The Effect of Suction and Injection on Unsteady Flow of a Dusty Conducting Fluid in Rectangular Channel

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In the present study, the unsteady Hartmann flow of a dusty viscous incompressible electrically conducting fluid under the influence of an exponentially decreasing pressure gradient is studied without neglecting the ion slip. The parallel plates are assumed to be porous and subjected to a uniform suction from above and injection from below. The fluid is acted upon by an external uniform magnetic field which is applied perpendicular to the plates. An analytical solution for the governing equations of motion is obtained to yield the velocity distributions for both the fluid and dust particles.

Key Words : Fluid Mechanics, Magnetofluid Mechanics, Two Phase Flow, Dust Particles, Hall Current, Ion Slip

1. Introduction

The study of the flow of dusty fluids has been considered by many researchers due to its wide applications in the fields of fluidization, combustion, use of dust in gas cooling systems, centrifugal separation of matter from fluid, petroleum industry, purification of crude oil, electrostatic precipitation, polymer technology, and fluid droplet sprays. The hydrodynamic flow of dusty fluids was studied by a number of authors (Saffman, 1962 ; Gupta et al., 1976 ; Prasad et al., 1979 ; Dixit, 1980 ; Ghosh et al., 1984 ; Kim, 2002 ; Ahn et al., 2002). Later the hydromagnetic flow of dusty fluids was studied (Singh, 1976 ; Mitra et al., 1981 ; Borkakotia et al., 1983 ; Megahed et al., 1988 ; Aboul-Hassan et al., 1991 ; Chamkha, 1997, 2000 ; Attia, 2002 ; Seddeek, 2002, 2003). In the above mentioned work the Hall and ion slip terms were ignored in applying Ohm's law. In fact, the Hall effect is important when the Hall parameter, which is the ratio between the electron-cyclotron frequency and the electron-atom-collision frequency is high (Crammer et al., 1973). This happens when the magnetic field is high or when the collision frequency is low (Crammer et al., 1973). Furthermore, the masses of the ions and electrons are different and, in turn, their motions will be different. Usually, the diffusion velocity of electrons is larger than that of ions and, as a first appriximation, the electric current density is determined mainly by the different velocity of the electrons. However, when the electromagnetic force is very large (such as in the case of strong magnetic field), the diffusion velocity of the ions may not be negligible (Crammer et al., 1973). If we include the diffusion velocity of ions as well as that of electrons, we have the phenomena of ion slip. In the above mentioned work, the Hall and ion slip terms were ignored in applying Ohm's law, as they have no marked effect for small and moderate values of the magnetic field. However, the current trend for the application of magnetohydrodynamics is towards a strong magnetic field, so that the influence of electromagnetic force is noticeable under these conditions, and the Hall current and the ion slip

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are important and they have a marked effect on the magnitude and direction of the current density and consequently on the magnetic force term (Crammer et al., 1973). The effect of the Hall current or the ion slip on the Hartmann flow of a clean fluid was studied by a number of authors (Sutton et al., 1965; Soundalgekar et al., 1979, 1986; Attia, 1998, 2002). Aboul-Hassan and Attia (2002) studied the influence of the Hall current on the flow of a dusty conducting fluid in a rectangular channel neglecting the ion slip.

In the present work, the transient Hartmann flow of an electrically conducting, viscous, incompressible, dusty fluid is studied with the consideration of both the Hall current and ion slip. The fluid is flowing between two infinite electrically insulating porous plates. The fluid is subjected to a uniform suction from above and a uniform injection from below. An external uniform magnetic field is applied perpendicular to the plates while the induced magnetic field is neglected by assuming a very small magnetic Reynolds number. The fluid is acted upon by an exponentially decaying with time pressure gradient. This configuration is a good approximation of some practical situations such as flow meters and pipes that connect system components. The equations of motion are solved analytically using the method of Laplace Transform to obtain the velocity distributions for both the fluid and dust particles as functions of space and time.

2. Description of the Problem

The dusty fluid is assumed to be flowing between two infinite horizontal electrically nonconducting plates located at the $y=\pm h$ planes, as shown in Fig. 1. The plates are subjected to a uniform suction from above and a uniform injection from below. Thus the y-component of the velocity of the fluid is assumed constant and denoted by v_o. The dust particles are assumed to be spherical in shape and uniformly distributed throughout the fluid. A uniform pressure gradient, which is taken to be an exponentially decaying with time, is applied in the horizontal direction. A uniform magnetic field $\mathbf{B}_o$ is applied

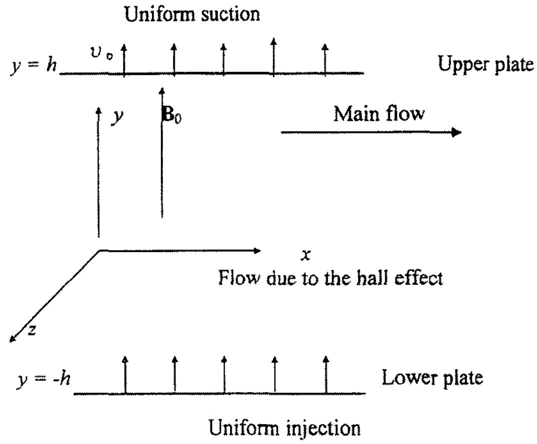


Fig. 1 The geometry of the problem

in the positive y-direction. This is the only magnetic field in the problem as the induced magnetic field is neglected by assuming a very small magnetic Reynolds number (Crammer et al. 1973). The fluid motion starts from rest at $t=0$, and the no-slip condition at the plates implies that the fluid and dust particles velocities have neither a z nor an x-component at $y=\pm h$. It is required to obtain the time varying velocity distributions for both fluid and dust particles. Due to the inclusion of the Hall current term, a z-component of the velocities of the fluid and of dust particles is expected to arise. Since the plates are infinite in the x and z-directions, the physical quantities do not change in these directions that is $\partial/\partial x=\partial/\partial z=0$ and the problem is essentially one-dimensional.

3. The Velocity Distribution

The flow of fluid is governed by the momentum equation

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{V}.\nabla)\mathbf{V}\right) = -\nabla P+\mu\nabla^2\mathbf{v}+\mathbf{J}x\mathbf{B}_o-KN(\mathbf{v}-\mathbf{v}_p) \tag{1}$$

where ρ is the density of clean fluid, μ is the viscosity of clean fluid, $\mathbf{v}$ is the velocity of the fluid $=u(y,\ t)\mathbf{i}+v_o\mathbf{j}+w(y,\ t)\mathbf{k}$, $\mathbf{v}_p$ is the velocity of dust particles $=u_p(y,\ t)\mathbf{i}+w_p(y,\ t)\mathbf{k}$, $\mathbf{J}$ is the current density, N is the number of dust particles per unit volume, K is the Stokes cons tant $=6\pi\mu a$, and "a" is the average radius of dust particles.

The first three terms in the right-hand side of Eq. (1) are, respectively, the pressure gradient, viscosity, and Lorentz force terms. The last term represents the force due to the relative motion between fluid and dust particles. It should be noted that in the present work the hydrodynamic interactions between the phases are limited to the drag force. This assumption is feasible when the Reynolds number of the relative velocity is small (Saffman, 1962; Chamkha, 1997). Other interactions such as the virtual mass force, the shear force, and the spin-lift force are assumed to be negligible compared to the drag force (Chamkha, 1997).

If the Hall and ion slip terms are retained, the current density $\mathbf{J}$ from the generalized Ohm's law is given by (Crammer et al., 1973; Sutton et al., 1965)

$$\mathbf{J}=\sigma\left[\mathbf{v}x\mathbf{B}_o-\beta_e(\mathbf{J}x\mathbf{B}_o)+\frac{\beta_e\beta_i}{B_o}(\mathbf{J}x\mathbf{B}_o)x\mathbf{B}_o\right] \quad (2)$$

where σ is the electric conductivity of the fluid, β_e is the Hall factor and β_i is the ion slip parameter (Crammer et al., 1973; Sutton et al., 1965). Solving Eq. (2) for $\mathbf{J}$ gives

$$\mathbf{J}x\mathbf{B}_o=\frac{\sigma B_o^2}{(1+\beta_i\beta_e)^2+\beta_e^2}[((1+\beta_i\beta_e)u+\beta_e w)\mathbf{i} + ((1+\beta_i\beta_e)w-\beta_e u)\mathbf{k}] \quad (3)$$

where $\beta_e=\sigma\beta B_o$, is the Hall parameter (Crammer et al., 1973; Sutton et al., 1965). Thus, in terms of Eq. (3), the two components of Eq. (1) read

$$\rho\frac{\partial u}{\partial t}+\rho v_o\frac{\partial u}{\partial y}=\frac{dP}{dx}+\mu\frac{\partial^2 u}{\partial y^2} -\frac{\sigma B_o^2}{(1+\beta_i\beta_e)^2+\beta_e^2}((1+\beta_i\beta_e)u+\beta_e w)-KN(u-u_p) \quad (4)$$

$$\rho\frac{\partial w}{\partial t}+\rho v_o\frac{\partial w}{\partial y}=\mu\frac{\partial^2 w}{\partial y^2} -\frac{\sigma B_o^2}{(1+\beta_i\beta_e)^2+\beta_e^2}((1+\beta_i\beta_e)w-\beta_e u)-KN(w-u_p) \quad (5)$$

The motion of the dust particles is governed by Newton's second law applied in the x and z-directions

$$m_p\frac{\partial u_p}{\partial t}=K(u-u_p) \quad (6)$$

$$m_p\frac{\partial w_p}{\partial t}=K(w-w_p) \quad (7)$$

where m_p is the average mass of dust particles. It is assumed that the pressure gradient is applied at $t=0$ and the fluid starts its motion from rest. Thus,

$$u=u_p=w=w_p=0 \text{ at } t\leq 0$$

The no-slip condition at the plates implies that

$$u=u_p=w=w_p=0 \text{ at } y=\pm h, \text{ for } t>0$$

The problem is simplified by writing the equations in the non-dimensional form. The characteristic length is taken to be h, and the characteristic time is $\rho h^2/\mu$ while the characteristic ve locity is $\mu/h\rho$. We define the following non-dimensional quantities

$$(\hat{x},\hat{y},\hat{z})=(x,y,z)/h$$

$$\hat{t}=t\mu/\rho h^2,\ \hat{P}=P\rho h^2/\mu^2$$

$$(\hat{u},\hat{v},\hat{w})=(u,v,w)\rho h/\mu$$

$$(\hat{u}_p,\hat{v}_p,\hat{w}_p)=(u_p,v_p,w_p)\rho h/\mu$$

$S=\rho h v_o/\mu$ is the suction parameter
$G=m_p\mu/\rho h^2 K$ is the particle mass parameter
$Ha^2=\sigma B_o^2 h^2/\mu$ is the Hartmann number squared
$R=KNh^2/\mu$ is the particle concentration parameter.

In terms of the above non-dimensional quantities the velocity equations read

$$\frac{\partial u}{\partial t}+S\frac{\partial u}{\partial y}=-\frac{dP}{dx}+\frac{\partial^2 u}{\partial y^2} -\frac{Ha^2}{(1+\beta_i\beta_e)^2+\beta_e^2}((1+\beta_i\beta_e)u+\beta_e w) -R(u-u_p) \quad (8)$$

$$\frac{\partial w}{\partial t}+S\frac{\partial u}{\partial y}=\frac{\partial^2 w}{\partial y^2}-\frac{Ha^2}{(1+\beta_i\beta_e)^2+\beta_e^2}((1+\beta_i\beta_e)w-\beta_e u) -R(w-w_p) \quad (9)$$

$$G\frac{\partial u_p}{\partial t}=u-u_p \quad (10)$$

$$G\frac{\partial w_p}{\partial t}=w-w_p \quad (11)$$

where the hats are dropped for convenience. The above four equations may be reduced to two coupled complex equations by introducing the quantities $V=u+iw$ and $V_p=u_p+iw_p$. To perform this we multiply Eq. (9) by i and add it to Eq. (8) and multiply Eq. (11) by i and add it to Eq. (10) and get after rearrangement

$$\frac{\partial^2 V}{\partial y^2}-S\frac{\partial V}{\partial y}-\frac{Ha^2((1+\beta_i\beta_e)-i\beta_e)}{(1+\beta_i\beta_e)^2+\beta_e^2}V-\frac{\partial V}{\partial t}-R(V-V_p)=\frac{dP}{dx} \quad (12)$$

$$G\frac{\partial V_p}{\partial t}=V-V_p \quad (13)$$

with the initial and boundary conditions, $V=V_p=0$ at $t\le 0$, $V=V_p=0$ at $y=\pm 1$, for $t>0$.

Equations (12) and (13) may be solved using the method of Laplace Transform (LT) (Spiegel, 1986) to obtain V and V_p as functions of y and t. The real part of V or V_p represents the x-component of the velocity while the imaginary part represents the z-component. Taking LT with respect to time of Eqs. (12) and (13) we have

$$\frac{d^2\bar{V}}{dy^2}-S\frac{d\bar{V}}{dy}-A\bar{V}-s\bar{V}-R(\bar{V}-\bar{V}_p)=-F(s) \quad (14)$$

$$Gs\bar{V}_p=\bar{V}-\bar{V}_p \quad (15)$$

where $\bar{V}=\bar{V}(y, s)$ and $\bar{V}_p=\bar{V}_p(y, s)$ are respectively, the LT of $V(y, t)$ and $\bar{V}_p(y, t)$, $A=Ha^2((1+\beta_i\beta_e)-i\beta_e)/((1+\beta_i\beta_e)^2+\beta_e^2)$, and $-F(s)$ is the LT of the pressure gradient. V and V_p must satisfy the boundary conditions, $V=V_p=0$ at $y=\pm 1$. All the bars will be dropped for convenience. Eliminating V_p gives

$$\frac{d^2V}{dy^2}-S\frac{dV}{dy}-K_1V=-F(s) \quad (16)$$

where $K_1\equiv K_1(s)=A+s+R(1-1/(1+Gs))$.

The solution of the above equation gives

$$V(y, s)=F(s)/K_1\left(1-\frac{\cosh(qy)}{\cosh(q)}\right)$$

and from Eq. (12) we obtain

$$V_p(y, s)=F(s)/K_2\left(1-\frac{\cosh(qy)}{\cosh(q)}\right)$$

where $q^2=(S+4K_1)/4$ and $K_2=K_1(1+Gs)$. Using the complex inversion formula and the residue and convolution theorems (Spiegel, 1986), the inverse transforms of $V(y, s)$ and $V_p(y, s)$ are given as

$$\begin{aligned}V(y, t)=C\sum_{n=1}^{\infty}\Big(&\frac{PI1}{PN1+a}(\exp(PN1xt)-\exp(-at))\\&+\frac{PI2}{PN2+a}(\exp(PN2xt)-\exp(-at))\\&+\frac{PI3}{PN3+a}(\exp(PN3xt)-\exp(-at))\\&+\frac{PI4}{PN4+a}(\exp(PN4xt)-\exp(-at))\Big)\end{aligned} \quad (17)$$

$$\begin{aligned}V_p(y, t)=C\sum_{n=1}^{\infty}\Big(&\frac{PIP1}{PN1+a}(\exp(PN1xt)-\exp(-at))\\&+\frac{PIP2}{PN2+a}(\exp(PN2xt)-\exp(-at))\\&+\frac{PIP3}{PN3+a}(\exp(PN3xt)-\exp(-at))\\&+\frac{PIP4}{PN4+a}(\exp(PN4xt)-\exp(-at))\Big)\end{aligned} \quad (18)$$

where

$-dP/dx=C\exp(-at)$

$$PN1=\frac{NN1xG-B+\sqrt{(NN1xG-B)^2-4xGx(A-NN1)}}{2xG}$$

$$PN2=\frac{NN1xG-B-\sqrt{(NN1xG-B)^2-4xGx(A-NN1)}}{2xG}$$

$$PN3=\frac{NN2xG-B+\sqrt{(NN2xG-B)^2-4xGx(A-NN2)}}{2xG}$$

$$PN4=\frac{NN2xG-B-\sqrt{(NN2xG-B)^2-4xGx(A-NN2)}}{2xG}$$

$PI1=NN3/KPN1xDPN1$

$PI2=NN3/KPN2xDPN2$

$PI3=NN4/KPN3xDPN3$

$PI4=NN4/KPN4xDPN4$

$PIP1=PI1/(1+GxPN1)$

$PIP2=PI2/(1+GxPN2)$

$PIP3=PI3/(1+GxPN3)$

$PIP4=PI4/(1+GxPN4)$

$B=1+G(A+R)$

$NN1=-\Pi^2(n-1)^2-S^2/4$

$NN2=-\Pi^2(n-0.5)^2-S^2/4$

$NN3=2\Pi(-1)^n(n-1)\exp(Sy/2)\sinh(S/2)\sin(\Pi(n-1)y)$

$NN4=2\Pi(-1)^{n+1}(n-0.5)\exp(Sy/2)\cosh(S/2)\cos(\Pi(n-0.5)y)$

$$KPN1=\frac{GxPN1^2+BxPN1+A}{1+GxPN1}$$

$$KPN2=\frac{GxPN2^2+BxPN2+A}{1+GxPN2}$$

$$KPN3=\frac{GxPN3^2+BxPN3+A}{1+GxPN3}$$

$$KPN4=\frac{GxPN4^2+BxPN4+A}{1+GxPN4}$$

$$DPN1=\frac{(1+GxPN1)x(2xGxPN1+B)-Gx(GxPN1^2+BxPN1+A)}{(1+GxPN1)^2}$$

$$DPN2=\frac{(1+GxPN2)x(2xGxPN2+B)-Gx(GxPN2^2+BxPN2+A)}{(1+GxPN2)^2}$$

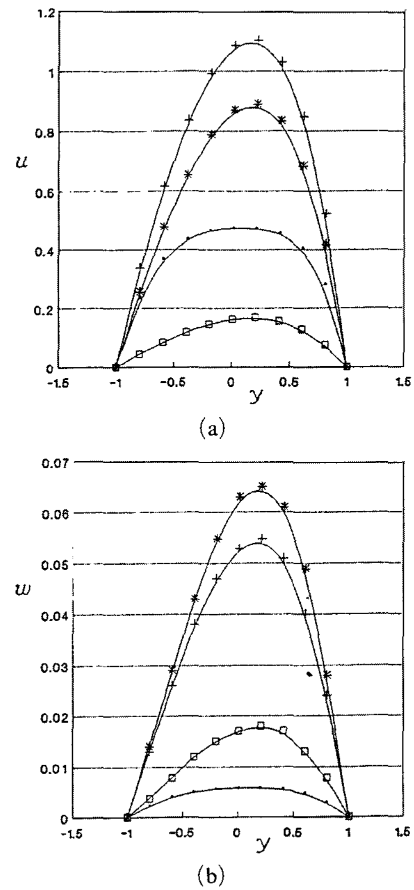
$$DPN3=\frac{(1+GxPN3)x(2xGxPN3+B)-Gx(GxPN3^2+BxPN3+A)}{(1+GxPN3)^2}$$

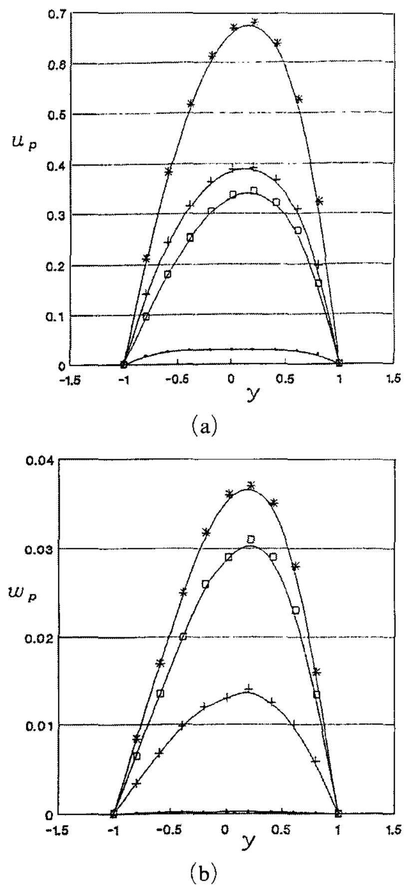
$$DPN4=\frac{(1+GxPN4)x(2xGxPN4+B)-Gx(GxPN4^2+BxPN4+A)}{(1+GxPN4)^2}$$

The summation parameter n and the coordinate y are included in the above quantities. It should be noted that the general exact results obtained herein reduce to those reported by Aboul-Hassan et al.(2002) when $\beta_i=0$. Also, the solutions reported by Attia (1998) are reproduced by setting $K=0$ and $\beta_i=0$ in the present results. While comparisons with previously published theoretical work on this problem were performed, no comparisons with experimental data were done because, as far as the author is aware, such data are lacking at the present time.

4. Results and Discussion

Figures 2 and 3 present, respectively, the pro-



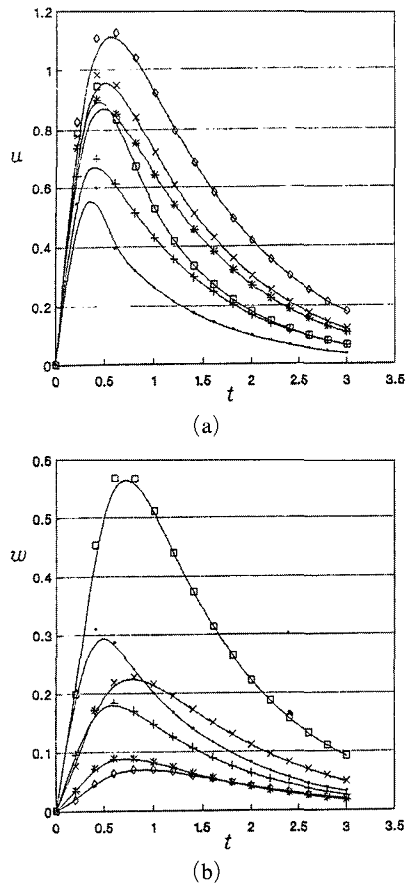


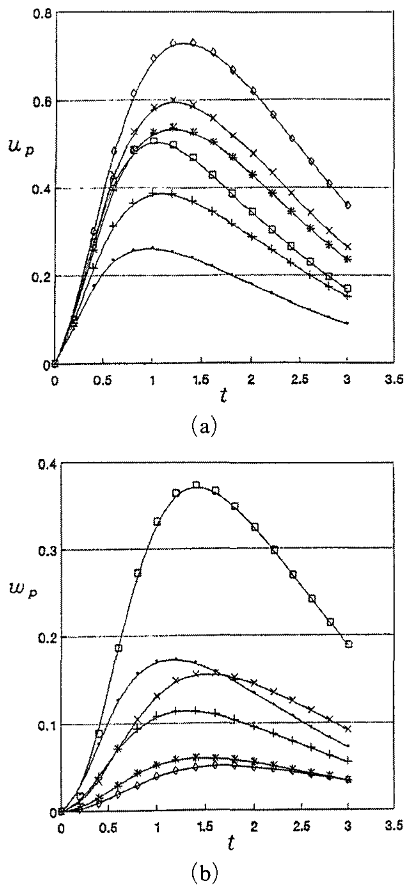
—•— $t=0.1$ —+— $t=0.5$ —*— $t=1$ —□— $t=3$

Fig. 2 Time development of the profile of (a) u; (b) w. (Ha=3, $\beta_e=3$, $\beta_i=3$, S=1)

Fig. 3 Time development of the profile of (a) u_p; (b) w_p. (Ha=3, $\beta_e=3$, $\beta_i=3$, S=1) files of the velocity components of the fluid u and w and particles u_p and w_p for various values of time t. The figures are plotted for $Ha=3$, $\beta_2=3$, $\beta_i=3$, and $S=1$. As shown in Fig. 2(a), 2(b), 3(a), and 3(b) the profiles of u, w, u_p and w_p are asymmetric about the plane $y=0$ because of the suction. It is observed that the velocity component u reaches the steady state faster than w which is expected, since u is the source of w. The same observation is clear in Fig. 3. Comparing Figs. 2 and 3 shows that the velocity components of the fluid phase reach the steady state faster than that of the particle phase. This is because the fluid velocity is the source for the dust particles' velocity. It is shown that the velocity components of the fluid and dust particles do not reach the steady state monotonically due to the effect of the pressure gradient.

Figures 4 and 5 show the time evolution of the velocity components at the centre of the channel ($y=0$), respectively, for the fluid and particle phases for various values of the Hall parameter β_e and the ion slip parameter β_i. In these figures, $Ha=3$ and $S=0$. It is clear from Figs. 4(a) and 5(a) that increasing the parameter β_e or β_i increases u and u_p. This is because the effective conductivity $(\sigma/((1+\beta_e\beta_i)^2+\beta_e^2))$ decreases with increasing β_e or β_i which reduces the magnetic damping force on u. In Figs. 4(b) and 5(b), the velocity components w and w_p increases with





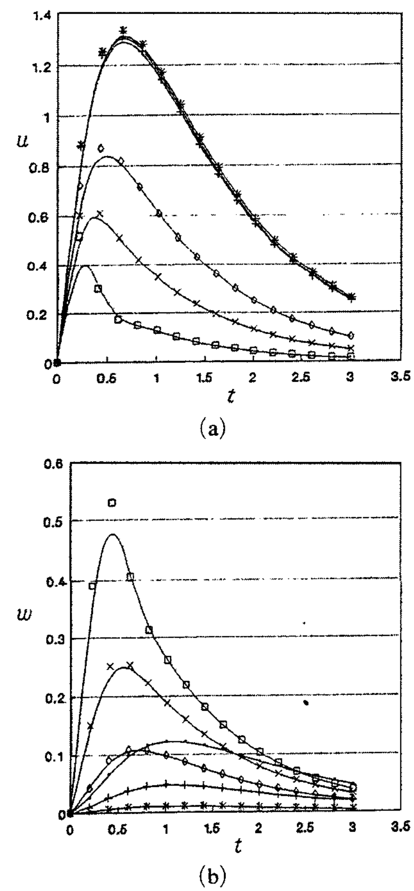
—•— $\beta_e=1$, $\beta_i=0$ —+— $\beta_e=1$, $\beta_i=1$ —*— $\beta_e=1$, $\beta_i=3$
—□— $\beta_e=3$, $\beta_i=0$ —×— $\beta_e=3$, $\beta_i=1$ —○— $\beta_e=3$, $\beta_i=3$

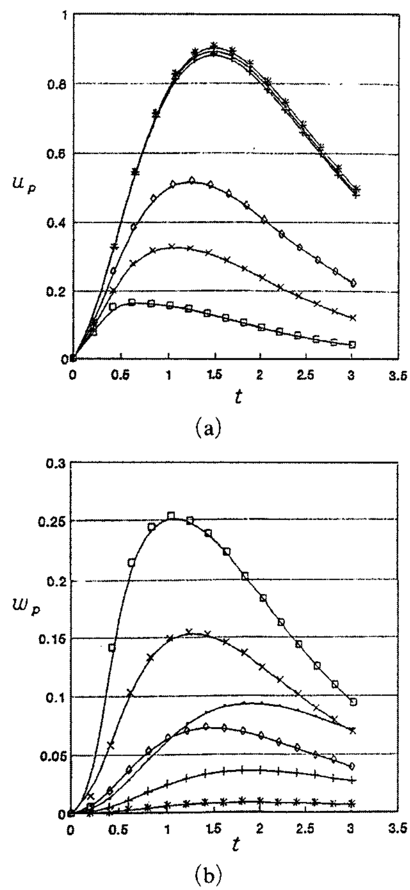
Fig. 4 Effect of β_e and β_i on the time development of (a) u at $y=0$; (b) w at $y=0$

Fig. 5 Effect of β_e and β_i on the time development of (a) u_p at $y=0$; (b) w_p at $y=0$

increasing β_e, since w is a result of the Hall effect. However, for large values of β_i, increasing β_e decreases w. On the other hand, increasing the ion slip parameter β_i decreases w for all values of β_e as a result of decreasing the source term of $w(\beta_e Ha^2 u/((1+\beta_e\beta_i)^2+\beta_e^2))$ and increasing its damping term $(Ha^2(1+\beta_e\beta_i)w/((1+\beta_e\beta_i)^2+\beta_e^2))$. The influence of the ion slip on w and w_p becomes more pronounced for higher values of β_e.

Figures 6 and 7 show the time evolution of the velocity components at the centre of the channel $(y=0)$, respectively, for the fluid and particle phases for various values of the Hartmann number Ha and the ion slip parameter β_i. In these figures, $\beta_e=3$ and $S=0$. Figures 6(a) and 7(a) indicate that the effect of β_i on u and u_p depends on Ha. For small values of Ha, increasing β_i slightly decreases u and u_p as a result of increasing the damping force on u which is proportional to β_i. Increasing β_i more increases the effective conductivity and, in turn, decreases the damping force on u which increases u and u_p. On the other hand, for larger values of Ha, u becomes small, and increasing β_i always decreases the effective conductivity and therefore increases u and u_p. It is also clear that the effect of β_i on u and u_p becomes more apparent for higher values of Ha. Figures 6(b) and 7(b) ensure that increasing the ion slip parameter β_i decreases w





—•— Ha=1, $\beta_i=0$ —+— Ha=1, $\beta_i=1$ —*— Ha=1, $\beta_i=3$
—□— Ha=5, $\beta_i=0$ —×— Ha=5, $\beta_i=1$ —◇— Ha=5, $\beta_i=3$

Fig. 6 Effect of Ha and β_i on the time development of (a) u at $y=0$; (b) w at $y=0$

Fig. 7 Effect of Ha and β_i on the time development of (a) u_p at $y=0$; (b) w_p at $y=0$

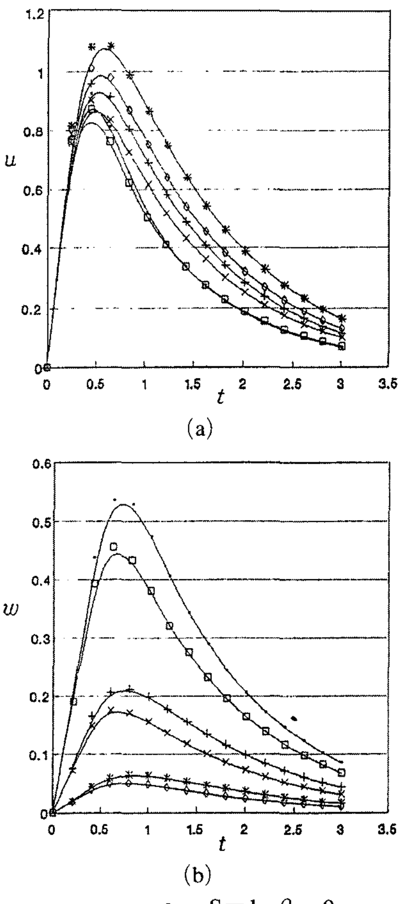
and w_p for all values of Ha and that its effect is more apparent for higher values of Ha.

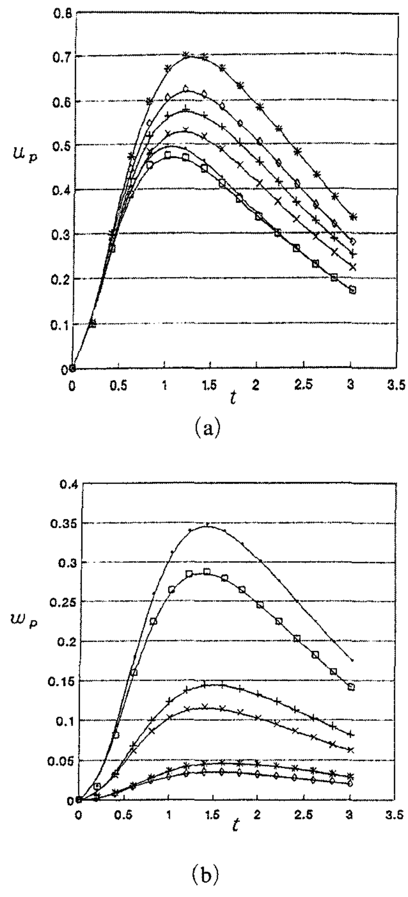
Figures 8 and 9 present the time evolution of the velocity components at the centre of the channel ($y=0$), respectively, for the fluid and particle phases for various values of the suction parameter S and the ion slip parameter β_i. In these figures $Ha=3$ and $\beta_e=3$. Figures 8(a), 8(b), 9(a), and 9(b) show that increasing the suction decreases u, w, u_p and w_p due to the convection of the fluid from regions in the lower half to the centre which has higher fluid speed. It is also clear from these figures that the effect of the suction parameter on u and u_p becomes more pronounced as β_i increases while its effect on w and w_p decreases as β_i increases. It should be mentioned that the results obtained herein reduce to those reported by Aboul-Hassan and Attia (2002) in the case $\beta_i=0$.

Table 1 presents the effect of variation of the particle concentration parameter R on the steady state velocity components u and w at the centre of the channel ($y=0$) for $Ha=1$, $\beta_e=3$, $\beta_i=3$ and

Table 1 Variation of u and w at $y=0$ for various values of R

R	0	0.25	0.5	0.75	1
u	2.4067	2.3855	2.3612	2.3345	2.3062
w	0.0644	0.0257	0.0249	0.0242	0.0234





S=1, $\beta_i=0$ S=1, $\beta_i=1$ S=1, $\beta_i=3$
S=3, $\beta_i=0$ S=3, $\beta_i=1$ S=3, $\beta_i=3$

Fig. 8 Effect of S and β_i on the time development of (a) u at $y=0$; (b) w at $y=0$

Fig. 9 Effect of S and β_i on the time development of (a) u_p at $y=0$; (b) w_p at $y=0$

$S=0$. The table indicates that increasing R decreases the steady state of both u and w.

5. Conclusions

The unsteady flow of a dusty conducting fluid under the influence of an applied uniform magnetic field has been studied, considering the Hall and ion slip effects in the presence of uniform suction and injection. The effect of the magnetic field, the Hall parameter, the ion slip parameter, and the suction and injection velocity on the velocity distributions for both the fluid and particle phases has been investigated. It is found that the effect of the ion slip on the main velocity of the fluid and particles u and u_p, respectively, depends upon the magnetic field. For large values of the magnetic field, increasing the ion slip increases u and u_p. For small values of the magnetic field, increasing the ion slip slightly decreases u and u_p, but increasing it more increases u and u_p. It is also shown that increasing the Hall parameter increases the velocity component w and w_p, while increasing the ion slip decreases w and w_p. The influence of the Hall current on w and w_p decreases greatly as the ion slip increases. The effect of the suction and injection velocity on u and u_p increases as the ion slip increases while its effect on w and w_p decreases when increasing the ion slip.

References

Aboul-Hassan, A. L. and Attia, H. A., 2002, "Hydromagnetic Flow of a Dusty Fluid in a Rectangular Channel with Hall Current and Heat Transfer," *Can. J. Phys.*, Vol. 80, pp. 579~589.

Aboul-Hassan, A. L., Sharaf El-Din H. and Megahed, A. A., 1991, "Temperature Due to the Motion of one of them," First International Conference of Engineering Mathematics and Physics, Cairo, pp. 723~732.

Ahn, S. W., Lee, B. -C., Kim, W. -C. and Bae, M. -W., 2002, "Characteristics of Fluid Flow and Heat Transfer in Fluidized Heat Exchanger with Circulating Solid Particles," *KSME International Journal*, Vol. 16, No. 9, pp. 1175~1182.

Attia, H. A., 2002, "A Transient Hartmann Flow with Heat Transfer of a Non-Newtonian Fluid with Suction and Injection, Considering the Hall Effect," *J. Plasma Physics*, Vol. 67, No. 1, pp. 27~47.

Attia, H. A., 1998, "Hall Current Effects on the Velocity and Temperature Fields of an Unsteady Hartmann Flow," *Can. J. Phys.*, Vol. 76, No. 9, pp. 739~748.

Attia, H. A., 2002, "Transient Hartmann Flow with Heat Transfer Consideration the Ion Slip," *Physics Scripta*, Vol. 66, No. 6, pp. 470~475.

Borkakotia, K. and Bharali, A., 1983, "Hydromagnetic Flow and Heat Transfer Between Two Horizontal Plates, the Lower Plate Being a Stretching Sheet," *Quarterly of Applied Mathematics*, pp. 461~472.

Chamkha, A. J., 1997 "Unsteady Flow of an Electrically Conducting Dusty-gas in a Channel Due to an Oscillating Pressure Gradient," *Appl. Math. Modelling*, Vol. 21, pp. 287~292.

Chamkha, A. J., 2000, "Unsteady Laminar Hydromagnetic Fluid-particle Flow and Heat Transfer in Channels and Circular Pipes," *International Journal of Heat and Fluid Flow*, Vol. 21, pp. 740~746.

Crammer, K. R. and Pai, S. -I., 1973, Magnetofluid dynamics for Engineer and scientists, McGraw-Hill.

Dixit, L. A., 1980, "Unsteady Flow of a Dusty Viscous Fluid Through Rectangular Ducts," *Indian J. of Theoretical Physics*, Vol. 28, No. 2, pp. 129~140.

Ghosh, A. K. and Mitra, D. K., 1984, "Flow of a Dusty Fluid Through Horizontal Pipes," *Rev. Roum. Phys.*, Vol. 29, No. 631.

Gupta, R. K. and Gupta, S. C., 1976, "Flow of a Dusty Gas Through a Channel with Arbitrary Time Varying Pressure Gradient," *Journal of Applied Mathematics and Physics*, Vol. 27, pp. 119~128.

Kim, D., 2002, "Improved Convective Heat Transfer Correlations for Two-Plase Two-Component Pipe," *KSME international Journal*, Vol. 16, No. 3, pp. 403~422.

Megahed, A. A., Aboul-Hassan, A. L. and Sharaf El-Din, H., 1988, "Effect of Joule and

Viscous Dissipation on Temperature Distributions Through Electrically Conducting Dusty Fluid," Fifth Miami International Symposium on Multi-Phase Transport and Particulate Phenomena ; Miami Beach, Florida, USA, Vol. 3, pp. 111~123.

Mitra, P. and Bhattacharyya, P., 1981, "Unsteady Hydromagnetic Laminar Flow of a Conducting Dusty Fluid Between Two Parallel Plates Started Impulsively From Rest," *Acta Mechanica*, Vol. 39, pp. 171~183.

Prasad, V. R. and Ramacharyulu, N. C. P., 1979, "Unsteady Flow of a Dusty Incompressible Fluid Between Two Parallel Plates Under an Impulsive Pressure Gradient," *Def. Sci. Journal*, Vol. 30, pp. 125~137.

Saffman, P. G., 1962, "On the Stability of a Laminar Flow of a Dusty Gas," *Journal of Fluid Mechanics*, Vol. 13, pp. 120~131.

Seddeek, M. A., 2002, "Effects of Magnetic Field, Variable Viscosity and Non-Darcy on Forced Convection Flow About a Flat Plate with Wariable wall Temperature in Porous Media in the Presence of Suction and Blowing," *J. of App. Mech. and Tech. Physics*, Vol. 43, No. 1, pp. 13~17.

Seddeek, M. A., 2003, "Flow of a Magneto-Micropolar Fluid Past a Continuously Moving Plate," *J. of Physics Letters A*, Vol. 306, pp. 255~257.

Singh, K. K., 1976, "Unsteady Flow of a Conducting Dusty Fluid Through a Rectangular Channel with Time Dependent Pressure Gradient," *Indian Journal of Pure and Applied Mathematics*, Vol. 8, No. 9, pp. 1124~1136.

Soundalgekar, V. M., Vighnesam N. V. and Takhar, H. S., 1979, "Hall and Ion-Slip Effects in MHD Couette Flow with Heat Transfer," *IEEE Transactions on Plasma Science*, Vol. PS-7, No. 3, pp. 178~182.

Soundalgekar, V. M. and Uplekar, A. G., 1986, "Hall effects in MHD Couette Flow with Heat Transfer," *IEEE Trans. on Plasma Science*, Vol. PS-14, No. 5, pp. 579~583.

Spiegel, M. R., 1986, *Theory and problems of Laplace Transforms*, McGraw-Hill.

Sutton, G. W. and Sherman, A., 1965, *Engineering Magnetohydrodynamics*, McGraw-Hill.