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# Analysis of The Behavior of Kurtosis By Simplified Model of One Sided Affiliated Impact Vibration

**Kazuhiro Takeyasu**†
Osaka Prefecture University
1-1 Gakuencho, Sakai, 599-8531, Japan
Tel: +81-72-254-9582, Fax: +81-72-254-9925, E-mail: takeyasu@eco.osakafu-u.ac.jp

**Yuki Higuchi**
Osaka Prefecture University
1-2 Gakuencho, Sakai, 599-8531, Japan
Tel: +81-72-254-9582, Fax: +81-72-254-9925, E-mail: takeyasu@eco.osakafu-u.ac.jp

**Abstract.** Among many amplitude parameters, Kurtosis (4-th normalized moment of probability density function) is recognized to be the sensitive good parameter for machine diagnosis. Kurtosis has a value of 3.0 under normal condition and the value generally goes up as the deterioration proceeds. In this paper, simplified calculation method of kurtosis is introduced for the analysis of impact vibration with one sided affiliated impact vibration which occurs towards the progress of time. That phenomenon is often watched in the failure of such as bearings' outer race. One sided affiliated impact vibration is approximated by one sided triangle towards the progress of time and simplified calculation method is introduced. Varying the shape of one sided triangle, various models are examined and it is proved that new index is a sensitive good index for machine failure diagnosis. Utilizing this method, the behavior of kurtosis is forecasted and analyzed while watching machine condition and correct diagnosis is executed.

**Keywords:** Maintenance, Impact Vibration, Kurtosis, Deterioration

## 1. INTRODUCTION

In mass production firms such as steel making that have big equipments, sudden stops of production processes by machine failure cause severe damages such as shortage of materials to the later processes, delays to the due date and the increasing idling time.

To prevent these troubles, machine diagnosis techniques play important roles. So far, Time Based Maintenance (TBM) technique has constituted the main stream of the machine maintenance, which makes checks for maintenance at previously fixed time. But it has a weak point that it makes checks at scheduled time without taking into account whether the parts are still keeping good conditions or not. On the other hand, Condition Based Maintenance (CBM) makes maintenance checks by watching the condition of machines. Therefore, if the parts are still keeping good condition beyond its expected life, the cost of maintenance may be saved because machines can be used longer than planned. Therefore the use of CBM has become dominant. The latter one needs less cost of parts, less cost of maintenance and leads to lower failure ratio.

However, it is mandatory to catch a symptom of the failure as soon as possible of a transition from TBM to CBM is to be made. Many methods are developed and examined focusing on this subject. In this paper, we propose a method for the early detection of the failure on rotating machines which is the most common theme in machine failure detection field.

So far, many signal processing methods for machine diagnosis have been proposed (Bolleter, 1998; Hoffner, 1991). As for sensitive parameters, Kurtosis, Bicoherence and Impact Deterioration Factor (ID Factor) were examined (Yamazaki, 1977; Maekawa *et al*., 1997; Shao *et al*., 2001; Song *et al*., 1998; Takeyasu, 1987). In this paper we focus our attention to the index parameters of vibration.

† : Corresponding Author

Kurtosis is one of the sophisticated inspection parameters which calculates normalized 4th moment of Probability Density Function (PDF).

Formerly, we introduced a simplified calculation method for the analysis of impact vibration including both sided affiliated impact vibration (Takeyasu *et al.*, 2004). Both sided affiliated impact vibration was approximated by triangle and simplified calculation method was introduced.

In this paper, simplified calculation method of kurtosis is introduced for the analysis of impact vibration with one sided affiliated impact vibration which occurs towards the progress of time. That phenomenon is often watched in the failure of such as bearings' outer race. One sided affiliated impact vibration is approximated by one sided triangle towards the progress of time and simplified calculation method is introduced. Varying the shape of one sided triangle, various models can be examined. If new model state the observed facts well, new method would be utilized effectively in making machine diagnosis.

We survey each index of deterioration in section 2. Simplified calculation method of Kurtosis including one sided affiliated impact vibration is introduced in section 3. Numerical example is exhibited in section 4. Section 5 is a summary.

## 2. FACTORS FOR VIBRATION CALCULATION

In cyclic movements such as those of bearings and gears, the vibration grows larger whenever the deterioration becomes bigger. Also, it is well known that the vibration grows large when the setting equipment to the ground is unsuitable (Yamazaki, 1977). Assume the vibration signal is the function of time as $x(t)$. And also assume that it is a stationary time series with mean 0. Denote the probability density function of these time series as $p(x)$. Indices for vibration amplitude are as follows.

$$X_{root} = \left[ \int_{-\infty}^{\infty} |x|^{\frac{1}{2}} p(x)dx \right]^2 \tag{1}$$

$$X_{rms} = \left[ \int_{-\infty}^{\infty} x^2 p(x)dx \right]^{\frac{1}{2}} \tag{2}$$

$$X_{abs} = \int_{-\infty}^{\infty} |x| \, p(x)dx \tag{3}$$

$$X_{peak} = \lim_{n\to\infty} \left[ \int_{-\infty}^{\infty} x^n p(x)dx \right]^{\frac{1}{n}} \tag{4}$$

These are dimensional indices which are not normalized. They differ by machine sizes or rotation frequencies. Therefore, normalized dimensionless indices are required. There are four main categories for this purpose.

A. Normalized root mean square value
B. Normalized peak value
C. Normalized moment
D. Normalized correlation among frequency domain

A. Normalized root mean square value

a. Shape Factor : SF

$$SF = \frac{X_{rms}}{\overline{X}_{abs}} \tag{5}$$

( $\overline{X}_{abs}$ : mean of the absolute value of vibration)

B. Normalized peak value

b. Crest Factor: CrF

$$CrF = \frac{X_{peak}}{X_{rms}} \tag{6}$$

( $X_{peak}$ : peak value of vibration)

c. Clearance Factor: ClF

$$ClF = \frac{X_{peak}}{X_{root}} \tag{7}$$

d. Impulse Factor: IF

$$IF = \frac{X_{peak}}{\overline{X}_{abs}} \tag{8}$$

e. Impact Deterioration Factor: ID Factor

$$ID = \frac{X_{peak}}{X_c} \tag{9}$$

( $X_c$ : vibration amplitude where the curvature of PDF becomes maximum)

C. Normalized moment

f. Skewness: SK

$$SK = \frac{\int_{-\infty}^{\infty} x^3 p(x)dx}{\left[ \int_{-\infty}^{\infty} x^2 p(x)dx \right]^{\frac{3}{2}}} \tag{10}$$

g. Kurtosis: KT

$$KT = \frac{\int_{-\infty}^{\infty} x^4 p(x)dx}{\left[ \int_{-\infty}^{\infty} x^2 p(x)dx \right]^2} \tag{11}$$

D. Normalized correlation in the frequency domain

h. Bicoherence

Bicoherence means the relationship of a function at different points in the frequency domain and is expressed as

$$Bic,_{xxx}\left(f_1, f_2\right) = \frac{B_{xxx}\left(f_1, f_2\right)}{\sqrt{S_{xx}\left(f_1\right)\cdot S_{xx}\left(f_2\right)\cdot S_{xx}\left(f_1+f_2\right)}} \quad (12)$$

Here

$$B_{xxx}\left(f_1, f_2\right) = \frac{X_T\left(f_1\right)\cdot X_T\left(f_2\right)\cdot X_T^*\left(f_1+f_2\right)}{T^{\frac{3}{2}}} \quad (13)$$

means Bispectrum and

$$X_T(t) = \begin{cases} x(t) & (0 < t < T) \\ 0 & (else) \end{cases}$$

T : Basic Frequency Interval

$$X_T(f) = \int_{-\infty}^{\infty} X_T(t) e^{-j2\pi ft} dt \quad (14)$$

$$S_{xx}(f) = \frac{1}{T} X_T(f) X_T^*(f) \quad (15)$$

Range of Bicoherence satisfies

$$0 < Bic,_{xxx}\left(f_1, f_2\right) < 1 \quad (16)$$

When there exists a significant relationship between frequencies $f_1$ and $f_2$, Bicoherence is near 1 and otherwise comes close to 0.

These indices are generally used in combination and machine condition is judged totally. Among them, Kurtosis is recognized to be superior index (Noda, 1987) and many researches on this have been made (Maekawa *et al.*, 1997; Shao *et al.*, 2001; Song *et al.*, 1998).

Judging from the experiment we made in the past, we may conclude that Bicoherence is also a sensitive good index (Takeyasu, 1987, 1989).

In Maekawa *et al.* (1997), ID Factor is proposed as a good index. In this paper, focusing on the indices of vibration amplitude, simplified calculation method of Kurtosis including one sided affiliated impact vibration is introduced.

Varying the shape of triangle, various models are examined.

## 3. SIMPLIFIED CALCULATION METHOD OF KURTOSIS

### 3.1 Several facts on Kurtosis

KT may be transformed into the one for discrete time system as

$$KT = \frac{\int_{-\infty}^{\infty} x^4 p(x)\, dx}{\left[\int_{-\infty}^{\infty} x^2 p(x)\, dx\right]^2} = \lim_{N\to\infty} \frac{\frac{1}{N-1}\sum_{i=1}^{N}\left(x_i - \overline{x}\right)^4}{\left\{\frac{1}{N-1}\sum_{i=1}^{N}\left(x_i - \overline{x}\right)^2\right\}^2} \quad (17)$$

where

$$\{x_i\} : i = 1, 2, \cdots, N, \cdots$$

are the discrete signal data.

$\overline{x}$ is an average of $\{x_i\}$

$$\overline{x} = \lim_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} x_i$$

Here the variance, the mean, KT of N amount of data are stated as

$$\sigma_N^2, \quad \overline{x}_N, \quad KT_N$$

### 3.2 Simplified Calculation Method of Kurtosis

When there arise failures on bearings or gears, peak value arise cyclically. In the early stage of the defect, this peak signal usually appears clearly. Generally, defects will injure other bearings or gears by contacting the inner covering surface as time passes. When defects grow up, affiliated impact vibration arises. One sided affiliated impact vibration which occurs towards the progress of time occurs in the case that there is a failure of such as bearings' outer race (Yamazaki *et al.*, 1988).

Hereafter, we analyze these cases by utilizing simplified model.

Assume that the peak signal which has $p$ times magnitude from normal signals asises during $m$ times measurement of samplings. As for determining sampling interval, sampling theorem is well known (Tokumaru *et al.*, 1982). But in this paper, we do not pay much attention on this point in order to focus on the proposed theme.

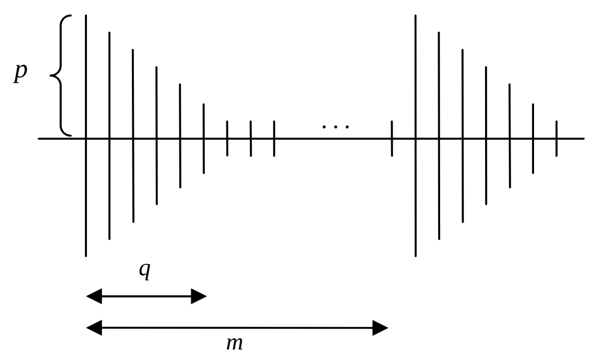


**Figure 1**. Impact vibration and affiliated vibration

Suppose that affiliated vibration can be approximated by triangle and set sampling count as $d$, then we can assume following triangle model (Figure 1).

When $d=1$, the peak signal which has $p$ times magnitude from normal signals arises.

When $d=i$, the peak signal which has $p-(i-1)\frac{p-1}{q}$ times magnitude from normal signals arises ($i=1,\cdots,q$).

When $d \geq q+1$, normal signal.

Let $\sigma_N^2$ state as $\overline{\sigma_N}^2$ when impact vibration occurs. As to 4th moment and Kurtosis, let them state as $\overline{MT_N}(4)$, $\overline{KT_N}$ in the same way.

$\overline{\sigma_N}^2$ can be calculated as follows.

$$\overline{\sigma_N}^2 = \frac{1}{m-1}\sum_{i=1}^{m}\left(x_i-\overline{x}\right)^2$$
$$= \left[\sum_{i=1}^{q}\left\{p-(i-1)\frac{(p-1)}{q}\right\}^2\right]\frac{\sigma_N^2}{m-1}+(m-1-q)\frac{\sigma_N^2}{m-1} \quad (18)$$
$$= \sigma_N^2+\frac{\sigma_N^2}{m-1}(q+1)(p-1)\left\{1+\frac{(p-1)(2q+1)}{6q}\right\}$$

As for $\overline{MT_N}(4)$, utilizing

$$\sum_{i=1}^{n} i^3 = \left\{\frac{n(n+1)}{2}\right\}^2$$

$$\sum_{i=1}^{n} i^4 = \frac{n}{30}(n+1)(2n+1)\left(3n^2+3n-1\right)$$

$\overline{MT_N}(4)$ can be calculated as follows.

$$\overline{MT_N}(4) = \frac{1}{m-1}\sum_{i=1}^{m}\left(x_i-\overline{x}\right)^4$$
$$= \frac{1}{m-1}\left[\sum_{i=1}^{q}\left\{p-(i-1)\frac{(p-1)}{q}\right\}^4\right]MT_N(4)+\frac{m-1-q}{m-1}MT_N(4)$$
$$= \left[1+\frac{1}{m-1}(q+1)(p-1)\left\{\frac{1}{30}(p-1)^3\frac{1}{q^3}(2q+1)\left(3q^2+3q-1\right)\right.\right.$$
$$\left.\left.+(p-1)^2\frac{1}{q}(q+1)+(p-1)\frac{1}{q}(2q+1)+2\right\}\right]MT_N(4) \quad (19)$$

Then we get $\overline{KT_N}$ as (20).

Set

$$A=1+\frac{1}{m-1}(q+1)(p-1)\left\{\frac{1}{30}(p-1)^3\frac{1}{q^3}(2q+1)\left(3q^2+3q-1\right)\right.$$
$$\left.+(p-1)^2\frac{1}{q}(q+1)+(p-1)\frac{1}{q}(2q+1)+2\right\}KT_N$$

$$B=\left[1+\frac{1}{m-1}(q+1)(p-1)\left\{\frac{2q+1}{6q}(p-1)+1\right\}\right]^2$$

Then

$$\overline{KT_N} = \frac{A}{B} \quad (20)$$

Here we introduce the following number. Each index is compared with normal index as follows.

$$Fa = \frac{P_{abn}}{P_{nor}} \quad (21)$$

Here

$P_{nor}$ : Index at normal condition
$P_{abn}$ : Index at abnormal condition

We get $F_a$ as (22).

Set

$$C=1+\frac{1}{m-1}(q+1)(p-1)\left\{\frac{1}{30}(p-1)^3\frac{1}{q^3}(2q+1)\left(3q^2+3q-1\right)\right.$$
$$\left.+(p-1)^2\frac{1}{q}(q+1)+(p-1)\frac{1}{q}(2q+1)+2\right\}$$

Then

$$Fa=\frac{\overline{KT_N}}{KT_N}$$
$$=\frac{C}{\left[1+\frac{1}{m-1}(q+1)(p-1)\left\{\frac{2q+1}{6q}(p-1)+1\right\}\right]^2} \quad (22)$$

## 4. NUMERICAL EXAMPLE

If the system is under normal condition, we may suppose $p(x)$ becomes a normal distribution function. Under this condition, $KT$ is always

KT = 3.0

Under the assumption of 3., let $m=12$. Considering the case $S=2,3,\cdots,6$ and $q=1,2,3,4$, we obtain Table 1 from the calculation of (22).

**Table 1**. $F_a$ by the variation of *p*, *q*

| | | *P* | | | | | |
|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 |
| *q* | 1 | 1.0 | 1.459 | 2.773 | 4.328 | 5.702 | 6.789 |
| | 2 | 1.0 | 1.422 | 2.409 | 3.425 | 4.254 | 4.883 |
| | 3 | 1.0 | 1.398 | 2.185 | 2.901 | 3.446 | 3.845 |
| | 4 | 1.0 | 1.201 | 2.006 | 2.531 | 2.912 | 3.184 |

As $KT_N \cong 3.0$, we show Table 2 as an approximation of $\overline{KT_N}$ by multiplying 3.0 for each item of Table 1.

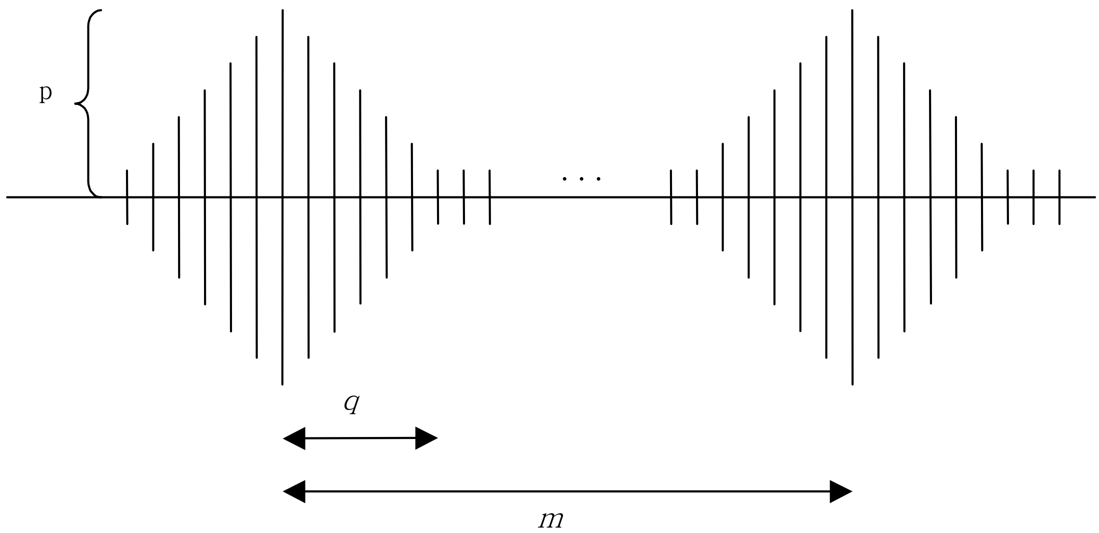


**Figure 2**. Impact vibration and both sided affiliated impact vibration

**Table 2**. $\overline{KT_N}$ for each case

| | | p | | | | | |
|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 |
| q | 1 | 3.0 | 4.377 | 8.319 | 12.984 | 17.106 | 20.367 |
| | 2 | 3.0 | 4.266 | 7.227 | 10.275 | 12.762 | 14.649 |
| | 3 | 3.0 | 4.194 | 6.555 | 8.703 | 10.338 | 11.535 |
| | 4 | 3.0 | 3.603 | 6.018 | 7.593 | 8.736 | 9.552 |

As $p$ increases, $F_a$ and $\overline{KT_N}$ increase.
On the other hand, $F_a$ and $\overline{KT_N}$ decrease ↗ ↙ as $q$ increases when $p$ is the same.

When damages increase or transfer to another place, peak level grows up and affiliated impact vibration spread.

This means that $\overline{KT_N}$ value shift from left-hand side upwards to right-hand side downwards in Table 2.

For example, following transition of $\overline{KT_N}$ can be supposed.

When $q=1$, $p=1$, $\overline{KT_N} = 3.0$
When $q=2$, $p=2$, $\overline{KT_N} = 4.266$
When $q=3$, $p=4$, $\overline{KT_N} = 8.703$
When $q=4$, $p=6$, $\overline{KT_N} = 9.552$

Here we compare the result of simplified calculation method of Kurtosis including one sided affiliated impact vibration and the result of simplified calculation method of Kurtosis (Takeyasu *et al*., 2004) including both sided affiliated impact vibration (Figure 2).

Previous calculation result (Takeyasu *et al*., 2004) is as follows.

**Table 3**. $\overline{KT_N}$ for each cases of both sided affiliated impact vibration

| | | p | | | | | |
|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 |
| q | 1 | 3.0 | 4.683 | 7.740 | 10.227 | 11.934 | 13.083 |
| | 2 | 3.0 | 4.263 | 6.090 | 7.431 | 8.325 | 8.964 |
| | 3 | 3.0 | 3.960 | 5.127 | 5.913 | 6.426 | 6.753 |
| | 4 | 3.0 | 3.165 | 4.336 | 4.932 | 5.247 | 5.463 |

Comparing these results, newly presented one is much more sensitive.

For example,
When $q=2$, $p=2$, $\overline{KT_N} = 4.266$ in Table 2,
to $\overline{KT_N} = 4.263$ in Table 3.
When $q=3$, $p=4$, $\overline{KT_N} = 8.703$ in Table 2
to $\overline{KT_N} = 5.913$ in Table 3.
When $q=4$, $p=6$, $\overline{KT_N} = 9.552$ in Table 2
to $\overline{KT_N} = 5.463$ in Table 3.

When the impact levels are same, the value of Kurtosis in simplified calculation method of one sided affiliated impact vibration is more sensitive than those of both sided affiliated impact vibration in general.

## 5. CONCLUSION

We proposed a simplified calculation method of Kurtosis for the analysis of impact vibration including one sided affiliated impact vibration. One sided affiliated impact vibration was approximated by triangle and simplified calculation method was introduced.

We compared them with those of both sided affiliated impact vibration. Newly proposed method proved to be much more sensitive.

Varying the shape of triangle, various models were examined and it was proved that new index was a sensitive good index for machine failure diagnosis. Utilizing this method, the behavior of Kurtosis would be forecasted and analyzed while watching machine condition and correct diagnosis would be executed.

The effectiveness of this method should be examined in various cases.

## REFERENCES


Bolleter, U. (1988), Blade Passage Tones of Centrifugal Pumps, *Vibration*, **4**(3), 8-13.

Hoffner, J. (1991), Preventive maintenance for No-Twist rod mills using vibration signature analysis, *Iron and Steel Engineer*, **68**(1), 55-61.

Maekawa, K., Nakajima, S., and Toyoda, T. (1997), New Severity Index for Failures of Machine Elements by Impact Vibration (in Japanese), *J.SOPE Japan*, **9**(3), 163-168.

Noda (1987), Diagnosis Method for a Bearing (in Japanese), *NSK Tec.J.*, **647**, 33-38.

Shao, Y., Nezu, K., Matsuura, T., Hasegawa, Y., and Kansawa, N. (2001), Bearing Fault Diagnosis Using an Adaptive Filter (in Japanese), *J.SOPE Japan*, **12**(3), 71-77.

Song, J. W., Tin, H., and Toyoda, T. (1998), Diagnosis Method for a Gear Equipment by Sequential Fuggy Neural Network (in Japanese), *J.SOPE Japan*, **10** (1), 15-20.

Takeyasu, K. (1987), Watching Method of Circulating Moving Object Patent by Japanese Patent Agency.

Takeyasu, K. (1989), Watching Method of Circulating Moving Object (in Japanese), Certified Patent by Japanese Patent Agency.

Takeyasu, K., Amemiya, T., Ino K., and Masuda, S. (2003), Machine Diagnosis Techniques by Simplified Calculation Method, *IEMS*, **2**(1), 1-8.

Hino, M. (1977), Spectrum Analysis(in Japanese), Asakura shoten Publishing.

Tokumaru, H., Soeda, T., Nakamizo, T., and Akizuki, K. (1982), Measurement and Calculation (in Japanese), Baifukan Publishing.

Yamazaki, H. (1977), Failure Detection and Prediction (In Japanese), Kogyo Chosakai Publishing.

Takeyasu, K. and Higuchi, Y. (2004), Analysis of the Behavior of Kurtosis by Simplified Model and its Application to Machine Diagnosis, The 5th Asia Pacific Industrial Engineering and Management systems Conference 2004 (Gold Coast Australia).