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쇼케이 거리측도와 응용에 관한 연구

A study on the Choquet distance measures and their applications

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요 약

구간치 퍼지집합은 Gorzalczang(1983)에 의해 처음 제의되었다. 이를 토대로 Wang과 Li는구간치 퍼지수에 관한 연산으로 일반화 하여 연구하였다. 최근에 홍(2002)는 왕과 리의 이론을 리만적분에 의해 구간치 퍼지수 상의 거리측도에 관한 연구를 하였다. 우리는 일반측도와 관련된 리만적분 대신에 퍼지측도와 관련된 쇼케이적분을 이용한 구간치 퍼지수 상의 쇼케이 거리측도를 연구하였다(2005). 본 논문에서는 퍼지수치 퍼지수 상의 쇼케이 거리측도를 정의하고 이와 관련된 성질들을 조사하였다.

Abstract

Interval-valued fuzzy sets were suggested for the first time by Gorzalczang(1983). Based on this, Wang and Li extended their operations on interval-valued fuzzy numbers. Recently, Hong(2002) generalized results of Wang and Li and extended to interval-valued fuzzy numbers with Riemann integral. By using interval-valued Choquet integrals with respect to a fuzzy measure instead of Riemann integrals with respect to a classical measure, we studied some characterizations of interval-valued Choquet distance(2005). In this paper, we define Choquet distance measure for fuzzy number-valued fuzzy numbers and investigate some properties of them.

Key Words : Fuzzy number-valued fuzzy number, Distance measure, Choquet integral.

1. Introduction

Interval-valued fuzzy sets were suggested for the first time by Gorzalczang([4]). Based on this, Wang and Li extended their operations on interval-valued fuzzy numbers. Recently, Hong([3]) generalized results of Wang and Li([13]) and extended to interval-valued fuzzy sets with Riemann integral. Using interval-valued Choquet integrals with respect to a fuzzy measure instead of Riemann integrals with respect to a classical measure, we studied some characterizations of interval-valued Choquet distance([1,2,5,6,7,8]). In this paper, we define Choquet distance measure for fuzzy number-valued fuzzy numbers and investigate some algebraic properties of them.

In section 2, we introduce interval-valued fuzzy numbers and basic properties which are used in the next sections. In section 3, we define fuzzy number-valued fuzzy numbers which are a generalized concept of interval-valued fuzzy numbers and discuss some basic properties of them. In section 4, we introduce Choquet integral and their properties which are used in the next sections. In section 5, we define a Choquet distance measures for interval-valued fuzzy numbers and investigate some properties of them(see[8]). In section 6, by using the definition of Choquet distance measures for interval-valued fuzzy numbers, we define a Choquet distance measures for interval fuzzy number-valued fuzzy numbers and investigate some properties of them.

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2. Interval-valued fuzzy numbers

Throughout this paper, I will denote the interval $[0,1]$,

$$[I]=\{\,[a,b]\mid a,b\in I \text{ and } a\le b\,\}.$$

Then an element in $[I]$ is called *an interval number*.

Definition 2.1. Let $[a_1,b_1]$, $[a_2,b_2]\in[I]$. We define

$[a_1,b_1]=[a_2,b_2]$ if and only if

$$a_1=a_2,\ b_1=b_2$$

$[a_1,b_1]\le[a_2,b_2]$ if and only if

$$a_1\le a_2 \text{ and } b_1\le b_2,$$

$[a_1,b_1]<[a_2,b_2]$ if and only if

$$[a_1,b_1]\le[a_2,b_2] \text{but } [a_1,b_1]\ne[a_2,b_2].$$

Definition 2.2. If $a_t\in I, t\in T$, then we define $\vee_{t\in T}a_t=sup\,\{a_t|t\in T\}$ and $\wedge_{t\in T}a_t=inf\,\{a_t|t\in T\}$. We also define for $[a_t,b_t]\in[I], t\in T$,

$$\vee_{t\in T}[a_t, b_t]=[\vee_{t\in T}a_t, \vee_{t\in T}b_t],$$
$$\wedge_{t\in T}[a_t, b_t]=[\wedge_{t\in T}a_t, \wedge_{t\in T}b_t].$$

Definition 2.3. Let X be an ordinary nonempty set. Then the mapping $A: X\to[I]$ is called *an interval-valued fuzzy set* on X. All interval-valued fuzzy sets on X are denoted by $IF(X)$.

Definition 2.4. Let $A\in IF(X)$ and $A(x)=[A^-(x), A^+(x)]$, for all $x\in X$. Then two ordinary fuzzy sets $A^-: X\to I$ and $A^+: X\to I$ are called *lower fuzzy set* and *upper fuzzy set* about A, respectively.

Definition 2.5. Let $A\in IF(X)$ and $[\lambda_1, \lambda_2]\in[I]$. Then we call

$A_{[\lambda_1,\lambda_2]}=\{x\in X| A^-(x)\geq\lambda_1, A^+(x)\geq\lambda_2\}$ and

$A_{(\lambda_1,\lambda_2)}=\{x\in X| A^-(x)>\lambda_1, A^+(x)>\lambda_2\}$

the $[\lambda_1, \lambda_2]$-*level set* of A and the (λ_1, λ_2)-*level set* of A, respectively. And let

$A^-_\lambda=\{x\in X| A^-(x)\geq\lambda\}$ and

$A^+_\lambda=\{x\in X| A^+(x)\geq\lambda\}$.

Definition 2.6. Let $A\in IF(X)$ and $[\lambda_1, \lambda_2]\in[I]$. We define

$$([\lambda_1, \lambda_2]A)(x)=[\lambda_1, \lambda_2]\wedge[A^-(x), A^+(x)].$$

Definition 2.7. Let $A\in IF(R^+)$, *i.e.*, $A: R^+\to[I]$. Assume the following conditions are satisfied:

(1) A is normal, *i.e.*, there exists $x_0\in R^+$ such that $A(x_0)=1$.

(2) For arbitrary $[\lambda_1, \lambda_2]\in[I]-\{\overline{0}\}$, $A_{[\lambda_1,\lambda_2]}$ is closed bounded interval.

Then we call A *an interval-valued fuzzy number* on R^+.

Let $IF^*(R^+)$ denote the set of all interval-valued fuzzy numbers on R^+, and we write $[I]^+=[I]-\{\overline{0}\}$.

Definition 2.8. Let $A\in IF(R^+)$. Then an interval-valued fuzzy set A is said to be *convex* if for any $x, y\in R^+$ and $\lambda\in[0,1]$,

$$A(\lambda x+(1-\lambda)y)\geq A(x)\wedge A(y).$$

Definition 2.9. Let $A\in IF(R^+)$ and $\cdot\in\{+,\times,\div\}$. We define their operations to

$$(A\cdot B)(z)=\vee_{z=x\cdot y}(A(x)\wedge B(y)).$$

For each $[\lambda_1, \lambda_2]\in[I]^+$, we write

$A_{[\lambda_1,\lambda_2]}\cdot B_{[\lambda_1,\lambda_2]}=\{x\cdot y| x\in A_{[\lambda_1,\lambda_2]}, y\in B_{[\lambda_1,\lambda_2]}\}$.

Based on above definitions, we have the following theorems.

Theorem 2.10.([3]) Let $A, B\in IF(R^+)$ and $\cdot\in\{+,\times,\div\}$. Then we have

$$(A\cdot B)(z)=[(A^-\cdot B^-)(z), (A^+\cdot B^+)(z)].$$

Since all interval-valued fuzzy numbers on R^+ are positive(see[3]), by Corollary 3.3([3]), we have the following three theorems.

Theorem 2.11. Let $A, B\in IF^*(R^+)$ and $\cdot\in\{+,\times,\div\}$. Then $A\cdot B\in IF^*(R^+)$.

Theorem 2.12. Let $A, B\in IF^*(R^+)$ and $\cdot\in\{+,\times\}$. Then $A\cdot B=B\cdot A$.

Theorem 2.13. Let $A, B, C\in IF^*(R^+)$ and $\cdot\in\{+,\times\}$. Then $(A\cdot B)\cdot C=A\cdot(B\cdot C)$.

3. Fuzzy number-valued fuzzy numbers

In this section, we introduce fuzzy number-valued fuzzy numbers which are the generalized concept of fuzzy numbers.

Definition 3.1. A fuzzy set u on I is called *a fuzzy number* if it is satisfying the following conditions:

(i) (normality) $u(x)=0$ for some $x\in I$,

(ii) (convexity) for arbitrary $\lambda\in(0,1]$,

$[u]^\lambda=\{x\in R^+ \mid u(x)\geq\lambda\}\in I(R^+)$, and

(iii) $[u]^0=\{x\in R^+ \mid u(x)>0\}\in I(R^+)$.

Let $F(I)$ denote the set of all fuzzy numbers. we define for each pair $u, v\in F(I)$,

$u=v$ if and only if $[u]^\lambda=[v]^\lambda$

for all $\lambda\in[0,1]$,

$u\leq v$ if and only if $[u]^\lambda\leq[v]^\lambda$

for all $\lambda\in[0,1]$,

$u<v$ if and only if $u\leq v$ and $u\neq v$.

Definition 3.2. Let X be an ordinary nonempty set. Then the mapping $\mathcal{A}: X\to F(I)$ is called *a fuzzy number-valued fuzzy set* on X. All fuzzy number-valued fuzzy sets on X are denoted by $FF(X)$.

Then, clearly we have the following theorem.

Theorem 3.3. Let $\mathcal{A}\in FF(X)$. If we define $\mathcal{A}_\alpha(x)=[\mathcal{A}(x)]_\alpha$, for each $\alpha\in[0,1]$, then $\mathcal{A}_\alpha\in IF(X)$.

Definition 3.4. Let $\mathcal{A}\in FF(R^+)$, *i.e.* $\mathcal{A}: X\to F(I)$. Then $\mathcal{A}$ is called *a fuzzy number-valued fuzzy number* on R^+ if it is satisfying the following condition:

$$\text{for each } \alpha\in[0,1],\quad \mathcal{A}_\alpha \in IF^*(R^+).$$

All fuzzy number-valued fuzzy numbers on R^+ are denoted by $FF^*(R^+)$.

Definition 3.5. Let $A\in IF(R^+)$. Then a fuzzy number-valued fuzzy set A is said to be *convex* if for each $\alpha\in[0,1]$, $\mathcal{A}_\alpha$ is convex.

Definition 3.6. Let $\mathcal{A}, \mathcal{B}\in FF^*(R^+)$ and $\cdot\in\{+,\times,\div\}$. Then the operation $\mathcal{A}\cdot\mathcal{B}$ is defined by

$$[\mathcal{A}\cdot\mathcal{B}]_\alpha=[\mathcal{A}]_\alpha\cdot[\mathcal{B}]_\alpha, \text{ for } \alpha\in[0,1].$$

Theorem 3.7. Let $\mathcal{A}, \mathcal{B}\in FF^*(R^+)$ and $\cdot\in\{+,\times,\div\}$. Then for each $\alpha\in[0,1]$,

$$[\mathcal{A}\cdot\mathcal{B}]_\alpha(z)=[(\widetilde{A_\alpha^-}\cdot\widetilde{B_\alpha^-})(z),(\widetilde{A_\alpha^+}\cdot\widetilde{B_\alpha^+})(z)]$$

Proof. Let $\alpha\in[0,1]$. By Theorem 2.10, we have

$$\begin{aligned}[\mathcal{A}\cdot\mathcal{B}]_\alpha(z) &= (\mathcal{A}_\alpha\cdot\mathcal{B}_\alpha)(z)\\ &=[(\widetilde{A_\alpha^-}\cdot\widetilde{B_\alpha^-})(z)(\widetilde{A_\alpha^+}\cdot\widetilde{B_\alpha^+})(z)].\end{aligned}$$

By Definition 3.6 and Theorem 2.11, clearly, we have the following three theorems.

Theorem 3.8. Let $\mathcal{A}, \mathcal{B}\in FF^*(R^+)$ and $\cdot\in\{+,\times,\div\}$. Then $\mathcal{A}\cdot\mathcal{B}\in FF^*(R^+)$.

Theorem 3.9. Let $\mathcal{A}, \mathcal{B}\in FF^*(R^+)$ and $\cdot\in\{+,\times\}$. Then $\mathcal{A}\cdot\mathcal{B}=\mathcal{B}\cdot\mathcal{A}$.

Theorem 3.10. Let $\mathcal{A}, \mathcal{B}, \mathcal{C}\in FF^*(R^+)$ and $\cdot\in\{+,\times\}$. Then

$$(\mathcal{A}\cdot\mathcal{B})\cdot\mathcal{C}=\mathcal{A}\cdot(\mathcal{B}\cdot\mathcal{C}).$$

4. Choquet integrals

In this section, we introduce Choquet integrals and their basic properties which are used in the remainder sections(see [9,10,11,12]).

Definition 4.1. (1) *A fuzzy measure* on a measurable space (X,Ω) is an extended real-valued function $\mu:\Omega\to[0,1]$ satisfying

(i) $\mu(\phi)=0, \mu(X)=1$

(ii) whenever $A,B\in\Omega, A\subset B$,

then $\mu(A)\le\mu(B)$.

(2) μ is said to be *continuous from below* it for every increasing sequence $\{A_n\}\subset\Omega$ of measurable sets, we have

$$\mu\left(\bigcup_{n=1}^{\infty}A_n\right)=\lim_{n\to\infty}\mu(A_n).$$

(3) μ is said to be *continuous from above* it for every decreasing sequence $\{A_n\}\subset\Omega$ of measurable sets, we have

$$\mu\left(\bigcap_{n=1}^{\infty}A_n\right)=\lim_{n\to\infty}\mu(A_n).$$

(4) if μ is said to be continuous from above and continuous from below, it is said to be *continuous*.

Recall that a function $f: X\to[0,\infty]$ is said to be measurable if $\{x|f(x)>r\}\in\Omega$ for all $r\in(-\infty,\infty)$.

Definition 4.2. (1) The Choquet integral of a measure μ is defined by

$$(C)\int f\,d\mu=\int_0^\infty \mu_f(r)dr$$

where $\mu_f(r)=\mu(\{x\mid f(x)>r\})$ and the integral on the right-hand side is an ordinary one.

(2) A measurable function f is called integrable if the choquet integral of f can be defined and its value is finite.

Definition 4.3. Let f,g be measurable nonnegative functions. We say that f and g are *comonotonic*, in symbol $f\sim g$ if and only if

$$f(x)<f(x')\Rightarrow g(x)\le g(x') \text{ for all } x,x'\in X.$$

Theorem 4.4. Let f,g,h be measurable nonnegative functions. Then we have

(1) $f\sim f$,

(2) $f\sim g \Rightarrow g\sim f$,

(3) $f\sim a$ for all $a\in R^+$,

(4) $f\sim g$ and $f\sim h \Rightarrow f\sim(g+h)$.

Theorem 4.5. Let f,g be nonnegative measurable functions.

(1) If $f\le g$, then $(C)\int fd\mu\le(C)\int gd\mu$.

(2) If $f\sim g$ and $a,b\in R^+$, then

$$(C)\int(af+bg)d\mu=a(C)\int fd\mu+b(C)\int gd\mu.$$

(3) If we define $(f\vee g)(x)=f(x)\vee g(x)$ for all $x\in X$, then

$$(C)\int f\vee g\,d\mu\ge(C)\int fd\mu\vee(C)\int gd\mu,$$

and if we define $(f\wedge g)(x)=f(x)\wedge g(x)$ for all $x\in X$, then

$$(C)\int f\wedge g\,d\mu\le(C)\int fd\mu\wedge(C)\int gd\mu.$$

Theorem 4.6. (1) If $\{f_n\}$ is an increasing sequence of nonnegative measurable functions, then we have

$$(C)\int\lim_{n\to\infty}f_nd\mu=\lim_{n\to\infty}(C)\int f_nd\mu.$$

(2) If $\{f_n\}$ is a decreasing sequence of nonnegative measurable functions and f_1 is Choquet integrable, then

we have

$$(C)\int \lim_{n\to\infty} f_n d\mu = \lim_{n\to\infty} (C)\int f_n d\mu.$$

5. Distance between interval-valued fuzzy numbers

In this section, we introduce the Choquet distance measure between interval-valued fuzzy numbers and their basic properties(see [8]).

Definition 5.1. For arbitrary interval-valued fuzzy numbers $A, B \in IF^*(R^+)$, the quantity

$$D_c(A,B) = (C)\int d_H(A(x),B(x))d\mu(x)$$
$$= \int_0^\infty \mu\{x \mid d_H(A(x),B(x)) > r\}dr$$

is the Choquet distance measure between A and B, where d_H is the Hausdorff metric between $A(x)$ and $B(x)$ which is defined as

$$d_H(A(x),B(x))$$
$$= d_H(A^-(x),B^-(x)) \vee d_H(A^+(x),B^+(x))$$

since $A^-(x)$ and $A^+(x)$ the lower and the upper endpoint of $A(x)$,

$$A(x)=[A^-(x),\ A^+(x)].$$

Theorem 5.2. ([8] Theorem 4.2) The Choquet distance measure D_c is a pseudo-metric.

Theorem 5.3. ([8] Theorem 4.3) Let $A, B \in IF^*(R^+)$ and A, B are continuous. Then $D_c(A,B)=0$ if and only if $A=B$ $\mu-a.e.$, that is,

$$\mu(\{x\in R^+ \mid A(x)\neq B(x)\})=0.$$

Theorem 5.4. ([8] Theorem 4.4) Let $\{A_n\}$ be an increasing sequence in $IF^*(R^+)$ and let $d_H-\lim_{n\to\infty} A_n(x)=A(x)$ for all $x\in R^+$. Then we have

$$\lim_{n\to\infty} D_c(A_n, A)=0.$$

Theorem 5.5. If $A, B \in IF^*(R^+)$ for $n=1,2,\cdots$ and $\overline{0}=[0,0]$, then we have

(1) $D_c(\vee_{n=1}^\infty A_n, \overline{0}) \geq \vee_{n=1}^\infty D_c(A_n, \overline{0})$, and

(2) $D_c(\wedge_{n=1}^\infty A_n, \overline{0}) \leq \wedge_{n=1}^\infty D_c(A_n, \overline{0})$.

Proof. (1) Since $\vee_{n=1}^\infty A_n \geq A_k$ for all $k=1,2,\cdots$, by Theorem 4.5(3),

$$D_c(\vee_{n=1}^\infty A_n, \overline{0}) \geq D_c(A_k, \overline{0})$$

for all $k=1,2,\cdots$. Thus

$$D_c(\vee_{n=1}^\infty A_n, \overline{0}) \geq \vee_{k=1}^\infty D_c(A_k, \overline{0}).$$

(2) The proof of (2) is similar to the proof of (1).

6. Distance between fuzzy number-valued fuzzy numbers

In this section, we define two Choquet distance measures $\triangle_1$ and $\triangle_2$ between fuzzy number-valued fuzzy numbers.

Definition 6.1. Let $\overline{A}, \overline{B} \in FF^*(R^+)$. We define two Choquet distance measures $\triangle_1$ and $\triangle_2$ as the followings:

(1) $\triangle_1(\overline{A}, \overline{B}) = \int_0^1 D_c(\overline{A}_\alpha, \overline{B}_\alpha)d\alpha$, and

(2) $\triangle_2(\overline{A}, \overline{B}) = \vee_{\alpha\in[0,1]} D_c(\overline{A}_\alpha, \overline{B}_\alpha)$.

Immediately, Definition 6.1 and Theorems 5.3, 5.4 and 5.5 implies the following two theorems.

Theorem 6.2. The above two Choquet distance measures $\triangle_1$ and $\triangle_2$ are pseudo metrics.

Theorem 6.3. Let $\overline{A}, \overline{B} \in FF^*(R^+)$ and $\overline{A}_\alpha$, $\overline{B}_\alpha$ are continuous for all $\alpha\in[0,1]$. Then, we have

(1) $\triangle_1(\overline{A}, \overline{B}) = 0$ if and only if $\overline{A}_\alpha = \overline{B}_\alpha$, $\mu-a.e.$ on R^+ and almost everywhere $\alpha\in[0,1]$, and

(2) $\triangle_2(\overline{A}, \overline{B}) = 0$ if and only if $\overline{A}_\alpha = \overline{B}_\alpha$, $\mu-a.e.$ on R^+ and for all $\alpha\in[0,1]$.

Finally we investigate some properties of two Choquet distance measures $\triangle_1$ and $\triangle_2$ which are applied to the fields of approximate inference, signal transmission and controller, etc.

Theorem 6.4. Let $\{\overline{A}_n\}$ is an increasing sequence of fuzzy number-valued fuzzy numbers in $FF^*(R^+)$ and $\overline{A}\in FF^*(R^+)$.

(1) If $d_H(\overline{A}_{n\alpha}(x), \overline{A}_\alpha(x))\to 0$ as $n\to\infty$ for all $x\in R^+$ and for all $\alpha\in[0,1]$, then

$$\triangle_1 - \lim_{n\to\infty} \overline{A}_n = \overline{A}.$$

(2) If $\vee_{\alpha\in[0,1]} d_H(\overline{A}_{n\alpha}(x), \overline{A}_\alpha(x))\to 0$ as $n\to\infty$ for all $x\in R^+$, then

$$\triangle_2 - \lim_{n\to\infty} \overline{A}_n = \overline{A}.$$

Proof. (1) Let $\varepsilon>0$ and $\alpha\in[0,1]$. Since $d_H(\overline{A}_{n\alpha}(x), \overline{A}_\alpha(x))\to 0$ as $n\to\infty$ for all $x\in R^+$, by

Theorem 5.4, there exists $K \in R^+$ such that

$$D_c(\overline{A}_{n\alpha}, \overline{A}_\alpha) < \varepsilon, \ \forall n \geq K.$$

Then, for all $n \geq K$, by

$$\triangle_1(\overline{A}_n, \overline{A}) = \int_0^1 D_c(\overline{A}_{n\alpha}, \overline{A}_\alpha) d\alpha \leq \int_0^1 \varepsilon \, d\alpha = \varepsilon.$$

That is, $\triangle_1 - \lim_{n \to \infty} \overline{A}_n = \overline{A}$.

(2) Let $\varepsilon > 0$. Since $\vee_{\alpha \in [0,1]} d_H(\overline{A}_{n\alpha}(x), \overline{A}_\alpha(x)) \to 0$ as $n \to \infty$ for all $x \in R^+$, by Theorem 5.4, there exists $K \in R^+$ such that

$D_c(\overline{A}_{n\alpha}, \overline{A}_\alpha) < \varepsilon$, $\forall n \geq K$ and for all $\alpha \in [0,1]$.

Then, for all $n \geq K$, by

$$\triangle_2(\overline{A}_n, A) = \vee_{n=1}^{\infty} D_c(\overline{A}_{n\alpha}, \overline{A}_\alpha) \leq \vee_{n=1}^{\infty} \varepsilon = \varepsilon.$$

That is, $\triangle_2 - \lim_{n \to \infty} \overline{A}_n = \overline{A}$.

Theorem 6.5. If $\overline{A}_n \in FF^*(R^+)$ for all $n = 1, 2, \cdots$, then for $i = 1, 2$,

(1) $\triangle_i(\vee_{n=1}^{\infty} \overline{A}_n, \{\overline{0}\}) \geq \vee_{n=1}^{\infty} \triangle_i(\overline{A}_n, \{\overline{0}\})$, and

(2) $\triangle_i(\wedge_{n=1}^{\infty} \overline{A}_n, \{\overline{0}\}) \leq \wedge_{n=1}^{\infty} \triangle_i(\overline{A}_n, \{\overline{0}\})$.

Proof. (1) We will only prove (1) with $i = 1$, because the proof of (1) with $i = 2$ is similar to the proof of (1) with $i = 1$. Since $[\vee_{n=1}^{\infty} \overline{A}_n]_\alpha = \vee_{n=1}^{\infty} \overline{A}_{n\alpha}$,

$$[\vee_{n=1}^{\infty} \overline{A}_n]_\alpha = \vee_{n=1}^{\infty} \overline{A}_{n\alpha} \geq \overline{A}_{k\alpha}.$$

Then we have

$D_c([\vee_{n=1}^{\infty} \overline{A}]_\alpha, \overline{0}) \geq D_c(\overline{A}_{k\alpha}, \overline{0})$, for $k = 1, 2, \cdots$.

This implies

$$\begin{aligned} &\triangle_1(\vee_{n=1}^{\infty} \overline{A}_n, \{\overline{0}\}) \\ &= \int_0^1 D_c([\vee_{n=1}^{\infty} \overline{A}]_\alpha, \overline{0}) d\alpha \\ &\geq \int_0^1 D_c(\overline{A}_{k\alpha}, \overline{0}) d\alpha = \triangle_1(\overline{A}_k, \{\overline{0}\}). \end{aligned}$$

Therefore,

$$\triangle_1(\vee_{n=1}^{\infty} \overline{A}_n, \{\overline{0}\}) \geq \vee_{n=1}^{\infty} \triangle_1(\overline{A}_n, \{\overline{0}\}).$$

(2) The proof of (2) is similar to the proof of (1).

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