erzurum, which is about 100 kilometers (km) from Erzincan at 10:00 am driving 70 kilometer per hour (kph) towards Ankara. At 10:00 am a bus leaves Erzincan driving at 50 kilometer per hour (kph) toward Ankara. How many hours later will the truck catch up with the bus? (The bus and truck are going in the same direction)

Algebraic Solution	Solution without Algebra
Suppose the truck catches the bus t hour(s) later. $V_1 = 70$ kph $V_2 = 50$ kph $t = ?$ 	 A picture technique based on the formula "distance = rate × time" can be used for solving this problem. The first question that needs to be asked is "are the vehicles in the same direction or are they going towards each other from opposite ends of the road?" The vehicles start to travel at the same time in the same direction. We can re-

$t = \frac{100 + \lvert BC \rvert}{V_1} = \frac{\lvert BC \rvert}{V_2}$ $\frac{100 + \lvert BC \rvert}{70} = \frac{\lvert BC \rvert}{50}$ $500 + 5 \cdot \lvert BC \rvert = 7 \cdot \lvert BC \rvert$ $500 = 2 \cdot \lvert BC \rvert$ $\lvert BC \rvert = 250\,\text{km}$ $t = \frac{\lvert BC \rvert}{V_2} = \frac{250}{50} = 5\,\text{hours}$	read the problem and ask ourselves questions based on "distance = rate*time", that is we start by asking questions about the distance traveled for each vehicle. Do we know how far the bus and truck travel? The truck and the bus travel at 70 and 50 kilometers per hour, respectively. The distance between the truck and the bus will decrease only 20 kilometers per hour: 70 km – 50 km = 20km. Since initial distance between the vehicles is 100 km, the truck had to travel 100 km more in order to catch up with the bus. Based on this explanation, the truck will catch up with the bus after 5 hours (100 kilometers/ 20 kilometers/hr).

Question 3: Cem is 42 years old. Cengiz is 12 years old. How many years from now will Cem be three times as old as Cengiz's present age?

Algebraic Solution	Solution without Algebra
$42 + x = 3 \cdot (12 + x)$ $42 + x = 36 + 3x$ $2x = 6$ $x = 3$ years	Now Later Cem's Age : 42 Cengiz's Age : 12 30  We can start to draw a diagram for this problem. The diagram needs to be divided into three equal pieces Age difference is 42 – 12 = 30 years. Since this 30 – year age difference will be constant, the two pieces of the diagram will equal to 30. Therefore, one of the pieces equals to: 30 / 2 = 15

	 15 years – 12 years = 3 years

Question 4: There are only chickens and dogs on a farm. The number of chickens and dogs together is 10. The number of their legs is 34. How many chickens and how many dogs are there on the farm?

Algebraic Solution	Solution without Algebra
Let the number of dogs be x. $4x + 2 \cdot (10 - x) = 34$ $4x + 20 - 2x = 34$ $2x = 14$ $x = 7$ dogs $10 - 7 = 3$ chickens	Each chicken has two legs and each dog has four, together the 10 animals have 34 legs. If the number of chicken's legs is assumed as 4, we will add 2 extra legs to each chicken. Therefore, the number of legs would be 40 (10 × 4 = 40) for all animals. Since the number of the legs on the farm is given as 34, 6 legs (40 – 34 = 6) are added up by us based on the assumption. As each chicken has 2 legs, the number of chickens can be found as: $6 / 2 = 3$ The number of dogs can be calculated by subtracting the number of chickens from the total number of the animals: The number of dogs: $10 - 3 = 7$

Question 5: A child has yellow, blue, and red marbles. One-sixth, two-third, and 5 of them are yellow, blue, and red respectively. Find the number of the marbles the kid has?

Algebraic Solution	Solution without Algebra
Let the number of marbles be x. $\frac{x}{6} + \frac{2x}{3} + 5 = x$	There are many different ways to solve this problem: you could draw a picture to represent the number of the marbles, and the boxes can be colored based on the information given in the

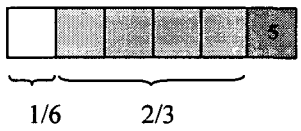
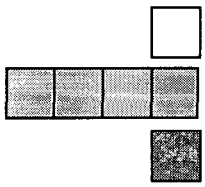
$\frac{5x}{6}+5=x$ $\frac{x}{6}=5$ $x=30$ marbles	question. Here is a drawing for the problem using colors:  1/6 2/3  Yellow: $1\times5=5$ Blue: $4\times5=20$ Red: $+\ 5$ 30 marbles

Table 1. The results of the test given to primary school teacher candidates

	Algebraic Solution N = 86		Solution without Algebra N=86	
	Correct answer (F)	Percentage (%)	Correct answer (F)	Percentage (%)
Question 1	72	84	11	13
Question 2	66	77	7	8
Question 3	70	81	5	6
Question 4	67	78	4	5
Question 5	76	88	14	16

Table 1 show that the primary school candidates are more successful in solving the problems with algebra. The number of teacher candidates that solve especially the questions 2, 3 and 4 without algebra is very low. Most of the primary school teacher candidates were not able to answer correctly the questions on the test unless they used straight forward equation to solve them. We also interviewed the primary school teacher

candidates who used the straight forward equation not the diagrams or schema to find the correct answers. Table 2 summarizes the reasons why the candidates were not able to apply simple mathematical operations to solve the problems.

Table 2. Candidate's opinions on failure

To solve the problem with drawing a picture or diagram takes time and to use straight forward equation for solution of the problem is so easy
They have been using equations to solve the problem during their undergraduate education. Therefore, they are not accustomed to use only simple mathematical operations or pictures of solutions of the problems.
Using figures and diagrams to solve the problems are time consuming processes.
Most of them were not ready to apply figures and diagrams in the solutions of the problems.
Only way to solve some problems is to use equations instead of figures and diagrams.

Results and Discussions:

The teachers of Primary School Education-Step I" (PSES-I) are responsible for the education of students 7–11 years old in Turkey. The teachers of PSES-I should use an appropriate problem solving method based on the student's ages. This study showed that future primary school teachers were not into using figures and diagrams to solve the problems. Most of the future primary school teacher candidates were not draw any figures or diagrams in order to solve the problems. They used straight forward equations to answer the questions and no explanations were found in their solutions. Their solutions were not suitable for any students whose age were 7–11 years old. Problem solving method using pictures and diagrams is required to be taught to the teacher candidates during their undergraduate educations. The following main conclusions are suggested by this study:

- Mathematical educations should focus on problem solving skills instead of theory. The solutions using only theoretical knowledge are not suitable for the students 7-11 years old.
- The future primary school teachers need to be educated with mathematics classes having many real world example problems. Teachers should have a proficiency in problem solving skills and they should know many ways to explain the problems to the students considering their age factors.
- To draw a picture or diagram in order to solve problems is time consuming process in

primary school education. However if the teachers explain the problems with figures and diagrams, the problems will be better understood by the primary school students.

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