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Understanding Student-Centered Teaching Practices in Elementary Mathematics Classrooms

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Although student-centered teaching practices have been advocated in mathematics education reform, implementing them at the classroom level remains challenging. This exploratory case study examined two unevenly successful student-centered approaches to see how teachers understand and characterize reform, and to articulate issues in implementing reform ideas. The comparison and contrast between the classrooms showed similar classroom social norms but dramatically different mathematical practices. This affords the possibility of exploring the challenges of reform for teachers and other personnel who are attempting to move teaching practices towards the student-centered ideals.

Keywords: student-centered teaching practice, mathematics education reform, students' learning opportunities
ZDM Classification: D42
MSC2000 Classification: 97D40

1. BACKGROUND TO THE STUDY

This study is based on the part of a cross-cultural investigation between Korea and the U. S. of how teachers understand the objectives of the current reform movements, and how they characterize reform relative to their own experiences and priorities. This study as a whole intends to articulate new issues and problems in implementing current mathematics education reform ideals. This paper describes the U. S. portion of the study.

In a typical U. S. mathematics class the teacher begins with a brief review of short-answer questions or homework problems, demonstrates how to solve routine problems through the use of rules and formulas, and then assigns similar problems. During the remainder of the class, students practice while the teacher moves around the room answering individual questions (TIMSS 1996). Against this common instructional practice, educational leaders are seeking to change the teacher-centered pedagogy towards a student-centered approach.

The National Council of Teachers of Mathematics has initiated a reform movement in which teachers are supposed to manage classroom teaching practices in ways that students' contributions and responses constitute the center of mathematics activity (NCTM 1991, 2000). Instead of listening and following teachers' instruction, students should have the opportunity to be enculturated into a mathematical discourse in which they invent, explain, and justify their own mathematical ideas, and critique others' ideas.

It seems that many U.S. teachers who are attempting to implement reform improvise methods that do not quite meet the intent and vision of reform ideas (Lester & Ferrini-Mundy 2003; Remillard & Bryans 2004). Many teachers report familiarity with and adherence to reform ideas, but their actual classroom teaching practices do not reflect a deep understanding of reform. Their methods tend to reflect only the surface social practices of the recommended methods (Cohen 1990; NCES 1996). They have not re-conceptualized the learning process in keeping with the new teaching processes.

Given this background, the purpose of this paper is to compare and contrast the classroom social norms and students' learning opportunities of two U. S. second grade classrooms that are attempting to implement student-centered instructional methods in mathematics.

The classes are selected because of their unequal success in implementing student-centered teaching methods. Comparing the two classes will provide an opportunity to reflect on the problems of implementing educational reform. The research questions are as follows:

(a) What are the processes that constitute more successful student-centered pedagogy in the elementary mathematics classroom? Specifically, in what ways do the teacher and students create such a mathematics classroom? What kinds of learning opportunities arise for the students in the classroom?
(b) What are the processes that constitute less successful student-centered pedagogy in the elementary mathematics classroom? Specifically, in what ways do the teacher and students create such a mathematics classroom? What kinds of learning opportunities arise for the students in the classroom?
(c) What are the differences and similarities between more successful and less successful student -centered classrooms? What are the challenges for reformers in changing the culture of primary level mathematics teaching?

2. RESEARCH METHODOLOGY

This study is an exploratory, cross-cultural, qualitative, comparative case study using the *grounded theory approach* based on the *constant comparative analysis* (Strauss & Corbin 1998) for which the primary data sources will be classroom video recordings and transcripts (Cobb & Whitenack 1996). This study fits into *holistic and multiple-case design* where single unit of analysis is used in various cases (Yin 2002).

Case study methods are an especially useful mode of inquiry when the boundaries between phenomenon and context are not clearly evident (Yin 2002). This is particularly apt for teaching practices, which can be broadly classified using terms like "student-centered" or "teacher-centered" but which only achieve their full definition within the context of particular classroom micro-cultures.

The purpose of exploratory research is to articulate new issues and problems, not to answer questions. The small number of classes, and small number of observations of each class, does not permit firm conclusions about the teaching practices. Rather the purpose of this exploratory research is to pave the way for further larger scale cross national studies involving Korea and the U. S. As Schmidt, Jorde, et al. (1996) observed, large scale cross national research can benefit greatly from previous exploratory research in which the international team has the opportunity to identify, dispute, and achieve consensus on the key issues of the investigation.

2.1 Data Collection

As a kind of purposeful sampling (Patton 1990), it was attempted to find second-grade mathematics teachers who aspire toward student-centered instructional methods in mathematics. Seventeen teachers were recommended by university professors, instructors, district supervisors, elementary mathematics specialists, primary school teachers, or persons from the state department of education. Each class was observed up to three times in order to confirm the possibility for this study. The classes were then analyzed by the extent to which students' ideas were solicited and became the center of mathematical discussion. Two classes were finally selected because of their unequal success. To be clear, both classes displayed a student-centered approach but one of them implemented it in ways that are compatible with current mathematics education reform initiatives.

A total of 15 mathematics lessons were video-taped using two or three camcorders in order to capture the teaching practices from different perspectives; one for the teacher, another for the students, and/or the third for the whole class. Additionally, audio-tapes were used to capture the students' conversation in small groups. Students' papers and projects were also collected. A total of 32 videotapes and 13 audiotapes were used for analyses of classroom teaching practices.

Following initial reviews of the videotapes and audiotapes, the two teachers, Ms. E and Ms. M were interviewed. The interviews had two purposes: One to clarify some points that were unclear from the videotaped lessons and the other to gain information on how they have constructed their own teaching methods. A total of 8 hours interview with each teacher was audio-taped. For the second purpose, significant influences on the teachers' conceptions of mathematics and its teaching were explored in order to better understand their current practices. This interview probed 12 topic areas:

(a) early influences on becoming a teacher;
(b) the decision to become a teacher;
(c) the teacher education years;
(d) early mathematics interests;
(e) early teaching experiences;
(f) career path;
(g) influence of peers within the school;
(h) influence of administrators;
(i) professional development
(retraining course, further courses and degrees, conferences and workshops);
(j) professional self-development;
(k) mathematics teaching; and (l) educational policies.

2.2 Data Analysis

The video-and audio-taped lessons were transcribed in a two-column format, the second column for notes related to the analysis. This study uses grounded qualitative methods based on the constant comparative methods. More specific guidance came from the methods that Cobb and Whitenack (1996) describes for their analyses of classroom video recordings and transcripts. Analyses of this study had two steps: analysis of individual classes for the first two research questions, and comparative analysis between the two classes for the third research questions. Interview data were incorporated into the teacher's approaches whenever they provided useful background information to understand her classroom teaching practices.

In keeping with the research questions, individual classes have been analyzed using the four categories and questions described in Table 1. An initial analysis was based on four categories from Cobb and Whitenack, which are "the children's social relationship, mathematical meanings, learning opportunities, and mathematical learning" (1996 pp. 219–220). As emerged in the process of analyses, however, it was apparent to modify such categories, partly because of the interest in teaching practices over small group mathematical activities.

Table 1. Interpretive framework for individual analysis of each classroom

Category	Main Questions
Classroom Flow	What is the general flow of classroom teaching practices, including classroom atmosphere, classroom activities, gross patterns of interaction, the roles and expectations adopted by the teacher and students, and sociomathematical norms?
Teacher's Approaches	What is the teacher's curricular intention as reflected in her participation in and organization of the class? How are the teacher's conceptions of mathematics and its teaching related to implemented teaching practices?
Students' Approaches	What is the students' learning intention as reflected in their patterns of classroom participation?
Students' Learning Opportunities	Given the approaches of teachers and students, what kinds of mathematical learning opportunities are likely to arise for students' concepts, connections, and tools?

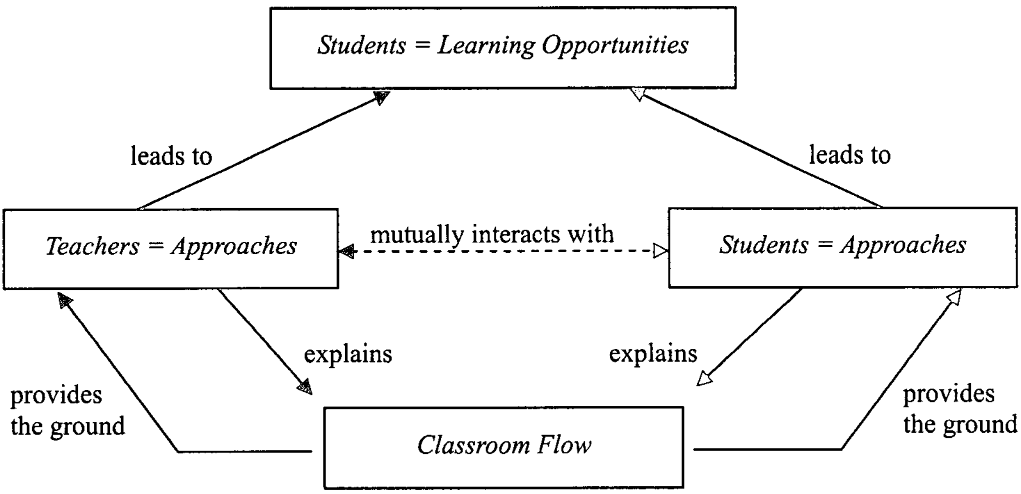

Figure 1. Relations among four categories in the interpretive framework

The four categories are closely interrelated. Classroom flow is specifically descriptive including important episodes. Both the teacher's approaches and students' approaches are based on the classroom flow in that each agenda is inferred from it and thus consistently grounded in the classroom activities. In other words, the teacher's and students' approaches are taken to be the most supportable inferences from the observations reported in classroom flow. Retrospectively, classroom flow can be consistently explained by the students' and teacher's agendas. Finally, the students' learning opportunity comes out of the teacher's and students' approaches. These relationships are summarized in the Figure 1.

3. RESULTS

3.1 Classroom Flow

The general atmosphere in the two classrooms was very similar. In both classrooms teachers established open and permissive atmospheres in which students' ideas and their mistakes were welcomed. Both classes were dynamic in that students actively responded to the teacher and one another. Both teachers tried to give each student an equal chance to present. Moreover, when students expressed a negative response in the middle of their friend's presentation, the teacher gave the presenting student an opportunity to finish his or her argument. Both teachers emphasized that working hard more important than getting a right answer. Students in both classrooms frequently expressed their excitement for various classroom activities. The teacher and the students in each classroom laughed a lot while engaged in mathematics activities.

Gross patterns of classroom activities were similar. Ms. E's classroom included estimation, problem solving, and collective activities with various manipulative materials. Ms. M's classroom included problem-solving, instruction of how to play a game, students' play, and whole-group discussion. The sequence of these classroom activities was consistent in Ms. M's classroom, whereas in Ms. E's classroom the sequence was not.

The usual pattern of social interaction in the two classrooms was similar in some respects: (a) the teacher initiated an activity or gave students a mathematical problem, (b) students independently solved the given problems, (c) the teacher asked students to report their solution methods to the whole class, (d) students presented their solution methods, and (e) the teacher mediated the classroom discussion. The difference was that Ms. E tended to control the discussion toward one direction, whereas Ms. M tended to facilitate the discussion. For instance, Ms. E often evaluated students' answers or expressed her expectation of what students would present. However, Ms. M provided further questions for clarification after a student's presentation, which often led to other students' participation in the discussion based on agreement or disagreement. The sequence of teacher-student-teacher-student turn taking was repeated in Ms. E's classroom, whereas the direct student-student interaction was found in the whole group discussion in Ms. M's classroom.

3.2 Teacher's Approaches

There were many similarities in the two classrooms with regard to the expectations, roles, and obligations adopted by each teacher (see Table 2). For instance, both teachers stressed group cooperation and provided encouragement with positive expectations. They often used manipulative materials and used an enjoyable activity format for students. There were also differences in that Ms. E emphasized the standard algorithm with line alignment and the order of computation, whereas Ms. M did not converge towards a standard method to solve problems. Difference was also noticeable when students had different answers. Ms. E usually explained with examples or chose the right answer with praise, whereas Ms. M let the students argue for a while and then mediated the discussion by summarizing the main argument in each position.

Table 2. Comparison: Teacher's Approaches

<table>
<tr><th>Degree of Similarity*</th><th>Ms. E</th><th>Ms. M</th></tr>
<tr><td>●</td><td colspan="2">- provided students with chances to solve problems for themselves.
- encouraged students to present their solution methods/strategies.
- repeated or amplified students' ideas to help disseminate them to the whole class.
- expressed excitement about students' novel ideas.
- sometimes changed the role of questioning and answering with students.
- shared her positive expectation to every child.
- took care of students' emotional states, especially when they made mistakes.
- emphasized the process of problem solving.
- often used manipulative materials and tried to connect symbolic/abstract representation with pictorial/concrete representation.
- encouraged students to work each other.
- frequently used an enjoyable activity format.
- asked students to pose a story problem for addition or subtraction.
- circulated and provided some help while students worked in their groups.
- lavishly provided praise and encouragement to support students' efforts.</td></tr>
<tr><td>◐</td><td>- when students were confused with directions, she provided detailed illustration with examples.
- sometimes asked for different methods.
- picked out something interesting from students responses, but it was not necessarily mathematically significant.</td><td>- posed a more challenging problem based on students' contributions.
- always emphasized different solution methods.
- frequently used observation of students' activities for productive mathematical discussion.</td></tr>
</table>

(Continue)

○	- sometimes provided her own solution strategies, instead of letting students invent them. - taught formal algorithm for subtraction with step-by-step instruction using students' contributions. - specifically emphasized line alignment and the order of computation. - provided direct explanation, a hint, an example or chose the right answer with praise, when students had different answers (or difficulties). - even after a student's novel idea, the teacher directly expressed interest in using algorithm. - emphasized memorization of basic addition/subtraction facts, and tested memorization periodically using a paper-and-pencil format. - very often set time in doing classroom activities.	- did not provide her own methods. - frequently checked whether the teacher's interpretation corresponded to what students meant after their presentation. - did not teach formal algorithm. - carefully checked whether students understood place value concept, when they used a vertical format of computation. - students were allowed to argue for a while, when there were debates among them. Later the teacher mediated by summarizing the main argument in each position. - emphasized the use any method which made sense to students. - connected students' contributions with previous activities they completed. - assessed individual student's understanding by conducting an interview. - sometimes led students to initiate the topics for whole-group discussion. - introduced multiplication responding to her students' interest and request.

* Note: Degree of similarity: ● = a lot; ◐ = somewhat; ○ = very little

3.3 Students' Approaches

There were many similarities with regard to the expectation, obligations, or roles adopted by students across the classrooms. The students solved given problems independently, presented their solution methods in the whole class, and complied with the teacher's instruction (see Table 3). The main difference lay in the concern about right answer. The students in Ms. E's classroom expressed their excitement when they got the answer. Some students waited for the teacher's confirmation while doing their group activities.

Table 3. Comparison: Students' Approaches

Degree of Similarity*	The Students in Ms. E's Classroom	The Students in Ms. M's Classroom
●	- solved given problems independently whenever they were supposed to. - presented their individual or group solution methods to the whole class. - usually listened carefully to their friends' explanations. - complied with the teacher's instruction. - collaborated with each other while working together. - pointed out mistakes made by others or the teacher.	
◐	- a few students invented their own solution methods for a given subtraction problem, even when the teacher indicated to use formal algorithm. - asked the teacher to clarify when her instructions were confusing. - in their independent problem solving phase, several students used specific strategies, with which the teacher provided them later. - gave more attention to the teacher whenever requested.	- focused on finding multiple solution methods to given mathematics problems - were eager to show their work to the teacher. - whenever students did not understand or had a question to the teacher's instruction, they asked for clarification. - attempted to support or dispute the ideas by other students or the teacher with specific examples. - were more engaged in their group activities and discussion.
○	- some students checked the teacher's response before finishing their presentation. - when students found the right answer, they expressed excitement. - while doing collective problem solving activities, some groups waited for the teacher's check or confirmation for their decisions.	- were often engaged in debate and argued for or against the ideas discussed without the teacher's initiation. - didn't seem to be concerned about right answer - articulated their peers' explanation, when the teacher did not understand the original contribution. - some students kept their ways of approaching a given task. - sometimes asked for more challenging problems.

* Note: Degree of similarity: ● = a lot; ◐ = somewhat; ○ = very little

However, Ms. M's classroom, the answer itself was not the main focus of discussion. For instance, students often argued for or against ideas without the teacher's initiation. Moreover, some students used their ways of approaching a given task in their small groups. Given that the expectations adopted by students stem from the teacher, this difference can be fully understood in conjunction with the different roles perceived by the teacher, which were described above.

3.4 Students' Learning Opportunities

Students' learning opportunities within the two classrooms were very much constrained by the mathematically significant distinctions embedded within the classroom discourse. Whereas socially both groups of students had many opportunities to participate and to experience success in their efforts, the mathematical content in Ms. E's classroom was mainly procedural. Those students had the opportunity to develop skills in solving problems (*e.g.*, using standard algorithms for computation), but they had very limited opportunities to learn the transition from informal to formal strategy of doing computation. In other words, they had little opportunity to reflect on the conceptual underpinnings of the arithmetic they were studying. In contrast, the students in Ms. M's classroom were continually exposed to mathematically significant ways of knowing, valuing and arguing. For instance, they had the chance to compare and contrast their own methods with peers, and to begin to see that certain methods had mathematically significant advantages over others. They had the opportunity to make conceptual sense of the arithmetic they were studying. Moreover, there was evidence that the students in Ms. M's classroom were becoming self-motivated in their pursuit for mathematical meaning.

Table 4. Comparison: Students' Learning Opportunities

Degree of Similarity*	Ms. E's Classroom	Ms. M's Classroom
○	- the mathematical content was mainly procedural. - learned mathematics as a static or fixed discipline. - developed skills in solving only routine mathematical problems. - had limited opportunities to present their novel strategies or ideas. - focused more on the teacher's instruction, without autonomous motivation for mathematical sense-making. - limited learning opportunities with regard to the transition from informal to formal strategy of doing computation.	- had the opportunity to develop conceptual basis of the arithmetic being studied. - learned mathematics as a dynamic discipline. - enculturated in the particular classroom microculture where specific mathematical ways of knowing, valuing, and arguing were emphasized. - self-motivated in their pursuit for mathematical meaning - had the chance to compare and contrast their own methods with peers, and to begin to see that certain methods had mathematically significant advantages over others. - had the chance to value themselves and their peers as mathematical problem solvers.

* Note: Degree of similarity: ● = a lot; ◐ = somewhat; ○ = very little

Table 4 shows the dramatic differences with regard to the learning opportunities in both classrooms.

4. CONCLUSIONS

The principal concern in implementing reform ideals has to do with the extent to which changes in teaching practice translate into changes in the learning opportunities that students will encounter in their mathematics classes. Despite the similarity of social practices, a close examination of mathematical discourse in the two classrooms showed dramatic differences with regard to the teachers' use of students' ideas. One teacher focused on a pre-given formal algorithm after eliciting students' ideas, whereas the other teacher carefully orchestrated the path of discourse towards conceptual understanding. This implies that the dynamic engine of learning opportunities is not located in the general social norms of the classroom. Rather learning opportunities arise from the ways in which mathematically significant distinctions are embedded within classroom social processes.

The teacher in a reform classroom in the U.S. is envisioned as a facilitator of students' individual and collective problem solving based on their own sense making about mathematics (NCTM 1991, 2000). However, reforming mathematics teaching is not simply a matter of changing the surface features of instructional practices. Rather it involves reconceptualizing the relationship between social processes of the class and the psychological states of the students. There is no reason to expect that this reconceptualization should come easily, even for teachers who are dedicated and committed to changing their teaching practices. The difficulties and successes of the two teachers afford the possibility of exploring the challenges of reform for teachers and other personnel who are attempting to move teaching practices towards the student-centered ideals. Real instructional change occurs only at the classroom level, as teachers grapple with their own values and priorities relative to the ideals promoted for the profession (Darling-Hammond & Sykes 1999). Reformers should be prepared to provide extensive resources for teachers to enable them to undertake this transformation in conjunction with more capable peers.

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