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Testing of Stochastic Trends, Seasonal and Cyclical Components in Macroeconomil Time Series[1]

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Abstract

We propose in this article a procedure for testing unit and fractional orders of integration, with the roots simultaneously occurring in the trend, the seasonal and the cyclical component of the time series. The tests have standard null and local limit distributions. However, finite sample critical values are computed, and several Monte Carlo experiments conducted across the paper show that the rejection frequencies against unit (and fractional) orders of integration are relatively high in all cases. The tests are applied to the UK consumption and income series, the results showing the importance of the roots corresponding to the trend and the seasonal components and, though the unit roots are found to be fairly suitable models, we show that fractional processes (including one for the cyclical component) may also be plausible alternatives in some cases.

Keywords : Long memory; Fractional integration; Seasonality; Stochastic cycles

1. Introduction

It is well-known that many macroeconomic time series contain trends, seasonal as well as cyclical components. However, while the literature on the trend and the seasonal components is quite extended, little attention has been paid to the cyclical component of the series. Initially, the trend was explained in terms of deterministic (linear) functions of time. Later on, however, it was observed that the trend component changed or evolved over time, and stochastic approaches (based on first or second differences of the data) were proposed, especially after the seminal paper of Nelson and Plosser (1982). In that paper, following the work and ideas of Box and Jenkins (1970), they showed that many US macroeconomic series could be specified in terms of unit root processes. Following that work, many test statistics were developed for testing unit roots, (e.g., Dickey and Fuller, 1979; Phillips and Perron, 1988; Kwiatkowski et al., KPSS, 1992; etc.). Similarly, for the seasonal component, deterministic models based on seasonal dummy

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variables were shown to be inappropriate in many cases, and seasonal unit-root tests were developed by Dickey, Hasza and Fuller (DHF, 1984); Hylleberg, Engle, Granger and Yoo (HEGY, 1990); Canova and Hansen (1995), and others. In relation to the cyclical component, the literature is scarce. A large number of publicatons exist on business cycles, chronology of peaks and troughts, etc. However, most of these papers do not consider the cyclical structure by itself, that is, independently of the trend and the seasonal components. Ahtola and Tiao (1987) developed tests for unit root cycles and, more recently, Bierens (2001) and Gil-Alana (2001) propose unit-root cycles for modelling several macroeconomic series. In this article we try to combine all these components in the same framework, testing for the presence of stochastic trends, seasonal and cyclical components in a unified testing procedure. The tests are due to Robinson (1994) and, unlike most of the procedures mentioned above, (which are based on autoregressive (AR) alternatives), the tests of Robinson (1994) are nested in fractional models. Additionally, they have standard null and local limit distributions.

The outline of the paper is as follows: Section 2 briefly describes the testing procedure of Robinson (1994), which allows us to test for the presence of unit roots (with integer or fractional orders of integration) in the trend, the seasonal and the cyclical components of the series. In Section 3 we compute finite sample critical values of the tests and a Monte Carlo experiment is conducted to examine the size and the power properties of the tests in finite samples. In Section 4, the tests are applied to the quarterly, seasonally unadjusted, consumption and income series in the UK, while Section 5 concludes.

2. Testing of Roots in the Trend, the Seasonal and the Cycles

A slight variation in the set up in Robinson (1994) leads to the model:

$$(1 - L)^{d_1+\theta_1} (1 - L^4)^{d_2+\theta_2} \prod_{j=3}^{h} (1 - 2\cos w_r L + L^2)^{d_j+\theta_j} x_t = u_t, \tag{1}$$

for a given number h, where $d_1, \cdots, d_h$ are given real numbers, x_t is the time series we observe, and u_t is an I(0) process, defined, for the purpose of the present paper, as a covariance stationary process with spectral density function that is positive and finite at any frequency on the spectrum. Under the null hypothesis, defined by:

$$H_o: \ \theta \equiv (\theta_1, \theta_2, \ldots, \theta_h)' = 0, \tag{2}$$

the model in (1) becomes:

$$(1 - L)^{d_1} \; (1 - L^4)^{d_2} \prod_{j=3}^{h} (1 - 2\cos w_r L + L^2)^{d_j} \, x_t \; = \; u_t . \tag{3}$$

We see that this is a very general specification, which allows us to consider several models of interest. For example, imposing $d_1 = 1$ and $d_j = 0$ for $j \neq 1$, we test for an I(1) process and, if $d_1 = d$, we test for fractionally integrated hypotheses. Empirical applications of this version of the tests can be found in Gil Alana and Robinson (1997), Gil Alana (2000a), and other empirical studies of I(d) processes at the long run or zero frequency are, for example, Diebold and Rudebusch (1989), Baillie and Bollerslev (1989) and Baillie (1996). Similarly, if $d_2 = 1$ and $d_j = 0$ for $j \neq 2$, we test for seasonal unit roots and, if $d_2 = d$, for seasonal fractional integration. (See, e.g., Porter Hudak, 1990 and Gil Alana and Robinson, 2001). Finally, unit root cycles will be tested if $d_3 = 1$ and $d_j = 0$ for $j \neq 3$ (Bierens, 2001; Gil Alana, 2001), and extensions to fractional models have been studied by Gray et al. (1989, 1994), Chung (1996a, b), Ferrara and Guegan (2001) amongst others. Each of these works tests for roots in the trend, the seasonal and the cycle separately. However, we can take h = 3 in (1) and (3) and consider the null model:

$$(1 - L)^{d_1} \; (1 - L^4)^{d_2} \; (1 - 2\cos w_r L + L^2)^{d_3} \, x_t \; = \; u_t , \tag{4}$$

for different real values d_1, d_2 and d_3, testing thus for roots at all the components simultaneously: d_1 will indicate the degree of integration of the trend component, i.e., with a root occurring at the long run or zero frequency; d_2 will be the order of integration of the seasonal component, i.e., implying roots at 0, π and $\pi/2$ ($3\pi/2$) (of a cycle 2π); while d_3 will be the order of integration of the cycle, with the periodicity determined by $w_r = 2\pi r/T$, $r = T/j$ and j indicating the number of periods per cycle. Specifically, the test statistic is:

$$\hat{R} = \frac{T}{\hat{\sigma}^4} \hat{a}' \hat{A}^{-1} \hat{a} , \tag{5}$$

where T is the sample size and

$$\hat{a} = \frac{-2\pi}{T} \sum_j^{*} \psi(\lambda_j) g(\lambda_j;\hat{\tau})^{-1} I(\lambda_j); \quad \hat{\sigma}^2 = \sigma^2(\hat{\tau}) = \frac{2\pi}{T} \sum_j^{*} g(\lambda_j;\hat{\tau})^{-1} I(\lambda_j); \quad \lambda_j = \frac{2\pi j}{T};$$

$$\hat{A} = \frac{2}{T}\left(\sum_j^{*} \psi(\lambda_j)\psi(\lambda_j)' - \sum_j^{*} \psi(\lambda_j)\hat{\varepsilon}(\lambda_j)' \, x \left(\sum_j^{*} \hat{\varepsilon}(\lambda_j)\hat{\varepsilon}(\lambda_j)' \right)^{-1} x \sum_j^{*} \hat{\varepsilon}(\lambda_j)\psi(\lambda_j)' \right);$$

$$\psi(\lambda_j) = \left(\log\left|2\sin\frac{\lambda_j}{2}\right|;\ \log\left|2\sin\frac{\lambda_j}{2}\right| + \log\left(2\cos\frac{\lambda_j}{2}\right) + \log\left|2\cos\lambda_j\right|;\ \log\left|2\left(\cos\lambda_j - \cos w_r\right)\right| \right)'$$

$$\hat{\varepsilon}(\lambda_j) = \frac{\partial}{\partial \tau} \log g(\lambda_j; \hat{\tau}),$$

where $g(\lambda;\ \tau)$ is a known function related to the spectral density of u_t: $f(\lambda;\tau) = (\sigma^2/2\pi)\ g(\lambda;\ \tau)$, evaluated at $\hat{\tau} = \arg\min \sigma^2(\tau)$. Note that these tests are purely parametric and therefore, they require specific modelling assumptions regarding the short memory specification of u_t. Thus, for example, if u_t is white noise, $g \equiv 1$ ($\hat{\varepsilon}(\lambda_j) = 0$), and if u_t is an AR process of form $\phi(L)u_t = \varepsilon_t$, $g = |(e^{i\lambda})|^{2}$, with $\sigma^2 = V(\varepsilon_t)$, so that the AR coefficients are a function of τ. $I(\lambda)$ is the periodogram of u_t defined as in (4), and the summation on * in the above expressions are over $\lambda \in M$ where $M = \{\lambda: -\pi < \lambda < \pi,\ \lambda \notin (\rho_k - \lambda_1,\ \rho_k + \lambda_1),\ k = 1,2,\cdots s\}$, such that ρ_k, $k = 1,2,\cdots s < \infty$ are the distinct poles of $\Psi(\lambda)$ on $(-\pi,\pi]$.

Based on H_o in (2), (with h = 3), Robinson (1994) showed that, under certain regularity conditions, the above test statistic has an asymptotic distribution given by:

$$\hat{R} \rightarrow \chi^2_3, \qquad as\ \ T \rightarrow \infty. \tag{6}$$

Thus, a test of (2) will reject H_o against the alternative H_a: $\theta \neq 0$ if $\hat{R} > \chi^2_{3.\alpha}$, where Prob $(\chi^2_3 > \chi^2_{3.\alpha}) = \alpha$. Moreover, Robinson (1994) shows that the test is efficient in the Pitman sense, i.e., that against local alternatives of form: H_a: $\theta = \delta\ T^{-1/2}$, with $\delta \neq 0$, the limit distribution is $\chi^2_3(\upsilon)$ with a non centrality parameter, υ, that is optimal, under Gaussianity of u_t, with respect to any other rival regular statistic. However, in spite of its standard limit distribution, we know that in finite samples, the results of the tests based on the asymptotic critical values can substantially differ from those obtained based on finite sample values (see, e.g. Gil Alana, 2000b). Thus, in the following section, we evaluate finite sample critical values of the test described just above.

3. A Finite Sample Experiment

In Table 1 we compute finite sample critical values of the test statistic given by $\hat{R}$ in (5), testing H_o (2) in a model given by:

$$(1-L)^{d_1+\theta_1}\ (1-L^4)^{d_2+\theta_2}\ (1-2\cos w_r L + L^2)^{d_3+\theta_3}\, x_t \;=\; u_t, \tag{7}$$

with white noise u_t. We generate Gaussian series, using the routines GASDEV and RAN3 of Press, Flannery, Teukolsky and Wetterling (1986), computing the values for w_r = 2πr/T, r = T/j, r = T/2, T/4, T/5, T/10 and T/20, and T = 100, 200 and 300. Note that since the model does not include deterministic regressors (like an intercept or a linear trend), there are not nuisance parameters, neither in the limit distribution nor in the finite sample one, and d does not affect the finite sample critical values, unlike the parameter r, which is required for the computation of the test statistic.

<Table 1> Finite sample critical values of $\hat{R}$ given by (5)

Size	α	Values of r					Asymptotic Crit. values
		T/2	T/4	T/5	T/10	T/20	
100	1%	24.53	17.99	17.75	18.73	18.75	11.35
	5%	11.83	10.53	9.93	9.50	9.87	7.82
200	1%	24.14	16.70	17.01	16.87	18.01	11.35
	5%	10.94	9.17	8.92	8.82	8.83	7.82
300	1%	23.11	15.36	15.69	16.08	15.79	11.35
	5%	10.80	8.87	8.64	8.42	8.71	7.82

10,000 replications were used in each case.

We observe in this table that the critical values are in all cases higher than those given by the χ^2_3 distribution, implying that when testing H_o: θ = 0 against H_a: θ ≠ 0, the tests based on the asymptotic critical values will reject the null more often than those based on the finite sample ones. We also observe that these values substantially change with r, though in a non monotonic way and, as we increase the number of observations, all of them tend to approximate to the asymptotic values given by the χ^2_3 distribution.

In Table 2 we examine the size and the power properties of the tests. We assume that the true model is given by (4) with white noise u_t and r = T/10. The choice of r is completely arbitrary. Other values were also tried and the results were very similar to those reported in this table. The alternatives are in all cases of form as in (7) with θ_1, θ_2 and θ_3 equal to -1, -0.5 and 0, i.e., corresponding to processes with orders of integration of 0, 0.5 and 1. The same value of w_r is taken under both the null and the alternative hypotheses. Thus, the rejection frequencies corresponding to $\theta_1 = \theta_2 = \theta_3 = 0$ will indicate the size of the tests. The nominal size is 5% and 10,000 replications are used in each case.

<Table 2> Rejection frequencies of $\hat{R}$ given by (5)

True model: $(1 - L)(1 - L^4)(1 - 2\cos w_r L + L^2)\, x_t = \varepsilon_t; \quad r = T/10.$

Alternatives: $(1 - L)^{d_1} (1 - L^4)^{d_2} (1 - 2\cos w_r L + L^2)^{d_3}\, x_t = \varepsilon_t; \quad r = T/10.$

d_1	d_2	d_3	T = 100		T = 200		T = 300	
			FSCV	ASYMP	FSCV	ASYMP	FSCV	ASYMP
0.00	0.00	0.00	1.000	1.000	1.000	1.000	1.000	1.000
0.00	0.00	0.50	0.780	0.827	0.823	0.931	0.998	1.000
0.00	0.00	1.00	1.000	1.000	1.000	0.999	1.000	1.000
0.00	0.50	0.00	1.000	1.000	1.000	1.000	1.000	1.000
0.00	0.50	0.50	0.772	0.821	0.973	0.995	1.000	1.000
0.00	0.50	1.00	1.000	1.000	0.999	1.000	1.000	1.000
0.00	1.00	0.00	1.000	1.000	1.000	1.000	1.000	1.000
0.00	1.00	0.50	0.763	0.817	0.953	0.964	1.000	1.000
0.00	1.00	1.00	1.000	1.000	1.000	1.000	1.000	1.000
0.50	0.00	0.00	1.000	1.000	1.000	1.000	1.000	1.000
0.50	0.00	0.50	0.821	0.858	0.999	1.000	1.000	1.000
0.50	0.00	1.00	0.992	0.992	1.000	1.000	1.000	1.000
0.50	0.50	0.00	1.000	1.000	1.000	1.000	1.000	1.000
0.50	0.50	0.50	0.821	0.856	1.000	1.000	1.000	1.000
0.50	0.50	1.00	0.998	0.998	1.000	1.000	1.000	1.000
0.50	1.00	0.00	1.000	1.000	0.999	0.998	1.000	1.000
0.50	1.00	0.50	0.821	0.857	0.953	0.959	0.999	0.999
0.50	1.00	1.00	0.987	0.995	1.000	0.999	1.000	1.000
1.00	0.00	0.00	0.995	0.998	1.000	1.000	1.000	1.000
1.00	0.00	0.50	0.825	0.867	1.000	1.000	1.000	1.000
1.00	0.00	1.00	0.836	0.893	0.943	0.993	0.999	0.999
1.00	0.50	0.00	1.000	1.000	1.000	1.000	1.000	1.000
1.00	0.50	0.50	0.831	0.865	0.995	0.999	1.000	1.000
1.00	0.50	1.00	0.638	0.708	0.875	0.905	0.995	0.999
1.00	1.00	0.00	1.000	1.000	1.000	1.000	1.000	1.000
1.00	1.00	0.50	0.836	0.866	0.958	0.976	0.999	1.000
1.00	1.00	1.00	0.052	0.103	0.052	0.087	0.051	0.079

The sizes are in bold. The nominal size is 5% and 10,000 replications were used in each case.

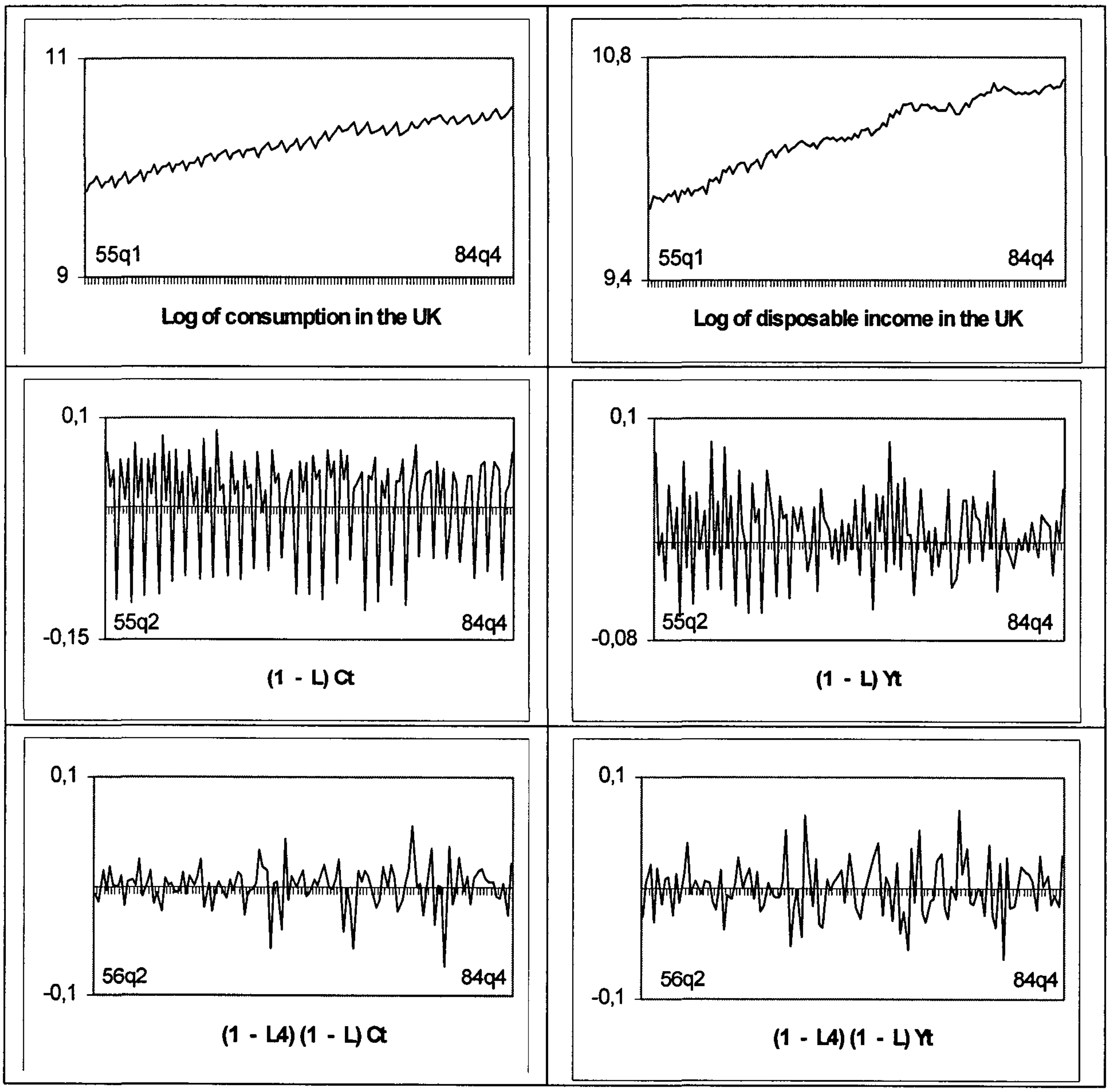
We see that the sizes of the tests based on the asymptotic critical values are in all cases too large though they tend to approximate to the nominal value of 5% as T increases. Thus, it is 10.3% when T = 100; it becomes 8.7% with T = 200, and reduces to 7.9% when T = 300. The larger sizes of the asymptotic tests are also associated with some superior rejection frequencies relative to the finite sample tests. (Note that in case of $\theta_i \neq 0$ for any i = 1, 2, 3, the rejection probabilities can be considered as misspecification tests to the specified null). However, we observe that even if the sample size is 100, the rejection probabilities are relatively high for both tests, exceeding 0.700 in practically all cases. Increasing the sample size, the rejection frequencies become even higher, and if T = 300, they approximate to 1 for all types of alternatives. The results presented in this table suggest that the optimal local power properties of the tests of Robinson (1994) may also hold reasonable well against non local departures from the null, and given the lack of empirical work in this context, an empirical study of fractionally based models for each of these components seems overdue.

4. An Empirical Application

The time series data analysed in this section correspond to the logarithmic transformations of the quarterly, seasonally unadjusted, consumption expenditure on non durables (c_t), and personal disposable income (y_t), in the UK, for the time period 1955q1 - 1984q4. These data were used by HEGY (1990) for testing seasonal unit roots and also by Gil Alana and Robinson (2001) in the context of seasonal fractional integration. In the first of these articles, they found that c_t could be I(1) at each of the frequencies 0, π and $\pi/2$ ($3\pi/2$), while y_t may contain only two unit roots, at 0 and π. Gil-Alana and Robinson (2001) extend these results and find that the roots could also be fractional. However, in both articles, the authors concentrate exclusively on the seasonal structure of the series and do not pay any attention to the trend and the cyclical components. The data are shown in Figure 1.

We see that the two series have strong trend and seasonal components. In fact, taking first differences, we observe (in the second part of the figure) that a seasonal structure is still present in the data, with possible changing patterns, especially for y_t. Taking seasonal differences, the resulting series appear to be stationary, though a cyclical structure could still be present in the data.

Denoting any of the series by y_t, we employ throughout model (7), testing H_o (2) for values d_1, d_2 and d_3 equal to 0, 0.50 and 1, and r = T/j, j = 20, 21, 22, 23 and 24. The choice of j was made based on the fact that the cycles in economics seem to occur every 5 or 6 years (i.e., corresponding to 20 or 24 quarters). Moreover, when testing H_o (2) with j < 20 or j > 24, the null was rejected in practically all cases for all values of the d's. The results for white noise u_t are given in Tables 3 and 4 and they correspond to the test statistic given by $\hat{R}$ in (5).



<Figure 1> The time series data in the UK for the time period 1955q1 - 1984q4

<Table 3> Testing H_o (2) in (7) with $\hat{R}$ of (5) and white noise u_t in the consumption series

Orders of integration			Values of r				
d_1	d_2	d_3	T/20	T/21	T/22	T/23	T/24
0.00	0.00	0.00	5515949053	5369284635	6200944268	648762025	7902631590
0.00	0.00	0.50	11.35	15.69	12.99	13.51	15.32
0.00	0.00	1.00	331.09	219.30	172.50	129.35	107.48
0.00	0.50	0.00	1895.67	1875.72	2168.57	2293.54	2770.66
0.00	050	0.50	11.20	15.54	12.93	13.47	15.26
0.00	0.50	1.00	27.41	26.89	27.06	26.71	27.65
0.00	1.00	0.00	93.74	93.69	93.62	93.68	92.71
0.00	1.00	0.50	11.02	15.40	12.88	13.42	15.20
0.00	1.00	1.00	31.45	29.95	29.73	28.85	29.83
0.50	0.00	0.00	834.71	827.58	957.82	1014.73	1224.10
0.50	0.00	0.50	7.42’	4.06’	3.29’	3.07’	4.21’
0.50	0.00	1.00	37.29	36.08	36.04	35.37	36.39
0.50	0.50	0.00	30.46	30.48	30.37	30.40	29.85
0.50	0.50	0.50	7.29’	3.94’	3.27’	3.00’	4.15’
0.50	0.50	1.00	37.31	35.61	35.45	34.50	35.73
0.50	1.00	0.00	15.83	15.23	14.40	13.68	13.03
0.50	1.00	0.50	7.17’	3.82’	3.24’	2.94’	4.08’
0.50	1.00	1.00	38.02	36.20	36.00	34.98	36.26
1.00	0.00	0.00	0.17’	0.13’	0.13’	0.12’	0.20’
1.00	0.00	0.50	5.32’	3.47’	2.97’	3.21’	3.59’
1.00	0.00	1.00	43.35	41.85	41.75	40.94	42.04
1.00	0.50	0.00	1.91’	1.94’	1.89’	1.88’	1.82’
1.00	0.50	0.50	5.22’	3.37’	3.05’	3.17’	3.60’
1.00	0.50	1.00	41.07	39.23	39.10	38.08	39.46
1.00	1.00	0.00	3.27’	3.27’	3.18’	3.14’	3.05’
1.00	1.00	0.50	5.11’	3.26’	3.13’	3.12’	3.60’
1.00	1.00	1.00	41.56	39.61	39.45	38.37	39.83

‘ and in bold: Non rejection values at the 5% significance level.

Starting with consumption (Table 3), and looking first at the non fractional cases, we observe that there are only two non rejection values, corresponding to $d_1 = 1$ and $d_2 = d_3 = 0$, and $d_1 = d_2 = 1$ and $d_3 = 0$, i.e., a unit root at the long run (trend) or zero frequency, and two unit roots, at zero and the seasonal component. However, we also observe several non rejection values corresponding to fractional models. The set of these values for the d's are: (0.5, 0, 0.5); (0.5, 0.5, 0.5); (0.5, 1, 0.5); (1, 0, 0.5); (1, 0.5, 0); (1, 0.5, 0.5) and (1, 1, 0.5). Thus, we see that the unit root cycles always are rejected, and the root at the long run or zero frequency seems to be more important than those corresponding to the seasonal or the cycles.

<Table 4> Testing H_o (2) in (7) with $\hat{R}$ given by (5) and white noise u_t in the income series

Orders of integration			Values of r				
d_1	d_2	d_3	T/20	T/21	T/22	T/23	T/24
0.00	0.00	0.00	3454123075	3362304599	3883134573	4062578719	4948868232
0.00	0.00	0.50	11.36	15.70	12.99	13.52	15.32
0.00	0.00	1.00	341.65	226.24	178.07	133.48	111.6
0.00	0.50	0.00	1967.26	1945.76	250.19	2380.03	2872.74
0.00	0.50	0.50	11.22	15.55	12.93	13.47	15.26
0.00	0.50	1.00	27.24	26.74	26.92	26.57	27.51
0.00	1.00	0.00	94.03	94.01	93.95	94.04	92.94
0.00	1.00	0.50	11.08	15.41	12.88	13.43	15.21
0.00	1.00	1.00	31.33	29.83	29.62	28.74	29.71
0.50	0.00	0.00	861.74	854.02	988.71	1047.51	1262.63
0.50	0.00	0.50	7.42’	4.07’	3.29’	3.07’	4.21’
0.50	0.00	1.00	38.34	37.21	37.17	36.54	37.49
0.50	0.50	0.00	30.75	30.78	30.67	30.71	30.10
0.50	0.50	0.50	7.30’	3.95’	3.26’	3.00’	4.15’
0.50	0.50	1.00	37.22	35.52	35.93	34.41	35.64
0.50	1.00	0.00	15.97	15.40	14.57	13.86	13.16
0.50	1.00	0.50	7.18’	3.83’	3.23’	2.94’	4.08’
0.50	1.00	1.00	37.94	36.12	35.93	34.90	36.19
1.00	0.00	0.00	0.15’	0.11’	0.11’	0.10’	0.17’
1.00	0.00	0.50	5.32’	3.47’	2.98’	3.21’	3.59’
1.00	0.00	1.00	45.09	43.67	43.57	42.81	43.84
1.00	0.50	0.00	1.88’	1.91’	1.86’	1.85’	1.78’
1.00	0.50	0.50	5.22’	3.37’	3.06’	3.17’	3.60’
1.00	0.50	1.00	41.01	39.17	39.04	38.03	39.40
1.00	1.00	0.00	3.23’	3.24’	3.14’	3.10’	3.01’
1.00	1.00	0.50	5.12’	3.26’	3.14’	3.12’	3.61’
1.00	1.00	1.00	41.51	39.56	39.41	38.33	39.78

‘ and in bold: Non rejection values at the 5% significance level.

Finally, we should remark that the results are quite robust to the different values of j. In fact, the non-rejection values coincide in all cases. This it not at all surprising if we take into account the similarity of the processes for different values of j when they are close to each other. The results for y_t are given in Table 4 and we observe that the non-rejection values coincide with those reported in Table 3. Thus, the same conclusions as in the previous table hold here: the root at the long run or zero frequency seems to be the most important one, though fractional orders of integration, especially for the seasonal and for the cyclical components, may also be plausible in some cases.

<Table 5> Testing H_o (2) in (7) with $\hat{R}$ given by (5) and AR(1) u_t in the consumption series

Orders of integration			Values of r				
d_1	d_2	d_3	T/20	T/21	T/22	T/23	T/24
0.00	0.00	0.00	7731124531	709827980	815666830	832193364	983585067
0.00	0.00	0.50	6.05'	2.60'	58.74	8.26'	14.03
0.00	0.00	1.00	434.83	284.79	222.48	163.28	136.11
0.00	0.50	0.00	160.76	148.62	164.81	166.50	185.88
0.00	0.50	0.50	5.82'	2.56'	60.20	7.78'	13.81
0.00	0.50	1.00	10.45	10.79	11.16	11.31	11.75
0.00	1.00	0.00	13.80	12.59	11.45	10.43	9.75'
0.00	1.00	0.50	5.61'	2.31'	61.57	7.28'	13.58
0.00	1.00	1.00	14.67	14.19	14.11	13.77	14.11
0.50	0.00	0.00	47.12	46.46	47.48	47.95	46.64
0.50	0.00	0.50	7.87'	8.80'	70.33	35.28	33.05
0.50	0.00	1.00	21.15	20.53	20.59	20.21	20.97
0.50	0.50	0.00	143.67	139.94	135.78	131.41	132.51
0.50	0.50	0.50	7.54'	8.53'	75.14	35.87	34.37
0.50	0.50	1.00	24.03	22.98	22.83	22.19	22.91
0.50	1.00	0.00	157.92	152.27	150.73	146.22	143.59
0.50	1.00	0.50	7.20'	8.24'	80.18	36.36	35.68
0.50	1.00	1.00	26.45	25.16	24.88	24.10	24.81
1.00	0.00	0.00	6.06'	5.21'	5.18'	4.73'	5.52'
1.00	0.00	0.50	21.48	33.24	31.34	47.68	29.28
1.00	0.00	1.00	28.81	27.54	27.42	26.68	27.66
1.00	0.50	0.00	3.21'	3.15'	3.19'	3.17'	3.33'
1.00	0.50	0.50	2175	34.47	34.63	51.41	31.82
1.00	0.50	1.00	33.06	31.49	31.28	30.37	31.44
1.00	1.00	0.00	5.05'	5.02'	5.05'	5.05'	5.17'
1.00	1.00	0.50	21.98	35.71	38.28	55.30	34.57
1.00	1.00	1.00	36.27	34.46	34.19	33.15	34.28

' and in bold: Non rejection values at the 5% significance level.

The significance of the above results may be in large part due to the un accounted for I(0) autocorrelation in u_t. Thus, in the following two tables we allow for an AR(1) structure on the disturbances. Higher AR orders were also considered and the results were very similar to those reported here for the AR(1) case, implying that this specification could be sufficient for describing the short-run dynamics underlying the series. (Finite sample critical values were also computed for the case of AR(1) u_t, and though not reported in the paper, they were employed in Tables 5 and 6).

<Table 6> Testing H_o (2) in (7) with $\hat{R}$ given by (5) and AR(1) u_t in the income series

Orders of integration			Values of r				
d_1	d_2	d_3	T/20	T/21	T/22	T/23	T/24
0.00	0.00	0.00	7356550689	7119024736	8195288956	8557125904	10203242215
0.00	0.00	0.50	6.05'	2.61'	58.59	8.19'	13.96
0.00	0.00	1.00	439.56	287.70	224.70	164.78	137.36
0.00	0.50	0.00	167.74	155.29	172.42	174.37	194.87
0.00	0.50	0.50	5.82'	2.56'	60.03	7.70'	13.75
0.00	0.50	1.00	10.38	10.72	11.09	11.25	11.67
0.00	1.00	0.00	13.50	12.32	1.20'	10.19	9.50'
0.00	1.00	0.50	5.61'	2.54'	61.39	7.21'	13.51
0.00	1.00	1.00	14.54	14.07	13.99	13.66	14.00
0.50	0.00	0.00	45.45	44.89	45.94	4.46'	45.26
0.50	0.00	0.50	7.83'	8.72'	70.51	35.16	33.05
0.50	0.00	1.00	20.96	20.36	20.42	20.04	20.79
0.50	0.50	0.00	138.35	134.50	130.24	125.79	126.64
0.50	0.50	0.50	7.50'	8.45'	75.32	35.74	34.35
0.50	0.50	1.00	23.87	22.83	22.68	22.05	22.76
0.50	1.00	0.00	159.95	154.68	153.32	149.12	146.25
0.50	1.00	0.50	7.17'	8.16'	80.35	36.22	35.66
0.50	1.00	1.00	26.27	24.99	24.72	23.94	24.65
1.00	0.00	0.00	37.59	33.34	32.57	30.16	32.48
1.00	0.00	0.50	21.43	33.12	31.56	47.76	29.41
1.00	0.00	1.00	28.60	27.34	27.22	26.49	27.47
1.00	0.50	0.00	3.44'	3.36'	3.40'	3.38'	3.57'
1.00	0.50	0.50	21.68	34.35	34.86	51.48	31.96
1.00	0.50	1.00	32.90	31.33	31.13	30.22	31.28
1.00	1.00	0.00	5.16'	5.12'	5.16'	5.26'	5.29'
1.00	1.00	0.50	21.91	35.56	38.53	55.37	34.71
1.00	1.00	1.00	36.10	34.31	34.03	33.00	34.12

' and in bold: Non rejection values at the 5% significance level.

Table 5 displays the results for consumption. Imposing r = T/20 or T/21, we observe several non rejection values, some of them corresponding to $d_1 = 0$. However, in these cases, though it is not reported in the tables, the estimated AR coefficients were very close to 1, implying that they might be competing with d_1 in describing the nonstationary component of the trend. (These coefficients are Yule-Walker estimates, entailing roots that are automatically less than one in absolute value but that can be arbitrarily close to one). Apart from these cases, we also observe another six non rejection values: two of them corresponding to the unit roots at zero and at zero and the seasonal frequencies, and the remaining four corresponding to fractional models with (d_1, d_2, d_3) = (0.5, 0, 0); (0.5, 0.5, 0.5); (0.5, 1, 0.5) and (1, 0.5, 0). All these models were non-rejected in Table 3 for the case of white noise disturbances. Imposing r = T/22, we only observe three non rejected cases: (1, 0, 0); (1, 0.5, 0) and (1, 1, 0), and they are non rejected with r = T/23 and T/24 along with several other cases with d_1 = 0.

In Table 6 we report the results for y_t. When r = T/20 or T/21, the non rejection values coincide with those given in Table 5 and, for the remaining cases, some small differences appear. Thus, if r = T/22 and d = (0, 1, 0), H_o (2) cannot be rejected for y_t, though it was rejected for c_t. On the other hand, if r = T/23 or T/24 and d = (1, 0, 0), H_o cannot be rejected for c_t but it is now rejected for y_t. Apart from these cases, all the remaining non rejection values coincide for the two series and also with the case of white noise disturbances, implying that the results are quite robust across r and also across the different types of disturbances.

5. Conclusions

We have presented a procedure for testing I(d) statistical models in the trend, the seasonal and the cyclical components in raw time series. The tests, due to Robinson (1994), have standard null and local limit distributions, and several Monte Carlo experiments conducted across the paper show that the sizes of the tests based on the asymptotic critical values are too large. Thus, finite sample critical values were computed and the rejection frequencies against unit and fractional alternatives were relatively high in all cases.

The tests were applied to the quarterly, seasonally unadjusted, UK consumption and income series, the results showing the importance of the roots corresponding to the trend and the seasonal components. However, though the unit roots were found to be fairly suitable, we show that fractional models (including the cyclical component) may also be plausible alternatives in some cases.

This article can be extended in several directions. Thus, for example, the finite sample critical values obtained in Section 3 can be extended to the case of non normal disturbances. Also, it would be worthwhile proceeding to get point estimates of the fractional differencing parameters. For the cyclical part, some attempts have been made by Arteche and Robinson (2000) and Arteche (2002). However, not only would this be computationally more expensive, but it is then in any case confidence intervals rather than point estimates which should be stressed, while available rules of inference seem to require preliminary integer differencing to achieve stationarity at each of these components. The approach used in this article generates simply computed diagnostics for departures from real values of d and thus, it is not at all surprising that when fractionally hypotheses are entertained, several non rejection values may appear. In that respect, a model selection criterion, (based on diagnostic tests on the residuals), should be established in order to determine which might be the best model specification for these and other macroeconomic time series. Finally, in relation with the cyclical component, it might also be of interest to compare the performance of the fractional model presented in this paper with the more traditional approaches based on ARIMA (or even ARFIMA) models. Work in these directions is now under progress.

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