다특성 동시최적화를 위한 통합배열과 교차배열 접근의 비교연구

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Combined and Product Array Approaches in Simultaneous Optimization of Multiple Responses

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Key Words : Robust Parameter Design, Product Array, Combined Array, Multiple Responses, Simultaneous Optimization

Abstract

Robust parameter design is an off-line production technique for reducing variation and improving the quality of products and processes by using product arrays. However, the use of the product arrays usually requires a large number of runs. To overcome the drawback of the product array, the combined array can be used. Also optimizing multiple responses is increasingly important in industry. Using simultaneous optimization measures, we can deal with the multiple response case. In this paper we compare the simultaneous optimization using the Taguchi's product array with using the combined array. And models possible to set on combined arrays are also investigated and compared with the cases of product arrays.

1. Introduction and Review of Literature

1.1 Product & Combined Array

Robust parameter design (RPD) has been successfully used to improve the quality of products since the mid-1980s (see Taguchi, 1986 ; Wu, 1985 ; Nair, 1992). The technique consists of determining the levels of some set of controllable factors that reduce the sensitivity of the quality characteristic in the process to variations in another set of uncontrollable or noise factors, thus increasing the robustness of the quality characteristic. Taguchi has highlighted the need for considering both mean and variance of the characteristic of interest. Through the robust parameter design method and the use of SN ratio, Taguchi has developed a total package for approaching these problems. But his approach has many disadvantages and draws much criticism.

There have been efforts at integrating Taguchi's important notion of heterogeneous variability with the standard experimental design and modeling technology provided by response surface methodology (RSM). This approach was first proposed by Welch et al. (1990). They combined control and noise factors in a single design ma-

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trix, which we call a combined array. Noise factors are treated as control factors in a combined array. The run-size saving allowed by the combined array format comes from the flexibility to estimate their effects. The combined array design can reduce the total run-size extent to half or one third or more compared with that of the product array. And the combined array allows one to use dual response surface modeling. Also the combined array allows a sequential investigation. But results using the combined array approach depend critically on how well the model fits.

1.2 Multiple Response Optimization

One limitation of Taguchi's method is that the method can only be applied to optimize single response problems. However, optimizing multiple responses are increasingly important in industry today. Moreover, correlations among multiple responses always exist and these correlations may create conflicts in determining optimal parameter settings when employing the Taguchi's method to optimize each response individually. For instance, assume a product has two quality characteristics, say y_1 and y_2, with three control factors, A, B, and C, and with each factor having three levels (low, medium, and high). If the optimal level combination for y_1 is A = low, B = low, and C = high and the optimal level combination for y_2 is A = low, B = high, and C = medium, then the optimal setting levels for control factors B and C conflict in these two optimal combinations. Vining and Myers (1990) has some discussions on this matter. In optimization of multiple responses, Taguchi et al. (2001) proposed a new method using MAHALANOBIS distance.

1.3 Second Order Polynomial Model in the Combined Array Approach

In the combined array, if the quadratic terms about control factors(x_i's) and the linear terms about noise factors(z_i's) are included in the model, we can set on the polynomial regression model, which may be expressed as

$$y_i(\mathbf{x},\mathbf{z}) = \beta_{0i} + \mathbf{x}'\boldsymbol{\beta}_i + \mathbf{x}'\mathrm{B}_i\,\mathbf{x} + \mathbf{z}'\boldsymbol{\gamma}_i + \mathbf{z}'\mathrm{D}_i\,\mathbf{x} + \boldsymbol{\varepsilon}_i \qquad (1)$$

where $i=1, 2, \cdots, r$, $\mathbf{x}'=(x_1, x_2, \cdots, x_l)$, $\mathbf{z}'=(z_1, z_2, \cdots, z_m)$, β_i is $l\times1$, γ_i is $m\times1$, $\mathrm{B}_i'=\mathrm{B}_i$ is $l\times l$, D_i is $m\times l$ and ϵ_i is the random error associated with the response(See Box and Jones, 1990).

Let N be the number of experimental runs and model (1) can be expressed in matrix notation as

$$\mathbf{y}_i = \mathbf{X}\boldsymbol{\theta}_i + \boldsymbol{\varepsilon}_i,\ i = 1,2,\ldots,r,$$

where $\mathbf{y}_i$ is and $N\times1$ vector of observations on the ith response, X is an $N\times p$ matrix of known constants, θ_i is $p\times1$ matrix of parameters, p is number of parameters, and ϵ_i is a vector of random errors associated with the ith response. An unbiased estimator of the $r\times r$ variance-covariance matrix Σ is given by

$$\hat{\Sigma} = \frac{\mathrm{Y}'[\mathrm{I}_N - \mathrm{X}(\mathrm{X}'\mathrm{X})^{-1}\mathrm{X}']\mathrm{Y}}{N-p},$$

where $\mathrm{Y}=(\mathrm{y}_1, \mathrm{y}_2, \cdots, \mathrm{y}_r)_{N\times r}$, and I_N is identity matrix of order $N\times N$.

Among the various simultaneous-optimization measures, we will use two kinds of measure, P_M and P_V (which were partially discussed by Kwon (1994)). To define these measures, we assume the following fitted model by the least square method.

$$\hat{y}_i(\mathbf{x},\mathbf{z}) = b_{0i} + \mathbf{x}'\mathbf{b}_i + \mathbf{x}'\hat{\mathbf{B}}_i\,\mathbf{x} + \mathbf{z}'\mathbf{r}_i + \mathbf{z}'\hat{\mathbf{D}}_i\,\mathbf{x},\ i = 1,2,\ldots,r \quad (2)$$

The noise variables z_i's are not controllable and they are random variables. In the absence of other knowledge, z_i's would be usually uniformly distributed over R_z. By means of proper linear transformations on $\mathbf{x}$ and $\mathbf{z}$, we usually define the R_x and R_z by

$R_{\mathbf{x}} = \{\mathbf{x} : -1 \le x_i \le +1, i = 1,2,\ldots,l\}$ and
$R_{\mathbf{z}} = \{\mathbf{z} : -1 \le z_i \le +1, i = 1,2,\ldots,m\}$.

Then the mean response can be obtained as

$$\begin{aligned}\hat{m}_i(\mathbf{x}) &= \int_{R_z} \hat{\mathbf{y}}_i(\mathbf{x},\mathbf{z})p(\mathbf{z})d\mathbf{z} \\ &= b_{0i} + \mathbf{x}'\mathbf{b}_i + \mathbf{x}'\hat{\mathrm{B}}_i\,\mathbf{x} \quad ,i = 1,2,\ldots,r. \end{aligned} \tag{3}$$

Also, we can find the variance response as

$$\begin{aligned}\hat{v}_i(\mathbf{x}) &= \int_{R_z} [\hat{y}_i(\mathbf{x},\mathbf{z}) - \hat{m}_i(\mathbf{x})]^2 p(\mathbf{z})d\mathbf{z} \\ &= (\mathbf{r}_i + \hat{\mathrm{D}}_i\,\mathbf{x})'(\mathbf{r}_i + \hat{\mathrm{D}}_i\,\mathbf{x}) \quad ,i = 1,2,\ldots,r. \end{aligned} \tag{4}$$

$\hat{\mathbf{m}}(\mathbf{x})$ and $\hat{\mathbf{v}}(\mathbf{x})$ denote $(\hat{m}_1(\mathbf{x}),\ldots,\hat{m}_r(\mathbf{x}))'$ and $(\hat{v}_1(\mathbf{x}),\ldots,\hat{v}_r(\mathbf{x}))'$, respectively. The P_M measure optimizes $\hat{\mathbf{m}}(\mathbf{x})$ for the target value τ, while constraining $\hat{\mathbf{v}}(\mathbf{x})$ over the region of interest $R_{\mathbf{x}}$. A distance function of $\hat{\mathbf{m}}(\mathbf{x})$ for the target τ may be expressed as

$$D[\hat{\mathbf{m}}(\mathbf{x}),\boldsymbol{\tau}] = [(\hat{\mathbf{m}}(\mathbf{x}) - \boldsymbol{\tau})'\{Var[(\hat{\mathbf{m}}(\mathbf{x})]\}^{-1} (\hat{\mathbf{m}}(\mathbf{x}) - \boldsymbol{\tau})]^{1/2}$$

An unbiased estimator of $Var[\hat{\mathrm{m}}(\mathrm{x})]$ is given by

$$\hat{V}ar[\hat{\mathbf{m}}(\mathbf{x})] = [\mathbf{h}'(\mathbf{x})(\mathrm{X}'\mathrm{X})_0^{-1}\mathbf{h}(\mathbf{x})]\hat{\Sigma}$$

where $\mathbf{h}'(\mathbf{x}) = (1, x_1, \ldots, x_l, x_1^2, \ldots, x_l^2, x_1x_2, \ldots, x_{l-1}x_l)$, $(\mathrm{X}'\mathrm{X})^{-1}$ is $p \times p$, $(\mathrm{X}'\mathrm{X})_0^{-1}$ is $q \times q$, $q = (l+1)(l+2)/2$. Here $(\mathrm{X}'\mathrm{X})_0^{-1}$ is the submatrix of $(\mathrm{X}'\mathrm{X})^{-1}$ as follows

$$(\mathrm{X}'\mathrm{X})^{-1} = \begin{pmatrix} (\mathrm{X}'\mathrm{X})_0^{-1} & (\mathrm{X}'\mathrm{X})_{01}^{-1} \\ (\mathrm{X}'\mathrm{X})_{10}^{-1} & (\mathrm{X}'\mathrm{X})_1^{-1} \end{pmatrix},$$

where $(\mathrm{X}'\mathrm{X})_0^{-1}$, $(\mathrm{X}'\mathrm{X})_{01}^{-1}$, $(\mathrm{X}'\mathrm{X})_{10}^{-1}$ and $(\mathrm{X}'\mathrm{X})_1^{-1}$ are submatrices of $(\mathrm{X}'\mathrm{X})^{-1}$. And calculating P_M, we generally consider variance constraints.

The P_M measure can be written as

$$P_M = \frac{(\hat{\mathbf{m}}(\mathbf{x}) - \boldsymbol{\tau})'\hat{\Sigma}^{-1}(\hat{\mathbf{m}}(\mathbf{x}) - \boldsymbol{\tau})}{\mathbf{h}'(\mathbf{x})(\mathrm{X}'\mathrm{X})_0^{-1}\mathbf{h}(\mathbf{x})}$$

subject to $\hat{v}_i(\mathbf{x}) \le l_i$, $i = 1,2,\ldots r$.

If we have a prior knowledge about $\hat{\mathbf{m}}(\mathbf{x})$, it is possible to minimize $\hat{\mathbf{v}}(\mathbf{x})$ while constraining $\hat{\mathbf{m}}(\mathbf{x})$. Let

$$\hat{v}_i^*(\mathbf{x}) = \frac{\hat{v}_i(\mathbf{x}) - \min_{\mathbf{x}\in R_x}\hat{v}_i(\mathbf{x})}{\max_{\mathbf{x}\in R_x}\hat{v}_i(\mathbf{x}) - \min_{\mathbf{x}\in R_x}\hat{v}_i(\mathbf{x})} \quad ,i = 1,2,\ldots,r$$

Then the P_V measure is

$$P_V = \frac{1}{r}\sum_{i=1}^{r}\hat{v}_i^*(\mathbf{x})$$

$$\text{subject to} \begin{cases} m_{i*} \le \hat{m}_i(x) \le m_i^* & \text{: target-is-best} \\ \hat{m}_i(x) \ge m_{i*} & \text{: larger-the-better} \\ \hat{m}_i(x) \le m_i^* & \text{: smaller-the-better} \end{cases}$$

where m_{i*} is the minimum acceptable value of $\hat{m}_i(\mathbf{x})$, and m_i^* is the maximum acceptable value of $\hat{m}_i(\mathbf{x})$.

2. Possible Designs in the Combined Array

In most cases, it is enough to consider 2~6 control variables and 1~3 noise variables. According to the number of control variables and the number of noise variables, the total number of parameters used in the model changes. Then the form of design for setting on the model will be also changed. In this section, we will investigate these forms of design possible to use in the combined array, and compare with the cases of the product array.

2.1 Selection of Orthogonal Arrays

If the combined array consists of l control variables, and m noise variables, what is the total number of parameters used in the model (1)? To calculate it, we can put one term in intercept, l terms in $\mathbf{x}'\boldsymbol{\beta}_i$, $l(l+1)/2$ terms in $\mathbf{x}'\mathrm{B}_i\mathbf{x}$, m terms in $\mathbf{z}'\boldsymbol{\gamma}_i$ and $l\,m$ terms in $\mathbf{z}'\mathrm{D}_i\mathbf{x}$. So the total number of parameters used in the model is $1 + l + l(l+1)/2 + m + l\,m = (l+2m+2)(l+1)/2$.

There must be more number of necessary experiments than the number of parameters in order to estimate the parameters and the error term. Within this constraint, we can decide the most economic design of combined arrays. Note that the control variable should be allocated in the column of at least 3 levels, but the noise variable can be allocated in the column of at least 2 levels. We use standard orthogonal arrays and mixed orthogonal arrays for the design(See Park, 1996). The orthogonal array considered here are $L_9(3^4)$, $L_{18}(2^1 \times 3^7)$, $L_{27}(3^{13})$, $L_{36}(2^{11} \times 3^{12})$, $L_{36}(2^3 \times 3^{13})$, $L_{54}(2^1 \times 3^{25})$, and $L_{81}(3^{40})$.

For example, consider the cases where the number of noise variables is only 1.

i) If there are two control variables, the number of parameters is 9, and the most economic design is $L_{18}(2^1 \times 3^7)$.
ii) If there are three control variables, the number of parameters is 14, and the most economic design is $L_{18}(2^1 \times 3^7)$.
iii) In this way if there are four, five, and six control variables, the number of parameters is 20, 27 and 35, respectively. Considering the number of parameters we can easily decide the most economic design in each case.

Note that the usual number of levels for noise variables is two or three. For each case, we investigate how to allocate the control and noise variables. In each case, we select the most economic orthogonal array (OA) for the combined array and compare it with the OA for the product array. In the following <Table 1>~<Table 4>, C, N, n, p indicates control variable, noise variable, number of total experiments, and number of parameters, respectively.

Note that in the case of combined array, we need to investigate the number of parameters because we use modeling, but in the case of product array, we don't have to consider it because there are no model fitting in product array.

<Table 2> Product arrays for the 3-level noise variables (no model fitting)

# of C	# of N					
	1		2		3	
	n	OA	n	OA	n	OA
2	27	$L_9(3^4)$	81	$L_9(3^4)$	81	$L_9(3^4)$
3	27	$L_9(3^4)$	81	$L_9(3^4)$	81	$L_9(3^4)$
4	27	$L_9(3^4)$	81	$L_9(3^4)$	81	$L_9(3^4)$
5	54	$L_{18}(2^1 \times 3^7)$	162	$L_{18}(2^1 \times 3^7)$	162	$L_{18}(2^1 \times 3^7)$
6	54	$L_{18}(2^1 \times 3^7)$	162	$L_{18}(2^1 \times 3^7)$	162	$L_{18}(2^1 \times 3^7)$

<Table 1> Combined arrays for the 3-level noise variables (model fitting)

# of C	# of N					
	1		2		3	
	n(p)	OA	n(p)	OA	n(p)	OA
2	18(9)	$L_{18}(2^1 \times 3^7)$	18(12)	$L_{18}(2^1 \times 3^7)$	18(15)	$L_{18}(2^1 \times 3^7)$
3	18(14)	$L_{18}(2^1 \times 3^7)$	27(18)	$L_{27}(3^{13})$	27(22)	$L_{27}(3^{13})$
4	27(20)	$L_{27}(3^{13})$	27(25)	$L_{27}(3^{13})$	36(30)	$L_{36}(2^{11} \times 3^{12})$ or $L_{36}(2^3 \times 3^{13})$
5	36(27)	$L_{36}(2^{11} \times 3^{12})$ or $L_{36}(2^3 \times 3^{13})$	36(33)	$L_{36}(2^{11} \times 3^{12})$ or $L_{36}(2^3 \times 3^{13})$	81(39)	$L_{81}(3^{40})$
6	36(35)	$L_{36}(2^{11} \times 3^{12})$ or $L_{36}(2^3 \times 3^{13})$	81(42)	$L_{81}(3^{40})$	81(49)	$L_{81}(3^{40})$

<Table 3> Combined arrays for the 2-level noise variables (model fitting)

# of C	# of N					
	1		2		3	
	n(p)	OA	n(p)	OA	n(p)	OA
2	18(9)	$L_{18}(2^1 \times 3^7)$	18(12)	$L_{18}(2^1 \times 3^7)$ (dummy-level)	18(15)	$L_{18}(2^1 \times 3^7)$ (combination)
3	18(14)	$L_{18}(2^1 \times 3^7)$	27(18)	$L_{27}(3^{13})$ (combination)	27(22)	$L_{27}(3^{13})$ (combination, dummy-level)
4	27(20)	$L_{27}(3^{13})$ (dummy-level)	27(25)	$L_{27}(3^{13})$ (combination)	36(30)	$L_{36}(2^{11} \times 3^{12})$ or $L_{36}(2^3 \times 3^{13})$
5	36(27)	$L_{36}(2^{11} \times 3^{12})$ or $L_{36}(2^3 \times 3^{13})$	36(33)	$L_{36}(2^{11} \times 3^{12})$ or $L_{36}(2^3 \times 3^{13})$	54(39)	$L_{54}(2^1 \times 3^{25})$ (combination)
6	36(35)	$L_{36}(2^{11} \times 3^{12})$ or $L_{36}(2^3 \times 3^{13})$	54(42)	$L_{54}(2^1 \times 3^{25})$ (dummy-level)	54(49)	$L_{54}(2^1 \times 3^{25})$ (combination)

<Table 4> Product arrays for the 2-level noise variables (no model fitting)

# of C	# of N					
	1		2		3	
	n	OA	n	OA	n	OA
2	18	$L_9(3^4)$	36	$L_9(3^4)$	36	$L_9(3^4)$
3	18	$L_9(3^4)$	36	$L_9(3^4)$	36	$L_9(3^4)$
4	18	$L_9(3^4)$	36	$L_9(3^4)$	36	$L_9(3^4)$
5	36	$L_{18}(2^1 \times 3^7)$	72	$L_{18}(2^1 \times 3^7)$	72	$L_{18}(2^1 \times 3^7)$
6	36	$L_{18}(2^1 \times 3^7)$	72	$L_{18}(2^1 \times 3^7)$	72	$L_{18}(2^1 \times 3^7)$

2.2 Comparison between Combined Array and Product Array

The run-size of the combined array is generally smaller than that of the product array. In <Table 1> and <Table 2>, the run-size is the same in one case that the number of control variable is 4 and the number of noise variable is 1.

Suppose (Ci, Nj) means that the number of control variables is i and the number of noise variable is j, respectively. In <Table 3> and <Table 4>, the cases where the run-size is the same are (C2, N1), (C3, N1), (C5, N1), (C6, N1), (C4, N3). There exists one case where the run-size of combined array is bigger than that of product array : (C4, N1).

When we need to allocate more than two 2-level factors with some 3-level factors, there are only two appropriate orthogonal arrays, $L_{36}(2^{11} \times 3^{12})$ and $L_{36}(2^3 \times 3^{13})$. Because of such a limit of form of orthogonal arrays, we used the dummy-level technique or the combination design in some cases. In <Table 3>, we can see what kind of technique is used in that cases.

3. Comparative Study : Example

In this section, we will compare the product array approach with the combined array approach through the example.

We want to show that for many situations, the combined-array approaches is better than the product array approach in the sense that the former needs fewer experiments than the latter, and the results are approximately the same.

3.1 Product Array Approach

Suppose that the objective is to find the simultaneous optimum conditions for increasing the strength of plastic product and reducing the wear on the plastic product. Suppose there are three control factors A, B and C which are assigned to the orthogonal array, $L_{18}(2^1 \times 3^7)$. Also suppose there is a noise factor N with three levels. (N_0 : good condition, N_1 : normal condition, N_2: bad condition). The control factors are listed in <Table 5>. <Table 6> gives a set of strength data y_1 and wear data y_2. The run-size in Table 6 is 108.

Suppose that the quality characteristics for y_1 and y_2 are the 'larger-the-better' and the 'smaller-the-better' characteristics, respectively. <Table 7> and <Table 8> give ANOVA tables for SN ratio of y_1, y_2 respectively.

We can find the simultaneous optimum condition, $A_0B_1C_0$ by summarizing the results of all the data as shown in <Table 9>.

<Table 5> Factors and levels of plastic experiment

Control factors	-1 level	0 level	1 level
A : time (min)	120	125	130
B : temperature (℃)	60	70	80
C : stir speed (rpm)	700	800	900

<Table 6> Product array design and data in plastic experiment (See and Park, 1996)

EXP #	Control factor assignment and column number								y_1				y_2			
	e 1	A 2	B 3	C 4	e 5	e 6	e 7	e 8	N_0	N_1	N_2	SN	N_0	N_1	N_2	SN
1	-1	-1	-1	-1	-1	-1	-1	-1	45	49	52	33.70	30	25	18	-29.96
2	-1	-1	0	0	0	0	0	0	65	64	60	35.97	15	11	10	-21.72
3	-1	-1	1	1	1	1	1	1	73	69	75	37.17	29	31	22	-28.82
4	-1	0	-1	-1	0	0	1	1	63	60	69	36.08	8	14	11	-23.69
5	-1	0	0	0	1	1	-1	-1	55	56	49	34.49	9	7	15	-20.73
6	-1	0	1	1	-1	-1	0	0	68	72	72	36.97	19	17	12	-25.77
7	-1	1	-1	0	-1	1	0	1	62	66	61	35.97	9	12	5	-19.21
8	-1	1	0	1	0	-1	1	-1	55	49	56	34.49	14	20	17	-24.70
9	-1	1	1	-1	1	0	-1	0	74	80	74	37.60	8	15	17	-22.85
10	1	-1	-1	1	1	0	0	-1	69	55	66	35.91	25	29	29	-28.75
11	1	-1	0	-1	-1	1	1	0	57	52	44	34.00	19	19	13	-23.09
12	1	-1	1	0	0	-1	-1	1	78	76	68	37.34	12	15	14	-22.75
13	1	0	-1	0	1	-1	1	0	50	52	46	33.83	9	12	8	-22.62
14	1	0	0	1	-1	0	-1	1	51	45	46	33.47	15	22	23	-26.16
15	1	0	1	-1	0	1	0	-1	66	75	69	36.87	12	13	8	-20.99
16	1	1	-1	1	0	1	-1	0	56	51	59	34.81	18	25	23	-26.93
17	1	1	0	-1	1	-1	0	1	50	45	48	33.54	11	19	13	-25.28
18	1	1	1	0	-1	0	1	-1	73	67	55	36.07	11	7	10	-19.54
Sum												638.89				-424.25

<Table 7> ANOVA table (SN for strength)

Source of variation	Sum of squares	Degrees of freedom	Mean square	F_0
A	0.51	2	0.255	0.36
B	26.05	2	13.025	18.19^{**}
C	1.07	2	0.535	0.75
e	7.88	11	0.716	
T	35.51	17		

<Table 8> ANOVA table (SN for wear)

Source of variation	Sum of squares	Degrees of freedom	Mean square	F_0
A	45	2	22.50	15.85^{*}
B	1.67	2	0.84	0.59
C	106.79	2	53.40	37.63^{**}
e	15.85	11	1.42	
T	169.06	17		

<Table 9> Summarized table for optimal condition

Factor	Level	Sum of SN for Strength B^{**}	Sum of SN for Wear $A^{*}C^{**}$	Overall optimum
A (Time)	-1	214.09	-154.67	
	0	212.28	<u>-132.99</u>	O
	1	213.52	-136.59	
B (Temperature)	-1	210.87	-143.67	
	0	205.96	-141.40	
	1	<u>223.06</u>	-139.18	O
C (Stir speed)	-1	211.79	-140.87	
	0	215.28	<u>-123.79</u>	O
	1	212.82	-159.59	

3.2 Combined Array Approach

In the product array design in <Table 6>, if we allocate the noise variable z into column 5 in the inner array, this is a typical combined array for this experiment. The model fitted in the combined array can be the quadratic full model in the equation (1), but we used a reduced model using comparatively significant factor to give more correct results. We want to show that the combined array approach using P_M and P_V measures gives similar results compared with the product array approach in spite of fewer run-size.

We are assuming that y_1 and y_2 are modeled by functions of the same form. <Table 10> gives the data we used in combined array approach. The run-size in <Table 10> is 36. The run-size is one third of the run-size of product array approach.

<Table 10> Combined array design and data

EXP #	Control factor assignment and column number									
	e1	x_1 (A) 2	x_2 (B) 3	x_3 (C) 4	z (N) 5	e 6	e 7	e 8	y_1	y_2
1	-1	-1	-1	-1	-1	-1	-1	-1	45	30
2	-1	-1	0	0	0	0	0	0	64	11
3	-1	-1	1	1	1	1	1	1	75	22
4	-1	0	-1	-1	0	0	1	1	60	14
5	-1	0	0	0	1	1	-1	-1	49	15
6	-1	0	1	1	-1	-1	0	0	68	19
7	-1	1	-1	0	-1	1	0	1	62	9
8	-1	1	0	1	0	-1	1	-1	49	20
9	-1	1	1	-1	1	0	-1	0	74	17
10	1	-1	-1	1	1	0	0	-1	66	29
11	1	-1	0	-1	-1	1	1	0	57	19
12	1	-1	1	0	0	-1	-1	1	76	15
13	1	0	-1	0	1	-1	1	0	46	8
14	1	0	0	1	-1	0	-1	1	51	15
15	1	0	1	-1	0	1	0	-1	75	13
16	1	1	-1	1	0	1	-1	0	51	25
17	1	1	0	-1	1	-1	0	1	48	13
18	1	1	1	0	-1	0	1	-1	71	11

The optimal condition can be obtained by minimizing P_M and P_V, respectively. To implement computations, we used the IML procedure in SAS.

We obtained the optimal conditions when the values of x_i's are assumed to be an integer, and when the value of x_i's are assumed to be the first place of decimal as follows. Here we assume that the variance constraints as '$\leq$ 2.1, $\leq$ 5.5' and the mean response constraints as '$\geq$ 78.5, $\leq$ 9.0' for two responses respectively by the experimenter's requirements.

$P_M(\hat{v}_1(x) \geq 2.1, \hat{v}_2(x) \leq 5.5)$

$\mathbf{x}^*$	$\hat{m}_1(x)$	$\hat{m}_2(x)$	$\hat{v}_1(x)$	$\hat{v}_2(x)$	P_M
[0 1 0]	79.13	7.89	1.75	5.06	0.596

$\mathbf{x}^*$	$\hat{m}_1(x)$	$\hat{m}_2(x)$	$\hat{v}_1(x)$	$\hat{v}_2(x)$	P_M
[.2 1 -.2]	79.10	7.64	1.61	5.06	0.511

$P_V(\hat{m}_1(x) \geq 78.5, \hat{m}_2(x) \leq 9.0)$

$\mathbf{x}^*$	$\hat{m}_1(x)$	$\hat{m}_2(x)$	$\hat{v}_1(x)$	$\hat{v}_2(x)$	P_V
[0 1 0]	79.13	7.89	1.75	5.06	0.300

$\mathbf{x}^*$	$\hat{m}_1(x)$	$\hat{m}_2(x)$	$\hat{v}_1(x)$	$\hat{v}_2(x)$	P_V
[-.1 1 -.3]	78.52	8.84	1.29	2.95	0.205

Note that we can get the similar results as the results of the product array approach in Table 9, even though using the only one third of all data. The optimal condition $\mathbf{x}^*$ is approximately [0 1 0]. If we let the value of x_i's to be the first place of decimal, we can get more detailed optimum condition. So we can say the combined array approach using P_M and P_V gives similar results compared with the product array approach in spite of fewer run-size and give more information about optimality than the product array approach.

However, according to the constraints on variance and mean, the optimal condition using P_M and P_V varies to some extend. The following tables show the results.

$P_M(\hat{v}_1(x) \geq 1.1, \hat{v}_2(x) \leq 3.3)$

$\mathbf{x}^*$	$\hat{m}_1(x)$	$\hat{m}_2(x)$	$\hat{v}_1(x)$	$\hat{v}_2(x)$	P_M
[1 0 0]	57.17	6.63	0.07	1.76	7.462

$\mathbf{x}^*$	$\hat{m}_1(x)$	$\hat{m}_2(x)$	$\hat{v}_1(x)$	$\hat{v}_2(x)$	P_M
[-.2].9 -.2]	75.13	8.78	1.06	2.23	1.491

$P_M(\hat{v}_1(x) \geq 4.5, \hat{v}_2(x) \leq 7.8)$

$\mathbf{x}^*$	$\hat{m}_1(x)$	$\hat{m}_2(x)$	$\hat{v}_1(x)$	$\hat{v}_2(x)$	P_M
[0 1 0]	79.13	7.89	1.75	5.06	0.596

$\mathbf{x}^*$	$\hat{m}_1(x)$	$\hat{m}_2(x)$	$\hat{v}_1(x)$	$\hat{v}_2(x)$	P_M
[.6 1 -.2]	79.5	6.54	1.89	7.73	0.147

$P_V(\hat{m}_1(x) \geq 74.5, \hat{m}_2(x) \leq 10.2)$

$\mathbf{x}^*$	$\hat{m}_1(x)$	$\hat{m}_2(x)$	$\hat{v}_1(x)$	$\hat{v}_2(x)$	P_V
[0 1 0]	79.13	7.89	1.75	5.06	0.300

$\mathbf{x}^*$	$\hat{m}_1(x)$	$\hat{m}_2(x)$	$\hat{v}_1(x)$	$\hat{v}_2(x)$	P_V
[-.7 .9 -.2]	74.63	10.15	0.81	0.69	0.105

$P_V(\hat{m}_1(x) \geq 80.1, \hat{m}_2(x) \leq 7.9)$

$\mathbf{x}^*$	$\hat{m}_1(x)$	$\hat{m}_2(x)$	$\hat{v}_1(x)$	$\hat{v}_2(x)$	P_V
[1 1 0]	80.13	5.14	2.51	12.81	0.545

$\mathbf{x}^*$	$\hat{m}_1(x)$	$\hat{m}_2(x)$	$\hat{v}_1(x)$	$\hat{v}_2(x)$	P_V
[1.0 1.0 0]	80.13	5.14	2.51	12.81	0.545

It will be of interest to decide a good constraint so that we get a good optimal condition if we have no prior knowledge about mean response and variance response.

4. Concluding Remarks

The combined array approach allows one to provide separate estimates for the mean response and for the variance response. Accordingly, we can apply the primary goal of the Taguchi methodology which is to obtain a target condition on the mean while constraining the variance, or to minimize the variance while constraining the mean.

We compared the simultaneous optimi- zation using the Taguchi's product array with using the combined array. We used simultaneous optimization measures, P_M and P_V in this paper.

In this article, we investigated designs possible to use in the combined array, and compared with the cases of the product array. When some control and noise variables exist, we can get similar result through combined array approach though the fewer run-size.

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