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A Synthetic Chart to Monitor The Defect Rate for High-Yield Processes

Etsuko Kusukawa†
Department of Industrial Engineering
Osaka Prefecture University Sakai 599-8531, JAPAN
Tel: +81-72-254-9349, E-mail: kusukawa@eis.osakafu-u.ac.jp

Hiroshi Ohta
Department of Industrial Engineering
Osaka Prefecture University Sakai 599-8531, JAPAN
Tel: +81-72-254-9349, E-mail: ohta@eis.osakafu-u.ac.jp

Abstract. Kusukawa and Ohta presented the $CS_{CQ\text{-}r}$ chart to monitor the process defect rate λ in high-yield processes that is derived from the count of defects. The $CS_{CQ\text{-}r}$ chart is more sensitive to monitor λ than the CQ (Cumulative Quantity) chart proposed by Chan *et al.*. As a more superior chart in high-yield processes, we propose a Synthetic chart that is the integration of the CQ-r chart and the $CS_{CQ\text{-}r}$ chart. The quality characteristic of both charts is the number of units y required to observe r ($\geq$2) defects. It is assumed that this quantity is an Erlang random variable from the property that the quality characteristic of the CQ chart follows the exponential distribution. In use of the proposed Synthetic chart, the process is initially judged as either in-control or out-of-control by using the $CS_{CQ\text{-}r}$ chart. If the process was not judged as in-control by the $CS_{CQ\text{-}r}$ chart, the process is successively judged by using the CQ-r chart to confirm the judgment of the $CS_{CQ\text{-}r}$ chart. Through comparisons of ARL (Average Run Length), the proposed Synthetic chart is more superior to monitor the process defect rate in high-yield processes to the stand-alone $CS_{CQ\text{-}r}$ chart.

Keywords: High-yield Process, Process Defect Rate, CQ(Cumulative Quantity)-*r* Chart, CS(Confirmation Sample) Chart, Synthetic Chart, ARL (Average Run Length)

1. INTRODUCTION

As a more superior chart to monitor the process fraction defectives in high-yield processes, Mishima *et al*. (2002) proposed a Synthetic chart for high-yield processes that is an integration of the CCC-r chart presented by Xie *et al*. (1998) and the $CS_{CCC\text{-}r}$ chart presented by Ohta and Kusukawa (2004).

As the counterpart for a charting technique to monitor high-yield processes, Kusukawa and Ohta (2004) presented the $CS_{CQ\text{-}r}$ chart to monitor the process defect rate that is derived from the count of defects. The $CS_{CQ\text{-}r}$ chart is an alternative of the CQ (Cumulative Quantity)-r chart which is extended naturally from the CQ chart proposed by Chan *et al*. (2000). The $CS_{CQ\text{-}r}$ chart can be designed by applying the CS charting procedure presented by Steiner (1999) to the CQ-r chart.

As a more superior chart to monitor the process defect rate in high-yield processes, we propose a Synthetic chart that is the integration of the CQ-r chart and the $CS_{CQ\text{-}r}$ chart. The quality characteristic of both the CQ-r chart and the $CS_{CQ\text{-}r}$ chart is the number of units (which is not necessary an integer) required to observe r ($\geq$2) defects. It is assumed that the quantity is an Erlang random variable in the case that $r \geq 2$, while is an exponential random variable in the case that $r = 1$. In use of the proposed Synthetic chart, the process is initially judged as either in-control or out-of-control by using the $CS_{CQ\text{-}r}$ chart. If the process was not judged as in-control by the $CS_{CQ\text{-}r}$ chart, the process is successively judged by using the CQ-r chart to confirm the judgment of the $CS_{CQ\text{-}r}$ chart. In numerical study, through ARL (Average Run Length), the performance of the proposed Synthetic chart is compared with that of the stand-alone $CS_{CQ\text{-}r}$ chart. Numerical examples demonstrate that the proposed Synthetic chart is the most superior to monitor the process defect rate in high-yield processes to the stand-alone $CS_{CQ\text{-}r}$ chart.

† : Corresponding Author

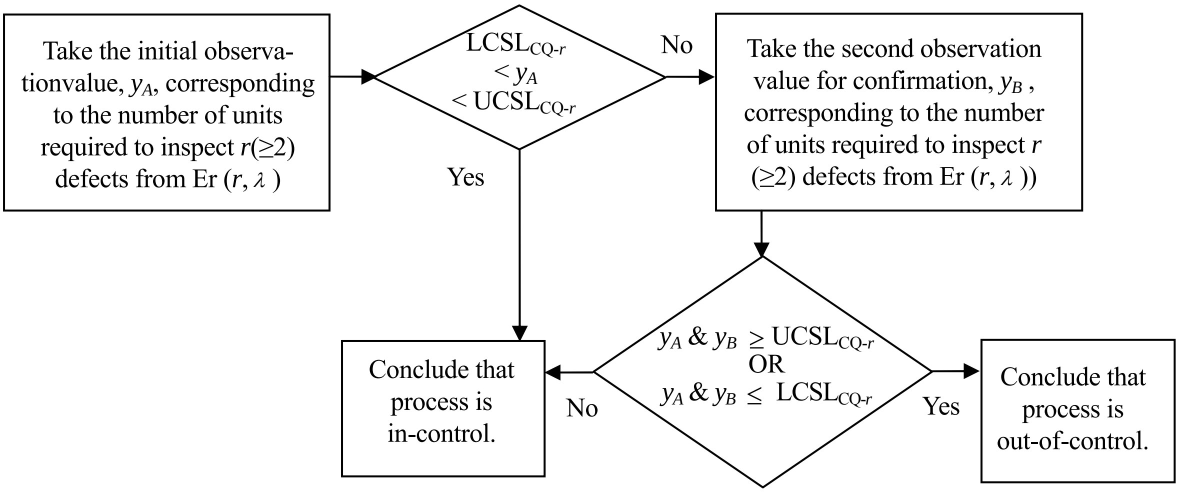

Figure 1. Decision procedure for the $\text{CS}_{CQ\text{-}r}$ chart

2. OUTLINE OF THE CQ-*r* CHART

The quality characteristic of the CQ chart is the number of units required to observe one defect. The CQ-*r* chart is designed naturally by extending the CQ chart.

Suppose that defects in a process occur according to a Poisson distribution with mean rate of occurrence equal to λ (>0) defects per unit quantity of product. In what follows, the parameter λ is referred to the process defect rate. Then the number of units, y, required to observe r (≥ 2) defects is an Erlang random variable with probability density function, cumulative distribution function and mean given by

$$f(y,r,r\lambda)=\frac{(r\lambda)^r}{(r-1)!}(y)^{r-1}\exp[-(r\lambda)y], \quad (y>0,\ r=1,2,.....,\lambda r>0) \tag{1}$$

$$F(y)=1-\sum_{j=0}^{r-1}\frac{((r\lambda)y)^j}{j!}\exp[-(r\lambda)y], \quad (y>0,\ r=1,2,.....,\lambda r>0) \tag{2}$$

$$E[y]=r/r\lambda=1/\lambda\,. \tag{3}$$

By using the probability limit method based on the Erlang distribution with given distribution parameters λ and r as shown in equation (2), the upper and lower control limits, $\text{LCL}_{CQ\text{-}r}$ and $\text{UCL}_{CQ\text{-}r}$, of the CQ-*r* chart are given as solutions of the following equations:

$$\sum_{j=1}^{r-1}\frac{((r\lambda_0)\text{UCL}_{CQ-r})^j}{j!}\exp[-(r\lambda_0)\text{UCL}_{CQ-r}]=\alpha/2\,, \tag{4}$$

$$\sum_{j=1}^{r-1}\frac{((r\lambda_0)\text{LCL}_{CQ-r})^j}{j!}\exp[-(r\lambda_0)\text{LCL}_{CQ-r}]=1-\alpha/2\,, \tag{5}$$

where α is type I error. Let λ_0 be the in-control process defect rate being monitored. The recommended value for r is about 2-5 depending on the process defect rate level and type of process being monitored.

The decision procedure to use the CQ-*r* chart applied in high-yield processes is made as follows: if the process defect rate λ decreases such that $\lambda<\lambda_0$, then a plotting point y on the CQ-*r* chart is expected to be larger, while if λ increases such that $\lambda>\lambda_0$, then it is expected to be smaller. According to the CQ-*r* chart, if a plotted point is above the upper control limit, the process is likely to have probably improved, while if a plotted point is below the lower control limit, the process has probably deteriorated.

3. OUTLINE OF THE $\text{CS}_{CQ\text{-}r}$ CHART

Kusukawa and Ohta (2004) presented the $\text{CS}_{CQ\text{-}r}$ chart to monitor the process defect rate in the high-yield processes. The $\text{CS}_{CQ\text{-}r}$ chart can be constructed by applying the CS charting procedure to the CQ-*r* chart.

3.1 Notation

Notation used in the $\text{CS}_{CQ\text{-}r}$ chart are defined as follows:

- λ_0 : the in-control process defect rate;
- y_A : the initial observation value corresponding to the number of units required to observe $r_{CS}(\geq 2)$ defects taken from a process which obeys the Erlang distribution Er(r_{CS}, λ);
- y_B : the second observation value for confirmation corresponding to the number of units required to observe $r_{CS}(\geq 2)$ defects taken from the same process which obeys the Erlang distribution Er(r_{CS}, λ);

α : type I error of the $\text{CS}_{\text{CQ-}r}$ chart;
$\text{UCSL}_{\text{CQ-}r}$: the upper confirmation control limit of the $\text{CS}_{\text{CQ-}r}$ chart;
$\text{LCSL}_{\text{CQ-}r}$: the lower confirmation control limit of the $\text{CS}_{\text{CQ-}r}$ chart.

3.2 Decision Procedure for the $\text{CS}_{\text{CQ-}r}$ Chart

The decision procedure for the $\text{CS}_{\text{CQ-}r}$ chart is summarized in flowchart form in Figure 1. In the $\text{CS}_{\text{CQ-}r}$ chart, the initial observation value is taken from the process. If

$$\text{LCSL}_{\text{CQ}-r} < y_A < \text{UCSL}_{\text{CQ}-r} \,, \tag{6}$$

conclude that the process is in-control. Otherwise, the second observation value for confirmation, y_B, is independently taken from the same process. Then, if

$$y_A \,\&\, y_B \le \text{LCSL}_{\text{CQ}-r} \;\; \text{OR} \;\; y_A \,\&\, y_B \ge \text{UCSL}_{\text{CQ}-r} \,, \tag{7}$$

conclude that the process is out-of-control. Otherwise, conclude that the process is in-control. Based on the decision procedure for the $\text{CS}_{\text{CQ-}r}$ chart given in Figure 1, the probability of type I error assuming fixed values for $\text{UCSL}_{\text{CQ-}r}$ and $\text{LCSL}_{\text{CQ-}r}$, is given as follows:

$$\begin{aligned}\alpha &= P(y_A \,\&\, y_B \ge \text{UCSL}_{\text{CQ}-r}) \\ &\quad + P(y_A \,\&\, y_B \le \text{LCSL}_{\text{CQ}-r})\end{aligned} \tag{8}$$

It is also assumed that type I error for each control limit is set equally. Based on equation (8), $\text{UCSL}_{\text{CQ-}r}$ and $\text{LCSL}_{\text{CQ-}r}$ are given as solutions of the following equations:

$$\sum_{j=1}^{r_{\text{CS}}-1} \frac{((r_{\text{CS}}\lambda_0)\text{UCSL}_{\text{CQ-}r})^j}{j!} \exp[-(r_{\text{CS}}\lambda_0)\text{UCSL}_{\text{CQ-}r}] = \sqrt{\alpha/2} \,, \tag{9}$$

$$\sum_{j=1}^{r_{\text{CS}}-1} \frac{((r_{\text{CS}}\lambda_0)\text{LCSL}_{\text{CQ-}r})^j}{j!} \exp[-(r_{\text{CS}}\lambda_0)\text{LCSL}_{\text{CQ-}r}] = 1 - \sqrt{\alpha/2} \,. \tag{10}$$

4. THE PROPOSED SYNTHETIC CHART FOR HIGH-YIELD PROCESSES

4.1 Notation

y_A and y_B are same variables as ones defined in section 3. The remainders of notation used in the proposed Synthetic chart are defined as follows:

λ_1 : the out-of-control process defect rate;
r_{CS} : the number of defects observed before a point is plotted on the $\text{CS}_{\text{CQ-}r}$ chart;
$r_{\text{CQ-}r}$: the number of defects observed before a point is plotted on the CQ-r chart;
α : the overall probability of type I error for the Synthetic chart;
α_{CS} : the probability of type I error for the $\text{CS}_{\text{CQ-}r}$ chart;
$\alpha_{\text{CQ-}r}$: the probability of type I error for the CQ-r chart;
LCSLs : the lower confirmation control limit of the $\text{CS}_{\text{CQ-}r}$ chart in the Synthetic chart;
UCSLs : the upper confirmation control limit of the $\text{CS}_{\text{CQ-}r}$ chart in the Synthetic chart;
$y_A \,\&\, y_B$: one nonconforming sample that both y_A and y_B are lower than or equal to LCSLs, or is higher than or equal to UCSLs;
N_{LCSLs} : the cumulative quantity of y_A observed on the $\text{CS}_{\text{CQ-}r}$ chart until observing $r_{\text{CQ-}r}$ nonconforming samples consisting of y_A & y_B are lower than or equal to LCSLs;
N_{UCSLs} : the cumulative quantity of y_A observed on the $\text{CS}_{\text{CQ-}r}$ chart until observing $r_{\text{CQ-}r}$ nonconforming samples consisting of y_A & y_B are higher than or equal to UCSLs;
$\text{LCL}_{\text{LCSLs}}$: the lower control limit of the CQ-r chart used to confirm the judgment of the process state when downward shifts in λ from the in-control process are detected by the $\text{CS}_{\text{CQ-}r}$ chart;
$\text{LCL}_{\text{UCSLs}}$: the lower control limit of the CQ-r chart used to confirm the judgment of the process state when upward shifts in λ from the in-control process are detected by the $\text{CS}_{\text{CQ-}r}$ chart;
Q_{LCSLs} : the lower type I error of the $\text{CS}_{\text{CQ-}r}$ chart;
Q_{UCSLs} : the upper type I error of the $\text{CS}_{\text{CQ-r}}$ chart;

4.2 Decision Procedure for the Proposed Synthetic Chart

The decision procedure for the proposed Synthetic chart is shown in Figure 2. The operation of the proposed Synthetic chart is as follows:

1) For the $\text{CS}_{\text{CQ-}r}$ chart in the Synthetic chart, y_A is initially taken from the process.
2) If y_A is higher than or equal to UCSLs, or lower than or equal to LCSLs, y_B would be independently taken from the process to confirm the judgment of the current process state made for y_A.
3) If both y_A & y_B are lower than or equal to LCSLs, or higher than or equal to UCSLs, the process is further successively judged by the CQ-r chart in the proposed Synthetic chart. Then y_A & y_B are counted as one nonconforming sample on the CQ-r chart. Otherwise, y_A & y_B are counted as one conforming sample on the CQ-r chart.

4) For upward shifts in λ, N_{UCSLs} is obtained as the cumulative quantity of y_A observed on the $CS_{CQ\text{-}r}$ chart until observing $r_{CQ\text{-}r}$ nonconforming samples consisting of y_A & y_B are higher than or equal to UCSLs.
5) For downward shifts in λ, N_{LCSLs} is obtained as the cumulative quantity of y_A observed on the $CS_{CQ\text{-}r}$ chart until observing $r_{CQ\text{-}r}$ nonconforming samples consisting of y_A & y_B are lower than or equal to LCSLs.
6) If N_{UCSLs} is lower than or equal to LCL_{UCSLs} or N_{LCSLs} is lower than or equal to LCL_{LCSLs}, it would be judged that the process is out-of-control. Otherwise, it would be judged that the process is in-control.

4.3 Probability of Type I Error of The Proposed Synthetic Chart

Based on Figure 2, the overall probability of type I error for the proposed Synthetic chart, α, is obtained as

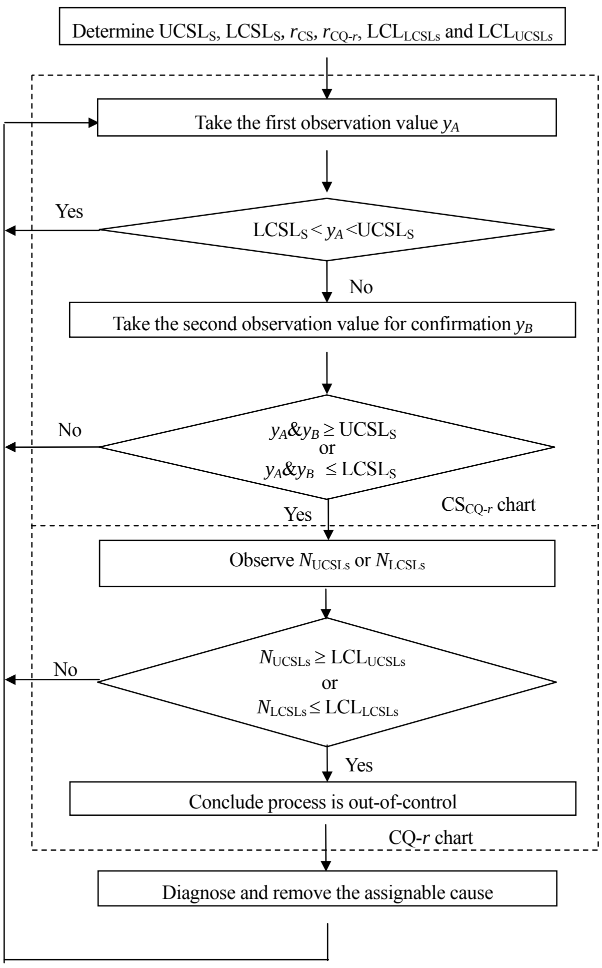

Figure 2. Decision procedure for the proposed synthetic chart

$$\alpha = P(y_A \,\&\, y_B \geq \text{UCSL}_\text{S})P(N_{\text{UCSL}_\text{S}} \leq \text{LCSL}_{\text{UCSL}_\text{S}})$$
$$+P(y_A \,\&\, y_B \leq \text{LCSL}_\text{S})P(N_{\text{LCSL}_\text{S}} \leq \text{LCSL}_{\text{LCSL}_\text{S}})$$
$$= \alpha_{\text{CS}} \times \alpha_{\text{CQ-}r} \quad (11)$$

For simplicity, in the proposed Synthetic chart α_{CS} and $\alpha_{\text{CQ-}r}$ in equation (10) are set equally as

$$\alpha_{\text{CS}} = \alpha_{\text{CQ-}r} = \sqrt{\alpha}\,. \quad (12)$$

4.4 Confirmation Control Limits of Actual Values of Type I Error of the $CS_{CQ\text{-}r}$ Chart in the Synthetic Chart

The upper and lower confirmation control limits of the $CS_{CQ\text{-}r}$ chart in the Synthetic chart, UCSLs and LCSLs, can be designed by the probability limit method based on the Erlang distribution Er(r_{CS}, λ). Based on the designing method of the stand-alone $CS_{CQ\text{-}r}$ chart in section 3.2, from equations (8), (11) and (12), UCSLs and LCSLs can be given as solutions of the following equations:

$$\sum_{j=1}^{r_{\text{CS}}-1} \frac{((r_{\text{CS}}\lambda_0)\text{UCSL}_\text{S})^j}{j!} \exp[-(r_{\text{CS}}\lambda_0)\text{UCSL}_\text{S}]$$
$$= \sqrt{\alpha_{\text{CS}}/2}\,, \quad (13)$$

$$\sum_{j=1}^{r_{\text{CS}}-1} \frac{((r_{\text{CS}}\lambda_0)\text{LCSL}_\text{S})^j}{j!} \exp[-(r_{\text{CS}}\lambda_0)\text{LCSL}_\text{S}]$$
$$= 1 - \sqrt{\alpha_{\text{CS}}/2}\,. \quad (14)$$

Then, the upper and lower probabilities of type I error of the $CS_{CQ\text{-}r}$ chart in the Synthetic chart, Q_{UCSLs} and Q_{LCSLs}, can be obtained as

$$Q_{\text{UCSL}_\text{S}} = \left(\sum_{j=1}^{r_{\text{CS}}-1} \frac{((r_{\text{CS}}\lambda_0)\text{UCSL}_{\text{CQ-}r})^j}{j!} \exp[-(r_{\text{CS}}\lambda_0)\text{UCSL}_{\text{CQ-}r}] \right)^2$$
$$\cong \alpha_{\text{CS}}/2 = \sqrt{\alpha}\,, \quad (15)$$

$$Q_{\text{LCSL}_\text{S}} = \left(1 - \sum_{j=1}^{r_{\text{CS}}-1} \frac{((r_{\text{CS}}\lambda_0)\text{LCSL}_{\text{CQ-}r})^j}{j!} \exp[-(r_{\text{CS}}\lambda_0)\text{LCSL}_{\text{CQ-}r}] \right)^2$$
$$\cong \alpha_{\text{CS}}/2 = \sqrt{\alpha}\,. \quad (16)$$

4.5 Control Limits and Actual Values of Type I Error of the CQ-*r* Chart in the Synthetic Chart

The CQ-*r* chart in the Synthetic chart can be designed by using Q_{UCSLs} and Q_{LCSLs} for the $CS_{CQ\text{-}r}$ chart. Based on α_{CQ-r} and Q_{LCSLs}, the lower control limit of the CQ-*r* chart, LCL_{LCSLs}, can be given as a solution of the following equation:

$$1-\sum_{j=1}^{r_{\text{CQ}-r}-1}\left(\frac{((r_{\text{CQ-}r}Q_{\text{UCSL}_\text{S}})\text{LCL}_{\text{UCSL}_\text{S}})^j}{j!}\exp[-(r_{\text{CQ-}r}Q_{\text{UCSL}_\text{S}})\text{LCL}_{\text{UCSL}_\text{S}}]\right)$$
$$\cong \alpha_{\text{CQ}-r}=\sqrt{\alpha}\,. \tag{17}$$

In the same way, based on $\alpha_{\text{CQ}-r}$ and Q_{UCSLs}, the lower control limit of the CQ-*r* chart, $\text{LCL}_{\text{UCSLs}}$, can be given as a solution of the following equation:

$$1-\sum_{j=1}^{r_{\text{CQ}-r}-1}\left(\frac{((r_{\text{CQ-}r}Q_{\text{LCSL}_\text{S}})\text{LCL}_{\text{LCSL}_\text{S}})^j}{j!}\exp[-(r_{\text{CQ-}r}Q_{\text{LCSL}_\text{S}})\text{LCL}_{\text{LCSL}_\text{S}}]\right)$$
$$\cong \alpha_{\text{CQ}-r}=\sqrt{\alpha}\,. \tag{18}$$

Note that from equations (17) and (18), the upper and lower control limits, $\text{LCL}_{\text{LCSLs}}$ and $\text{LCL}_{\text{UCSLs}}$, of the CQ-*r* chart in the proposed Synthetic chart can be set equally.

5. NUMERICAL EXPERIMENTS

For illustrative purpose, in this section we assess the performance of the proposed Synthetic chart by comparing with that of the stand-alone $\text{CS}_{\text{CQ-}r}$ chart in section 3. We may express the null hypothesis H_0 (process is in-control) and the alternative hypothesis H_1 (process is out-of-control) in a formal manner as follows:

$$H_0:\lambda=\lambda_0,\quad H_1:\lambda_1=\kappa\lambda_0, \tag{19}$$

where κ denotes a process state, e.g., $\kappa = 1$ indicates the in-control process with λ_0, while $\kappa \neq 1$ indicates the out-of-control process with λ_1.

For a process shift in λ, we compare ARL of the proposed Synthetic chart with that of the $\text{CS}_{\text{CQ-}r}$ chart. ARL of the $\text{CS}_{\text{CQ-}r}$ chart is defined as the expected number of plotting point y_A required on the $\text{CS}_{\text{CQ-}r}$ chart to detect the first out-of-control observation value on the chart. ARL of the proposed Synthetic chart is defined as the expected number of plotted point y_A required on the $\text{CS}_{\text{CQ-r}}$ chart in the Synthetic chart to detect the first out-of-control observation value on the CQ-*r* chart in the Synthetic chart.

The upper and lower ARLs of the stand-alone $\text{CS}_{\text{CQ-}r}$ chart under each condition of the Erlang distribution for several values of κ, $\text{Er}(r, \lambda_1=\kappa\lambda_0)$, are obtained respectively as

Upper ARL

$$=\frac{\lambda_1^{-1}}{\left(\sum_{j=1}^{r_{\text{CS}}-1}\frac{((r_{\text{CS}}\lambda_1)\text{UCSL}_{\text{CQ-}r})^j}{j!}\exp[-(r_{\text{CS}}\lambda_1)\text{UCSL}_{\text{CQ-}r}]\right)^2} \tag{20}$$

Lower ARL

$$=\frac{\lambda_1^{-1}}{\left(1-\sum_{j=1}^{r_{\text{CS}}-1}\frac{((r_{\text{CS}}\lambda_1)\text{LCSL}_{\text{CQ-}r})^j}{j!}\exp[-(r_{\text{CS}}\lambda_1)\text{LCSL}_{\text{CQ-}r}]\right)^2} \tag{21}$$

Here, let P_{LCSLs} be the lower detection power of the $\text{CS}_{\text{CQ-}r}$ chart for λ_1, and let P_{UCSLs} be the upper detection power of the $\text{CS}_{\text{CQ-}r}$ chart for λ_1.
Also, let $P_{\text{LCL}_{\text{UCSL}}}$ be the lower detection power of the CQ-*r* chart for an upward shift λ_1, and let $P_{\text{LCL}_{\text{LCSL}}}$ be the lower detection power of the $\text{CS}_{\text{CQ-}r}$ chart for an downward shift λ_1. The upper and lower ARLs of the proposed Synthetic chart are obtained respectively as

$$\text{Upper ANOS}=\frac{\lambda_1^{-1}}{P_{\text{UCSL}_\text{S}}\times P_{\text{LCL}_{\text{UCSL}_\text{S}}}}, \tag{22}$$

where

$$P_{\text{UCSL}_\text{S}}=\left(\sum_{j=1}^{r_{\text{CS}}-1}\frac{((r_{\text{CS}}\lambda_1)\text{UCSL}_\text{S})^j}{j!}\exp[-(r_{\text{CS}}\lambda_1)\text{UCSL}_\text{S}]\right)^2$$

$$P_{\text{LCL}_{\text{UCSL}_\text{S}}}=1-\sum_{j=1}^{r_{\text{CQ}-r}-1}\left\{\frac{((r_{\text{CQ-}r}P_{\text{UCSL}_\text{S}})\text{LCL}_{\text{UCSL}_\text{S}})^j}{j!}\times\exp[-(r_{\text{CQ-}r}P_{\text{UCSL}_\text{S}})\text{LCL}_{\text{UCSL}_\text{S}}]\right\},$$

$$\text{Lower ANOS}=\frac{\lambda_1^{-1}}{P_{\text{LCSL}_\text{S}}\times P_{\text{LCL}_{\text{LCSL}_\text{S}}}}, \tag{23}$$

where

$$P_{\text{LCSL}_\text{S}}=\left(1-\sum_{j=1}^{r_{\text{CS}}-1}\frac{((r_{\text{CS}}\lambda_1)\text{LCSL}_\text{S})^j}{j!}\exp[-(r_{\text{CS}}\lambda_1)\text{LCSL}_\text{S}]\right)^2$$

$$\text{P}_{\text{LCL}_{\text{LCSL}_\text{S}}}=1-\sum_{j=1}^{r_{\text{CQ}-r}-1}\left\{\frac{((r_{\text{CQ-}r}P_{\text{LCSL}_\text{S}})\text{LCL}_{\text{LCSL}_\text{S}})^j}{j!}\times\exp[-(r_{\text{CQ-}r}P_{\text{LCSL}_\text{S}})\text{LCL}_{\text{LCSL}_\text{S}}]\right\}.$$

In numerical study, each ARL for each chart obtained from equations (20)~(23) is transformed as

$$\ln\text{ARL}=\log_e(\text{ARL})\,. \tag{24}$$

We assume that the overall probability of type I error for the Synthetic chart, α, is set by 0.0027. The upper and lower confirmation control limits of the stand-alone $\text{CS}_{\text{CQ-}r}$ chart can be obtained from equations (8)~(10). On the other hand, the upper and lower control limits of the

Table 1. Confirmation control limits and actual values of type I error (%) of the stand-alone $CS_{CQ\text{-}r}$ chart.

$\lambda_0 = 0.001$	Stand-alone $CS_{CQ\text{-}r}$ chart		
r	$LCSL_{CQ\text{-}r}$	$UCSL_{CQ\text{-}r}$	Actual value of type I error (%)
2	299	5111	0.2695
5	1805	9640	0.2699

Table 2. Control limits and actual values of type I error (%) of the proposed synthetic chart.

$\lambda_0 = 0.001$	Synthetic chart				
	$CS_{CQ\text{-}r}$ chart		CQ-r chart		Atual value of type I error (%)
$r_{CS} = r_{CQ\text{-}r}$	$LCSL_S$	$UCSL_S$	LCL_{LCSLs}	LCL_{UCSLs}	
2	715	3276	14	14	0.2595
5	2856	7130	77	77	0.2595

Table 3. Comparison of ln ARLs of the proposed synthetic chart and the stand-alone $CS_{CQ\text{-}r}$ chart.

(a) Case 1: $r=r_{CS}=r_{CQ\text{-}r}=2$, $\lambda_0=0.001$.

κ	Actual values of ln ARL			
	Proposed synthetic chart		Stand-alone $CS_{CQ\text{-}r}$ chart	
	Lower ln ARL	Upper ln ARL	Lower ln ARL	Upper ln ARL
0.60	9. 55	18. 88	10. 75	15. 91
0.70	10. 26	17. 18	11. 38	15. 18
0.80	11. 17	15. 76	12. 06	14. 55
0.90	12. 28	14. 55	12. 77	14. 00
1.00	0.2694*		0.2700*	
1.10	14. 85	12. 63	14. 28	13. 08
1.20	16. 26	11. 85	15. 07	12. 68
1.30	17. 73	11. 18	15. 87	12. 32
1.40	19. 24	10. 59	16. 69	11. 99

* denotes the type I error (%).

(b) Case 2: $r=r_{CS}=r_{CQ\text{-}r}=5$, $\lambda_0=0.001$.

κ	Actual values of ln ARL			
	Proposed synthetic chart		Stand-alone $CS_{CQ\text{-}r}$ chart	
	Lower ln ARL	Upper ln ARL	Lower ln ARL	Upper ln ARL
0.60	8. 53	32. 45	9. 73	17. 98
0.70	8. 90	25. 87	10. 51	16. 57
0.80	9. 47	20. 66	11. 42	15. 40
0.90	10. 84	16. 59	12. 43	14. 39
1.00	0.2696*		0.2700*	
1.10	17. 27	11. 34	14. 67	12. 75
1.20	21. 76	9. 93	15. 89	12. 08
1.30	26. 71	9. 09	17. 16	11. 48
1.40	31. 99	8. 58	18. 48	10. 94

* denotes the type I error (%).

proposed Synthetic chart can be designed from equations (12)~(18). Table 1 shows the upper and lower confirma tion control limits and the actual values of type I error of the stand- alone $CS_{CQ\text{-}r}$ chart with following parameters: r = 2, 5 and λ_0 = 0.001. Table 2 shows the upper and lower control limits and the actual values of type I error of the proposed Synthetic chart with following parameters: $r_{CS} = r_{CQ\text{-}r}$ = 2, 5 and λ_0=0.001. As mentioned in section 4.5., it can be confirmed that LCL_{LCSLs} and LCL_{UCSLs} are set equally.

Tables 3(a) and 3(b) show ln ARLs of the $CS_{CQ\text{-}r}$ chart and the proposed Synthetic chart for process shifts in λ given as several values of κ. From Tables 3 (a) and 3 (b), in situations where κ<1, the upper ln ARL indicates the expected number of plotting point y_A required for the $CS_{CQ\text{-}r}$ chart until judging correctly that λ is getting smaller. Similarly, in situations where κ>1, the lower ln ARL indicates the expected number of plotting point y_A required for the $CS_{CQ\text{-}r}$ chart until judging correctly that λ is getting larger. It implies that the smaller the values of the upper and lower ln ARL are, the more superior the performance of a control chart is. On the other hand, in situations where κ<1, the lower ln ARL indicates the expected number of plotting point y_A required for the $CS_{CQ\text{-}r}$ chart until judging incorrectly that λ is getting larger. Similarly, in situations where κ>1, the upper ln ARL indicates the expected number of plotting point y_A required for the $CS_{CQ\text{-}r}$ chart until judging incorrectly that λ is getting smaller. It implies that the larger the values of the lower and upper ln ARL are, the more superior the performance of a control chart is. The following can be summarized from Tables 3 (a)~3 (b).

(1) On the correct detection for a process shift in λ

The upper ln ARLs of the proposed Synthetic chart in situations where κ<1 are smaller than those of the stand-alone $CS_{CQ\text{-}r}$ chart. The lower ln ARLs in situations where $\kappa > 1$ of the proposed Synthetic chart are smaller than those of the stand-alone $CS_{CQ\text{-}r}$ chart. It is demonstrated that the proposed Synthetic chart can detect a process shift in λ correctly than the $CS_{CQ\text{-}r}$ chart.

(2) On the incorrect detection for a process shift in λ
The lower ln ARLs of the proposed Synthetic chart in situations where $\kappa < 1$ are smaller than those of the stand-alone $CS_{CQ\text{-}r}$ chart. The upper ln ARLs in situations where $\kappa > 1$ of the proposed Synthetic chart are larger than those of the stand-alone $CS_{CQ\text{-}r}$ chart. It is illustrated that the proposed Synthetic chart is harder to make a misjudgement than the $CS_{CQ\text{-}r}$ chart.

It can be seen from results of numerical experiments that it is more adequate to apply the proposed Synthetic chart in high-yield processes than the $CS_{CQ\text{-}r}$ chart. Therefore, our purpose has been successfully achieved.

6. CONCLUSIONS

In this paper, we presented a Synthetic chart that is an integration of the $CS_{CQ\text{-}r}$ chart and the CQ-*r* chart in order to construct a more sensitive chart to detect process shifts in the process defect rate. It can be seen from results of numerical experiments that ARL performance of the proposed Synthetic chart is more superior to that of the stand-alone $CS_{CQ\text{-}r}$ chart. Therefore, it is more adequate to apply the proposed Synthetic chart in high-yield processes.

On the proposed Synthetic chart, the larger both values of r_{CS} and $r_{CQ\text{-}r}$ are, the more superior its ARL performance is, while more and more observations are required to obtain a plotting point on the chart. From trade-off problem, it is necessary to determine economically values of designing parameters such as number of defects observed before a point is plotted on the chart, the sampling (inspection) interval and the upper and lower control limits of the proposed Synthetic chart. The issue of economic design of the Synthetic chart will be left for future research.

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