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ON THE GENUS OF CERTAIN DRINFELD MODULAR CURVES

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Abstract. We determine the genera of the Drinfeld modular curves $X_\Delta(\mathfrak{n})$.

1. Introduction

Gekeler [1, 2] calculated genus formulas for some Drinfeld modular curves(e.g. $X(\mathfrak{n}), X_0(\mathfrak{n})$). Later, he obtained the same genus formulas by viewing the Drinfeld modular curves as the quotient graphs of the Bruhat-Tits building by congruence subgroups [3]. In this paper, we determine the genera of the Drinfeld modular curves $X_\Delta(\mathfrak{n})$(See Theorem 2.) by adopting Gekeler's former idea.

2. Some preliminaries

Let k be the rational function field $\mathbb{F}_q(T)$ over the finite field $\mathbb{F}_q$ and $A = \mathbb{F}_q[T]$. Let k_∞ be the completion of k at $\infty = (\frac{1}{T})$ and C the completion of the algebraic closure of k_∞. The group $GL_2(k_\infty)$ acts on the Drinfeld upper half plane $\Omega = C - k_\infty$ by the linear fractional transformation.

Let $\Gamma(1) = GL_2(A)$ and Δ a subgroup of $\mathbb{F}_q^*$. For $\mathfrak{n} \in A$, let

$$\begin{aligned}
\Gamma(\mathfrak{n}) &= \left\{\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in \Gamma(1) \,|\, \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \equiv \left(\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right) \mod \mathfrak{n}\right\} \\
\Gamma_\Delta(\mathfrak{n}) &= \left\{\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in \Gamma(1) \,|\, c \equiv 0, a \equiv 1, d \equiv \alpha \mod \mathfrak{n},\ \alpha \in \Delta\right\} \\
\Gamma_0(\mathfrak{n}) &= \left\{\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in \Gamma(1) \,|\, c \equiv 0 \mod \mathfrak{n}\right\}.
\end{aligned}$$

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We let $Y(1)$,$Y(\mathfrak{n})$,$Y_\Delta(\mathfrak{n})$,$Y_0(\mathfrak{n})$ be the corresponding quotients of Ω by $\Gamma(1)$,$\Gamma(\mathfrak{n})$,$\Gamma_\Delta(\mathfrak{n})$,$\Gamma_0(\mathfrak{n})$ respectively, and $X(1)$,$X(\mathfrak{n})$,$X_\Delta(\mathfrak{n})$,$X_0(\mathfrak{n})$ the compactions by adding cusps. By abuse of terminology, the cusp of infinity will be also denoted by ∞.

Let $\mathfrak{n} \in A$ be given as

$$\mathfrak{n} = \prod_{1 \leq i \leq s} \mathfrak{p}_i^{r_i},$$

where the $\mathfrak{p}_i$'s are different monic primes. We further put $q_i := |\mathfrak{p}_i| := |A/(\mathfrak{p}_i)|$. We will need some arithmetic functions related to $\mathfrak{n}$, i.e.,

$$\begin{aligned} \varphi(\mathfrak{n}) &= \prod_{1 \leq i \leq s} q_i^{r_i - 1}(q_i - 1) \\ \epsilon(\mathfrak{n}) &= \prod_{1 \leq i \leq s} q_i^{r_i - 1}(q_i + 1). \end{aligned}$$

Let Y be a complete nonsingular algebraic curve over C. Let G be a finite group of automorphisms of Y and $X = G \backslash Y$ the quotient curve. For a point Q of Y, let G_Q be the stabilizer, $\mathcal{O}_Q$ the local ring at Q and π_Q a local parameter. We define the *i-th ramification group* by

$$G_{Q,i} = \{\sigma \in G_Q \mid \sigma^*(f) \equiv f \mod \pi_Q^{i+1} \text{ for all } f \in \mathcal{O}_Q\}$$

for each $i \in \mathbb{Z}_{\geq 0}$. Then we get the following tower of normal subgroups:

$$G_Q = G_{Q,0} \supset G_{Q,1} \supset \cdots \supset G_{Q,m} = \{1\}$$

for sufficiently large m. The covering $Y \to X$ is called *tamely ramified* at Q if $G_{Q,1} = \{1\}$, and *wildly ramified* otherwise. We define

$$i_Q(\sigma) = \sup\{i \mid \sigma \in G_{Q,i}\} + 1,$$

where $\sigma \in G_Q - \{1\}$. For the Euler characteristic e and the genus g, we have the relation $e = 2 - 2g$. Also we get the Hurwitz formula

$$e(Y) = |G| e(X) - \sum_{Q \in Y} a_Q,$$

where $a_Q = \displaystyle\sum_{\sigma \in G_Q - \{1\}} i_Q(\sigma)$.

We set the following notations:

$$\begin{aligned} G &= \{(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}) \in GL_2(A/\mathfrak{n}) \mid ad - bc \in \mathbb{F}_q^*\} \\ T &= \{(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}) \in G \mid c = 0\} \\ U &= \{(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}) \in G \mid c = 0, a, d \in \mathbb{F}_q^*\} \\ E_\Delta &= \{(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}) \in G \mid a = 1, c = 0, d \in \Delta\} \\ E &= \{(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}) \in G \mid a = d = 1, c = 0\}. \end{aligned}$$

Throughout this paper, δ denote the order of Δ. By a simple calculation, we know that the order of G (resp. T, U, E_Δ, E) is $\varphi(\mathfrak{n})\epsilon(\mathfrak{n})|\mathfrak{n}|$ (resp. $\varphi(\mathfrak{n})|\mathfrak{n}|, (q-1)|\mathfrak{n}|, \delta|\mathfrak{n}|, |\mathfrak{n}|$).

The above group G is the Galois group of the covering $X(\mathfrak{n}) \to X(1)$, i.e., G is isomorphic to the quotient group $\Gamma(1)/\Gamma(\mathfrak{n})\mathbb{F}_q^*$. Gekeler [1] has shown that the above covering is ramified only at the elliptic points and cusps. Moreover, it is tamely ramified at the elliptic points, and the second ramification group is trivial for any cusp. Then applying Hurwitz formula to the covering $X(\mathfrak{n}) \to X(1)$, we can get the following result.

THEOREM 1. (Theorem 3.4.8, [1]) *The genus of the Drinfeld modular curve $X(\mathfrak{n})$ is given by*

$$g(X(\mathfrak{n})) = 1 + \frac{1}{q^2-1}(|\mathfrak{n}| - q - 1)\varphi(\mathfrak{n})\epsilon(\mathfrak{n}).$$

3. A genus formula for $X_\Delta(\mathfrak{n})$

To obtain a genus formula for $X_\Delta(\mathfrak{n})$, we will apply Hurwitz formula to the covering $X(\mathfrak{n}) \to X_\Delta(\mathfrak{n})$ with the Galois group E_Δ. First of all we investigate the ramification at cusps. Note that $G_\infty = U$. Thus the number of cusps of $X(\mathfrak{n})$ is equal to the order of G/U which denotes the set of left cosets. Let $\{\gamma\}$ be a representative system of G/U. Then $U^\gamma (= \gamma U \gamma^{-1})$ is the fixed group of another cusp of $X(\mathfrak{n})$. Let Q be the cusp corresponding to γ. By Theorem 1, one can get the following:

$$a_Q = \sum_{\sigma \in E_{\Delta Q} - \{1\}} i_Q(\sigma) = |E_\Delta \cap U^\gamma| + |E_\Delta \cap E^\gamma| - 2.$$

Let $\{\alpha\}$(resp. $\{\beta\}$) be a representative system of G/T(resp. $T/U \cong (A/\mathfrak{n})^*/\mathbb{F}_q^*$). Then $\{\alpha\beta\}$ is a representative system for G/U, and $U^{\alpha\beta} = (U^\beta)^\alpha = U^\alpha$, because U is normal in T. Thus it suffices to compute the order of $E_\Delta \cap U^\alpha$. The system $\{\alpha\}$ can be chosen componentwise, i.e., $\alpha = (\alpha_i)$, $1 \le i \le s$, where $\{\alpha_i\}$ is a representative system of $GL_2(A/\mathfrak{p}_i^{r_i})$. We will take $\{\alpha_i\}$ as

$$\{\left(\begin{smallmatrix} 1 & 0 \\ u_i & 1 \end{smallmatrix}\right) \mid u_i \in A/\mathfrak{p}_i^{r_i}\} \cup \{\left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right)\left(\begin{smallmatrix} 1 & 0 \\ u_i & 1 \end{smallmatrix}\right) \mid u_i \in \mathfrak{p}_i/\mathfrak{p}_i^{r_i}\}.$$

The element $\left(\begin{smallmatrix} 1 & 0 \\ u_i & 1 \end{smallmatrix}\right)$ will be called *type 1*, the other called *type 2*. Observe that $GL_2(\mathbb{F}_q)$ is imbedded in $GL_2(A/\mathfrak{n})$ diagonally. So $\left(\begin{smallmatrix} a & b \\ 0 & d \end{smallmatrix}\right)$ $(\in U)$ can be viewed as $(\left(\begin{smallmatrix} a & b_i \\ 0 & d \end{smallmatrix}\right))$ in $GL_2(A/\mathfrak{n})$. Then

$$\left(\begin{smallmatrix} 1 & 0 \\ u_i & 1 \end{smallmatrix}\right)\left(\begin{smallmatrix} a & b_i \\ 0 & d \end{smallmatrix}\right)\left(\begin{smallmatrix} 1 & 0 \\ -u_i & 1 \end{smallmatrix}\right) = \left(\begin{smallmatrix} a-b_iu_i & b_i \\ -b_iu_i^2+(a-d)u_i & d+b_iu_i \end{smallmatrix}\right) \tag{1}$$

and

$$\left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right)\left(\begin{smallmatrix} 1 & 0 \\ u_i & 1 \end{smallmatrix}\right)\left(\begin{smallmatrix} a & b_i \\ 0 & d \end{smallmatrix}\right)\left(\begin{smallmatrix} 1 & 0 \\ -u_i & 1 \end{smallmatrix}\right)\left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right) = \left(\begin{smallmatrix} d+b_iu_i & -b_iu_i^2+(a-d)u_i \\ b_i & a-b_iu_i \end{smallmatrix}\right). \tag{2}$$

For $u = (u_i) \in A/\mathfrak{n}$, let $v_i(u) = v(u_i)$ be the order of zero of u_i, i.e.,

$$\begin{aligned} v_i(u) &= l < r_i, \text{ if } u_i \in (\mathfrak{p}_i^l - \mathfrak{p}_i^{l+1})/\mathfrak{p}_i^{r_i}, \\ &= r_i, \text{ if } u_i = 0. \end{aligned}$$

Further, for $\alpha = (\alpha_i)$, we set the representation

$$\begin{aligned} v_i(\alpha) &= v_i(u), \quad \text{if } \alpha_i = \left(\begin{smallmatrix} 1 & 0 \\ u_i & 1 \end{smallmatrix}\right) \text{ is type 1}, \\ &= 0, \qquad\quad \text{if } \alpha_i \text{ type 2}. \end{aligned}$$

LEMMA 3.1. *The order of the group $E_\Delta \cap U^\alpha$ is given by*

$$\tau(\alpha)\prod_{i=1}^{s} q_i^{\inf(v_i(\alpha), r_i)},$$

where $\tau(\alpha) = \delta$ if $v_i(\alpha) = 0$ for all i or $v_i(\alpha) = r_i$ for all i, and 1 otherwise.

PROOF. Suppose $\left(\begin{smallmatrix} a & b \\ 0 & d \end{smallmatrix}\right) \in U$, so that $\left(\begin{smallmatrix} a & b \\ 0 & d \end{smallmatrix}\right)^\alpha \in E_\Delta$. From the equation (2), if α_i is type 2, it is satisfied that $b_i = 0$, $d+b_iu_i = 1$ and $a-b_iu_i \in \Delta$. Thus $b_i = 0, d = 1$, and $a \in \Delta$.

In other case, it must be satisfied that

$$u_i(a - d - b_iu_i) = 0, \tag{3}$$

$$a - b_iu_i = 1 \text{ and } d + b_iu_i \in \Delta. \tag{4}$$

If $v_i(u) = 0$, the condition (3) is equivalent to $b_i = u_i^{-1}(a-d)$, and then $d = 1$ and $a \in \Delta$. If $0 < v_i(u) < r_i$, the condition (3) is equivalent to the following:

$$a = d \text{ and } b_i \in \mathfrak{p}_i^{r_i - \inf(r_i, 2v_i(u))}/\mathfrak{p}_i^{r_i},$$

and then, from (4), we get

$$a = d \text{ and } b_i \in \mathfrak{p}_i^{v_i(u)}/\mathfrak{p}_i^{r_i}.$$

If $v_i(u) = r_i$, i.e., $u_i = 0$, then we have

$$a = 1, d \in \Delta \text{ and } b_i \in A/\mathfrak{p}_i^{r_i}.$$

Summing up the above, we can conclude the result. □

Now we compute the sum $\sum_\alpha (|E_\Delta \cap U^\alpha| + |E_\Delta \cap E^\alpha|)$, where α lies in a representative system of G/T.

$$\begin{aligned}
&\sum_\alpha (|E_\Delta \cap U^\alpha| + |E_\Delta \cap E^\alpha|) \\
&= (\delta+1) \sum_{\substack{\alpha \\ \tau(\alpha)=\delta}} \prod_{i=1}^{s} q_i^{\inf(v_i(\alpha), r_i)} + 2 \sum_{\substack{\alpha \\ \tau(\alpha)=1}} \prod_{i=1}^{s} q_i^{\inf(v_i(\alpha), r_i)} \\
&= 2 \sum_{\text{all } \alpha} \prod_{i=1}^{s} q_i^{\inf(v_i(\alpha), r_i)} + (\delta - 1) \sum_{\substack{\alpha \\ \tau(\alpha)=\delta}} \prod_{i=1}^{s} q_i^{\inf(v_i(\alpha), r_i)} \\
&= \text{the first term} + \text{the second term.}
\end{aligned}$$

The computation of the first term.
Note that

$$\sum_\alpha \prod_{i=1}^{s} q_i^{\inf(v_i(\alpha), r_i)} = \prod_{i=1}^{s} \sum_{\alpha_i} q_i^{\inf(v(\alpha_i), r_i)}.$$

For the calculation, we use the following table.

$v = v(\alpha_i)$	number of α_i	contribution of an α_i	sum
0	$q_i^{r_i}$	1	$q_i^{r_i}$
$0 < v < r_i$	$q_i^{r_i - v} - q_i^{r_i - v - 1}$	q_i^v	$q_i^{r_i} - q_i^{r_i - 1}$
r_i	1	$q_i^{r_i}$	$q_i^{r_i}$

Then $\sum_{\alpha_i} q_i^{\inf(v(\alpha_i), r_i)} = 2q_i^{r_i} + (r_i - 1)(q_i^{r_i} - q_i^{r_i - 1})$. Put

$$c(\mathfrak{n}) = \prod_{i=1}^{s} \left(2 + (r_i - 1)(1 - \frac{1}{q_i})\right).$$

Then the first term is equal to $2|\mathfrak{n}|c(\mathfrak{n})$.

The computation of the second term.
Note that

$$(\delta-1)\sum_{\substack{\alpha\\ \tau(\alpha)=\delta}}\prod_{i=1}^{s} q_i^{\inf(v_i(\alpha),r_i)} = (\delta-1)\prod_{i=1}^{s}\sum_{\substack{\alpha_i\\ \tau(\alpha)=\delta}} q_i^{\inf(v(\alpha_i),r_i)}.$$

Then $\sum_{\substack{\alpha_i\\ \tau(\alpha)=\delta}} q_i^{\inf(v(\alpha_i),r_i)} = 2q_i^{r_i}$, because the α with $\tau(\alpha)=\delta$ satisfy that $v(\alpha_i)=0$ for all i or $v(\alpha_i)=r_i$ for all i. Thus the second term is equal to $2(\delta-1)|\mathfrak{n}|$.

Therefore the sum $\sum_Q a_Q$ over the cusps is given by

$$\frac{\varphi(\mathfrak{n})}{q-1}\big(2|\mathfrak{n}|c(\mathfrak{n})+2(\delta-1)|\mathfrak{n}|-2\epsilon(\mathfrak{n})\big). \tag{5}$$

We now compute the contribution of elliptic ramification. Note that all the elliptic points of $\Gamma(1)$ are all conjugate under $\Gamma(1)$ and represented by an element of $\mathbb{F}_{q^2}-\mathbb{F}_q$. Let $z\in\mathbb{F}_{q^2}-\mathbb{F}_q$. Then

$$H := G_z = \{\left(\begin{smallmatrix} a & b\\ c & d\end{smallmatrix}\right) \mid a,b,c,d\in\mathbb{F}_q, cz^2+(d-a)z-b=0\}.$$

LEMMA 3.2. *$\Gamma_\Delta(\mathfrak{n})$ has no elliptic points.*

PROOF. We must show that $H^\alpha\cap E_\Delta=\{1\}$ for all $\alpha\in G$. One can check that the order of $H^\alpha\cap E_\Delta$ depends only on the class of α in $E_\Delta\backslash G$ which denotes the set of right cosets. First, we will take a representative system $\{\alpha\}=\{(\alpha_i)\}$ of $T\backslash G$ as follows:

$$\{\alpha_i\}=\{\left(\begin{smallmatrix} 1 & 0\\ u_i & 1\end{smallmatrix}\right) \mid u_i\in A/\mathfrak{p}_i^{r_i}\}\cup\{\left(\begin{smallmatrix} 1 & 0\\ u_i & 1\end{smallmatrix}\right)\left(\begin{smallmatrix} 0 & 1\\ 1 & 0\end{smallmatrix}\right) \mid u_i\in \mathfrak{p}_i/\mathfrak{p}_i^{r_i}\}.$$

Next, we will take a representative system $\{\beta\}=\{(\beta_i)\}$ of $E_\Delta\backslash T$ as follows:

$$\{\beta_i\}=\{\left(\begin{smallmatrix} 1 & 0\\ 0 & \xi\end{smallmatrix}\right) \mid \xi\in(A/\mathfrak{p}_i^{r_i})^*/\Delta\}.$$

Then $\{\beta\alpha\}=\{(\beta_i\alpha_i)\}$ is a representative system of $E_\Delta\backslash G$, where

$$\begin{aligned}\{\beta_i\alpha_i\} &= \{\left(\begin{smallmatrix} 1 & 0\\ \xi u_i & \xi\end{smallmatrix}\right) \mid \xi\in(A/\mathfrak{p}_i^{r_i})^*/\Delta, u_i\in A/\mathfrak{p}_i^{r_i}\}\\ &\quad\cup\{\left(\begin{smallmatrix} 1 & 0\\ \xi u_i & \xi\end{smallmatrix}\right)\left(\begin{smallmatrix} 0 & 1\\ 1 & 0\end{smallmatrix}\right) \mid \xi\in(A/\mathfrak{p}_i^{r_i})^*/\Delta, u_i\in \mathfrak{p}_i/\mathfrak{p}_i^{r_i}\}.\end{aligned}$$

By abuse of terminology, the element $\left(\begin{smallmatrix} 1 & 0\\ \xi u_i & \xi\end{smallmatrix}\right)$ will be called *type 1.* Suppose that $\left(\begin{smallmatrix} a & b\\ c & d\end{smallmatrix}\right)^\alpha\in E_\Delta$ for some $\left(\begin{smallmatrix} a & b\\ c & d\end{smallmatrix}\right)\in H$. Note that $bc\neq 0$. Consider the following:

$$\left(\begin{smallmatrix} 1 & 0\\ \xi u_i & \xi\end{smallmatrix}\right)\left(\begin{smallmatrix} a & b\\ c & d\end{smallmatrix}\right)\begin{pmatrix} 1 & 0\\ -u_i & \xi^{-1}\end{pmatrix} = \begin{pmatrix} a-bu_i & b\xi^{-1}\\ -b\xi u_i^2+(a-d)\xi u_i+c\xi & d+bu_i\end{pmatrix}$$

$$\left(\begin{smallmatrix} 1 & 0\\ \xi u_i & \xi\end{smallmatrix}\right)\left(\begin{smallmatrix} 0 & 1\\ 1 & 0\end{smallmatrix}\right)\left(\begin{smallmatrix} a & b\\ c & d\end{smallmatrix}\right)\left(\begin{smallmatrix} 0 & 1\\ 1 & 0\end{smallmatrix}\right)\begin{pmatrix} 1 & 0\\ -u_i & \xi^{-1}\end{pmatrix} = \begin{pmatrix} d-cu_i & c\xi^{-1}\\ -c\xi u_i^2+(d-a)\xi u_i+b\xi & a+cu_i\end{pmatrix}.$$

The equality $-c\xi u_i^2 + (d-a)\xi u_i + b\xi = 0$ with $u_i \in \mathfrak{p}_i/\mathfrak{p}_i^{r_i}$ implies $b = 0$, and so α_i must be type 1 for all i. Thus u_i satisfies $-bu_i^2 + (a-d)u_i + c = 0, a - bu_i = 1$ and $d + bu_i \in \Delta$. But it is impossible. $\square$

From the Hurwitz formula, Theorem 1, (5) and Lemma 3.2, we obtain

$$e(X_\Delta(\mathfrak{n})) = \frac{2\varphi(\mathfrak{n})}{(q^2-1)\delta}\big(- \varepsilon(\mathfrak{n}) + (q+1)(c(\mathfrak{n}) + \delta - 1)\big).$$

Now we obtain the following theorem.

THEOREM 2. *The genus of the Drinfeld modular curve $X_\Delta(\mathfrak{n})$ is given by*

$$g(X_\Delta(\mathfrak{n})) = 1 + \frac{\varphi(\mathfrak{n})}{(q^2-1)\delta}\big(\varepsilon(\mathfrak{n}) - (q+1)(c(\mathfrak{n}) + \delta - 1)\big).$$

4. Some consequences

Next, we will determine all $\mathfrak{n}$ for which $X_\Delta(\mathfrak{n})$ is rational or elliptic. Since there is a canonical covering $X_\Delta(\mathfrak{n}) \to X_0(\mathfrak{n})$, we only consider the cases for which $X_0(\mathfrak{n})$ is rational or elliptic. Let $\mathfrak{n} \in A$ be of degree d. Then Schweizer [4] proved that

(1) $X_0(\mathfrak{n})$ is rational if and only if $d \leq 2$.
(2) $X_0(\mathfrak{n})$ is elliptic if and only if $q = 2$ and $\mathfrak{n}$ is one of the polynomials T^3, $T^2(T+1)$, $T(T+1)^2$ and $(T+1)^3$.

PROPOSITION 4.1. (1) *$X_\Delta(\mathfrak{n})$ is rational if and only if $d = 1$ or $d = 2$ and $\delta = q - 1$.*
(2) *$X_\Delta(\mathfrak{n})$ is elliptic if and only if $d = 2, q = 3$ and $\delta = 1$.*

PROOF. Let g be the genus of $X_\Delta(\mathfrak{n})$. If $d = 1$, then it is clear that $g = 0$. Suppose $d = 2$. Then there are three cases.

Case(i) $s = 1, r_1 = 1$ and $q_1 = q^2$.

From the formula in Theorem 2, we obtain $g = \dfrac{q(q-\delta-1)}{\delta}$. Thus $g = 0$ if and only if $n = q-1$. The genus g cannot be equal to 1, since q doesn't divide δ.

Case(ii) $s = 1, r_1 = 2$ and $q_1 = q$.

We come up with $g = \dfrac{(q-1)(q-\delta-1)}{\delta}$. Thus $g = 0$ if and only if $\delta = q-1$. The genus g cannot be 1, either.

Case(iii) $s = 2, r_1 = r_2 = 1$ and $q_1 = q_2 = q$.

We have $g = \dfrac{(q-2)(q-\delta-1)}{\delta}$. Therefore $g = 0$ if and only if $\delta = q-1$. Also $g = 1$ if and only if $q = 3$ and $\delta = 1$.

By a simple calculation, if $q = 2$ and $\mathfrak{n}$ is one of the polynomials T^3, $T^2(T+1)$, $T(T+1)^2$ and $(T+1)^3$, then $g = 5, 3, 3$ and 5 respectively. □

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