Instructional Design in All (K–3) Students' Mathematical Achievement in Solving Word Problems

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This paper investigates instructional strategies with potential for improving students' achievement in word problem solving. This review compares and analyzes the direct instruction (DI) and cognitively guided instruction (CGI) research on K–3 word problem solving mathematics students in a demonstration of my position that teachers need to understand student mathematical thinking to enhance students' achievement in word problem solving. CGI provides a more appropriate instructional model than DI for teaching word problem solving. For example, student-centered, conceptual under-standing, and children's informal or invented problem solving strategies communicating with each other mathematically, etc. Korean teachers and teacher educators need to consider implementing CGI teaching strategies.

INTRODUCTION

Students learn mathematics through experiences that teachers provide. The National Council for the Teaching of Mathematics (NCTM 2000) indicates that effective teachers utilize a deep understanding of mathematics and instructional strategies as well as an understanding of their students as learners of mathematics. Differences in instructional strategies are directly related to students' achievement (Fennema & Carpenter 1996).

Currently widespread demands direct that mathematics instruction should be dramatically reformed so that students are able to learn mathematics with understanding by actively participating in tasks that incorporate important mathematics (NCTM 1991).

Many points of view consider what children should learn in school and how they should learn it. Since the early 1990s, research on teaching how has made great strides in developing theories and supported by research-based evidence about how to teach elementary school mathematics in a way that develops students' mathematical understanding. Much of this progress has grown out of research projects that engage teachers in learning to teach mathematics.

Mathematics word problems, the most frequent kind of mathematics-related problem solving, has been described by teachers as a frustrating and difficult exercise in the elementary grades (Kameenui & Griffin 1989). Two problems provide examples of word problems:

Our class has 3 pages with stickers on them. There are 4 stickers on each page. How many stickers do we have?

Nineteen children are going to the circus. Five children can ride in each car. How many cars will be needed to get all 19 children to the circus?

Solving word problems in grades K–3 has been studied in teaching mathematics. In an effort to teach a more generalizable strategy for solving word problem, instructional procedures have received much attention over the past decade. Two instructional strategies have been recognized as potential strategies for teaching word problem solving to lower grade (K–3) students.

Cognitively Guided Instruction (CGI) focuses on teachers' understanding the development of children's mathematical thinking by interacting with their students through a specific research-based model (Fennema & Carpenter 1996). In this model, the teacher engages students in mathematical thinking and in explaining their understandings. The focus in this strategy is on students' active involvement in the development of the mathematical ideas. A central idea of the approach is that children do not come to school as blank slates, that they already have certain intuitive understandings about mathematics and they can use a variety of fairly predictable strategies to solve problems. CGI research has focused on (a) the development of students' mathematical thinking, (b) instruction that influences that development, (c) teachers' knowledge and beliefs that influence their instructional practices, and (d) the way that teachers' knowledge, beliefs, and practices are influenced by their understanding of students' mathematical thinking (Carpenter, Fennema, Franke, Levi & Empson 2000).

Direct Instruction (DI) is an instructional approach that uses teacher explanation and modeling combined with student practice and feedback to teach concepts and skills. DI is teacher-centered in the sense that the teacher takes the responsibility for identifying the

lesson goals, and then plays an active role in explaining the content or skills to the students. Students are then given multiple opportunities to practice the concept or skill under the guidance of the teacher feedback (Eggen & Kauchak 1996).

WHAT RESEARCH LITERATURE SUPPORTS CGI AND DI?

Enhancing students' word problem solving is an important yet difficult task in classroom practices especially at the elementary level. It is important to consider how students learn to solve word problems. In the classroom, teachers must meet diverse student needs. These students use diverse strategies to attack word problems. Through instruction in the classroom, teachers need to understand their understanding of the mathematics in word problem solving.

Some researchers (Darch, Carnine & Gersten 1983; Gersten & Carnine 1981; Harper & Mallette 1993) claim that DI is superior in leading to positive student achievement with word problem solving. On the other hand, some researchers (Carpenter, Ansell, Frank, Fennema & Weisbeck 1993; Carpenter, Fennema, Peterson, Chiang & Loef 1988; Peterson, Fennema, Carpenter & Loef 1989; Villasenor & Kepner 1993; Staub & Stern 2002) claim that CGI provides significantly greater achievement with word problem solving for K–3 mathematics students. This review compares and analyzes the DI and CGI research on K–3 word problem solving mathematics students in a demonstration of my position in support of the CGI approach.

The two instructional approaches are different in many ways. Teacher-centered instruction focuses on the teacher talking, and students listening. The teacher has responsibility to choose the topics and evaluate student learning. Students typically work alone. Teacher-centered instruction was used in the most of the DI studies (Darch, Carnine & Gersten 1983; Gersten & Carnine 1981; Harper & Mallette 1993; Staub & Stern 2002). Teachers led instruction for 15–20 minutes and students worked independently at their seats on problem work sheets (Darch, Carnine & Gersten 1983). Students spent a lot of time on independent seat work in DI (Darch, Carnine & Gersten 1983; Harper & Mallette 1993). The main activities included explanation, demonstration, illustration, and step-by-step teacher modeling. Harper & Mallette (1993) studied an integrated instructional model that used teacher-centered instruction and peer mediation. Improvement in students' mathematical word problem solving was attained with the use of the combined procedures. Perhaps the teacher lead instruction alone was not enough in student mathematics word problem solving.

Many of studies in DI did not deal with traditional student classroom in grade K–3. DI studies were mostly conducted with low achievement students (Darch, Carnine &

Gersten 1983) or disadvantaged students from low income families (Gersten & Carnine 1981). DI might therefore, be effective primarily for special students and not the general students because of their ability levels.

DI is based in a behavioral perspective, spending more time on drill and practice, and feedback (Darch, Carnine & Gersten 1983; Gersten & Carnine 1981; Harper & Mallette 1993). In DI, it is premised that students can readily understand what is being presented and adequate practice with feedback is provided. According to this premise, students understand more if they have extended practice with feedback. However, there was no effect of extended practice in the study by Darch, Carnine & Gersten (1983). Through the drill and practice, and feedback students demonstrated computational skill but they did not improve much on their conceptual understanding in word problem solving.

DI does not directly address conceptual thinking skills; rather it is focused on addressing more basic skills (Staub & Sten 2002). In a DI classroom, the teacher explains how students need to solve word problems without any attention to conceptual thinking skills and reasoning (Darch, Carnine & Gersten 1983).

If you use the same number again and again you multiply.

If the big number is not given, the problem is a multiplication problem. If the big number is given it is a division problem.

DI pays little attention to student conceptual understanding rather is focused on procedural facility. While students retain some of general word problem solving strategies, they are not learning new strategies for solving word problems.

With student-centered instruction, students are actively engaged and construct their own mathematical knowledge. The focus is on both students and instructor. Students work in pairs, in groups, or alone depending on the purpose of the activity. Students talk without constant instructor monitoring.

Students answer each other's questions, using the instructor as an information resource. Students have some choice of topics. Students evaluate their own learning in addition to the instructor evaluation. Student-centered instruction was used in most of the CGI studies (Carpenter, Ansell, Frank, Fennema & Weisbeck 1993; Carpenter, Fennema, Peterson, Chiang & Loef 1988; Peterson, Fennema, Carpenter & Loef 1989; Villasenor & Kepner 1993; Staub & Stern 2002). Teachers facilitated students' meaningful understanding and problem solving. Teachers encouraged students to use a variety of problem solving strategies and listened to processes their students used (Carpenter, Fennema, Peterson, Chiang & Loef 1988).

Student-centered instruction is one of the current educational movements on how children learn. This approach allows children to identify the paths they find most fruitful in constructing their knowledge. Instruction is based on what children know and what they need to know, and they are encouraged to discuss how they solve problems. CGI is

well matched with this current educational movement.

CGI students used advanced and varied strategies to solve word problems studies (Carpenter, Ansell, Frank, Fennema & Weisbeck 1993; Carpenter, Fennema, Peterson, Chiang & Loef 1988; Peterson, Fennema, Carpenter & Loef 1989; Villasenor & Kepner 1993; Staub & Stern 2002). Peterson Fennema, Carpenter & Loef (1989) found that students in cognitively guided instruction demonstrated cognitive knowledge growth.

One of the most robust findings of this line of research has been that conceptual understanding is an important component of proficiency, along with factual knowledge and procedural facility. Students who memorize facts or procedures without under-standing often are not sure when or how to use what they know and such learning is often fragile. Learning arithmetic based on conceptual understanding is more advantageous than learning based on drill and practice (Staub & Stern 2002).

In CGI, children's informal or invented problem solving strategies were used often in the classroom (Carpenter, Ansell, Frank, Fennema & Weisbeck 1993; Peterson, Fennema, Carpenter & Loef 1989; Staub & Stern 2002). Student presentation of their knowledge in problem solving was abundant in CGI. Students communicated with each other mathematically (Carpenter, Ansell, Frank, Fennema & Weisbeck 1993; Staub & Stern 2002; Villasenor & Kepner 1993). Through the presentation of their knowledge in problem solving with mathematical communication, students demonstrated deeper understanding in word problem solving.

Many studies in CGI are conducted in typical student classrooms (Carpenter, Ansell, Frank, Fennema & Weisbeck 1993; Carpenter, Fennema, Peterson, Chiang & Loef 1988; Peterson, Fennema, Carpenter & Loef 1989; Villasenor & Kepner 1993; Staub & Stern 2002). Villasenor & Kepner (1993) studied minority students. Staub & Stern (2002) studied in Germany showing that CGI is effective internationally. It shows that CGI can be effective in Korea too.

CGI is based on the theory that students learn mathematics with understanding, actively building new knowledge from experience and prior knowledge and research on cognition (Carpenter, Ansell, Frank, Fennema & Weisbeck 1993; Carpenter, Fennema, Peterson, Chiang & Loef 1988; Peterson, Fennema, Carpenter & Loef 1989; Villasenor & Kepner 1993; Staub & Stern 2002).

In the CGI, researchers found cognitive knowledge growth, development of mathematics ideas, student's use of advanced strategies, and effectiveness at an international level in solving a wide range of problems. However DI studies use extended practice to enhance student academic improvement (Darch, Carnine & Gersten 1983; Gersten & Carnine 1981; Harper & Mallette 1993).

In CGI, teachers focus on students' mathematics thinking to enhance their reasoning and problem solving. They ask how students solved the problems, help them attend to

the details of the problem, and allowed them to engage in discussion among themselves about the problem and its solution.

The emphasis of CGI is on students' mathematical thinking and problem solving strategies and on the mathematical demands of the word problems. They develop deeper understanding of student thinking and problem solving strategies.

WHAT HAVE WE LEARNED FROM
THESE TWO INSTRUCTIONAL STRATEGIES?

Multiple studies identify that the DI instruction model is superior to other instructional models in leading to positive student achievement with word problem solving (Darch, Carnine & Gersten 1983; Gersten & Carnine 1981; Harper & Mallette 1993). Students used the step-by-step teacher modeling and they mastered the steps. Through drill and practice, students performed all the steps, but they did not develop their own strategies for word problem solving. Also, a later effect of DI indicated that students retained general word problem-solving strategies taught in the program. However, students did not learn new computational skills and new word problem solving strategies. The integration of two approaches, DI and the Classwide Student Tutoring Teams, did not show sufficient information about students' word problem-solving strategies. Apparently, the DI model did not develop students' word problem-solving skills.

Teachers whose students (K–3) demonstrated strong achievement in addition, subtraction, multiplication and division word problem solving tended to agree with a cognitively-based perspective that instruction needed to build upon children's existing knowledge and that teachers needed to help students to construct mathematical knowledge rather than to passively absorb that information (cf. Carpenter, Ansell, Frank, Fennema & Weisbeck 1993; Carpenter, Fennema, Peterson, Chiang & Loef 1988; Peterson, Fennema, Carpenter & Loef 1989; Villasenor & Kepner 1993; Staub & Stern 2002). Research has shown that CGI students outperformed non-CGI students in all categories.

As students advanced through the grades, the CGI students, including minority students, changed and outperformed the others in solving word problems that were not specifically taught in that approach. Through the CGI approach, children were able to solve a wide range of word problems much earlier than generally had been presumed. CGI students exceeded non-CGI students in number facts as well as in word problem solving.

DISCUSSION

Research has demonstrated that CGI provides a more appropriate instructional method than DI in K–3 grades for teaching word problem solving. Using students' understanding of mathematics is more effective than drill and practice to attain high achievement in word problem solving. In the research using the CGI procedures, students gained higher scores on word problem solving and used advanced strategies to solve word problems. In drill and practice classes, students used limited word problem-solving strategies. However, in CGI classes teachers incorporated word problems in instruction when teaching addition, subtraction, multiplication, and division more than DI classes. This might affect the students' score of word problem solving in CGI classes. Most studies did not directly compare CGI and DI in word problem solving. Fortunately, Staub & Stern (2002) directly compared student achievement in word problem solving in DI and CGI providing same word problems. This study showed that CGI is more appropriate approach to teach word problem solving.

According to DI studies (Darch, Carnine & Gersten 1983; Gersten & Carnine 1981; Harper & Mallette 1993), teachers were not involved in professional development for teaching with the DI model. DI approach is widespread and most teachers are used to it because they have learned through this model since they were students in this model and their teacher preparation program used the model. However, teachers are not familiar with the CGI approach. In the most of CGI studies (Carpenter, Ansell, Frank, Fennema & Weisbeck 1993; Carpenter, Fennema, Peterson, Chiang & Loef 1988; Peterson, Fennema, Carpenter & Loef 1989; Villasenor & Kepner 1993; Staub & Stern 2002) volunteer teachers participated in professional development for CGI. Volunteer teachers have a willingness to change their instruction and that agreement might have affected student achievement in word problem solving. The results suggest that teachers need to take part in a professional development program for CGI in supporting and developing their ability to involve students in mathematical thinking, changing their instructional methods and content.

Theoretical beliefs about children learning have described children as constructing their own mathematical knowledge. Mathematics instruction needs to be organized to facilitate and support children's construction of knowledge involving students in discussing their understanding and listening other students' word problem solving strategies.

CGI approach is the research-based program and applicable for all students in grade K–3. The approach is not a specifically defined set of activities, rather a principle that requires teachers to engage students in mathematical thinking. How teachers do that is

determined by the teacher. As a result, what teachers do to engage students in discussing their understandings of mathematics may be quite varied. Because of the variety, at the beginning of teacher education or professional development program teachers require practice in the approach. These differences need to be considered in order to clarify the CGI methods. To reduce the confusion, teachers may need to have knowledge about student mathematical thinking based on research and also may need appropriate teaching experiences. They need help in planning for their students to apply their ideas and to discuss their mathematical thinking.

The CGI approach has successfully focused on grades K–3. However it has not been extended to higher grades and other areas of mathematics. The approach needs to be expanded to work with children in grades four and above and needs to be considered for teaching a broader range of mathematics, including geometry, probability, algebra, and other advanced areas of mathematics. Fortunately Carpenter, Franke and Levi are investigating students' understandings of algebra and teaching in grade 1–6. CGI has potential to change how teachers gather their students' understandings and how they teach their students utilizing their personal knowledge and understandings. Korean teachers and teacher educators need to consider implementing CGI teaching strategies to enhance student achievement in word problem solving and facilitate student mathematical thinking skills.

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