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# DEA와 AHP 기법이 결합된 DMU의 효율성 분석[†]

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# The Efficiency Analysis for DMU Using the Integration Method of DEA and AHP

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This study proposes a new approach which combines Data Envelopment Analysis (DEA) and the Analytic Hierarchy Process (AHP) techniques to effectively evaluate Decision Making Units (DMUs). While DEA evaluates a quantitative data set, employs linear programming to obtain input and output weights and ranks the performance of DMUs, AHP evaluates the qualitative data retrieved from expert opinions and other managerial information in specifying weights. The objective of this research is to design a decision support process for managers to incorporate positive aspects of DEA's absolute numerical evaluations and AHP's human preference structure values. It is believed that a pragmatic manager will be more receptive to the results that include subjective opinions incorporated into the evaluation of the efficiency of each DMU efficiency. The WPDEA method provides better discrimination than the DEA method by reducing the number of efficient units.

Keywords : Data Envelopment Analysis, Analytic Hierarchy Process, Priority Weight, Discrimination, Efficiency

## 1. 서 론

In real world situations, there exist peer groups of Decision Making Units (DMUs) which use various resources (inputs) to generate various results (outputs). It may be of interest to know the overall performance of DMUs by their inputs consumed and outputs generated. Generally, it is desirable to combine various inputs and outputs into one measure so that it can be used to evaluate and rank the performance of DMUs by a corresponding single measure of performance. There are two possible ways to determine the weights to be included in the analysis. Expert opinions, Analytic Hierarchy Process (AHP) methods or other managerial valuation information obtained through Principal Component Analysis (PCA) could be used. Also, the mathematical programming approach known as Data Envelopment Analysis (DEA) is one popular optimization method used for measuring the relative efficiency of DMUs (Zhu, 1998).

The DEA method is a data-oriented approach for relatively evaluating the performance of a group of entities referred to as DMUs. The AHP evaluates the qualitative data retrieved through expert opinions and other managerial in-

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formation to specify weights. When applied to decision making, it assists one to describe the general decision operation by decomposing a complex problem into a multi-level hierarchic structure of objectives, criteria, sub-criteria and alternatives.

The AHP only aims to synthesize the target unit's efficiency into an overall quality measure of each unit. Accordingly, the AHP helps the decision-maker not only to identify the principal competitors of the DMU, but also to assess the service performance of the DMU relative to its principal competitors. The AHP permits the decision-maker to investigate the sensitivity of the quality measure to whatever kinds of changes in customer judgement may occur. The AHP carries out comparisons of the elements involved in a decision in an appropriate manner to derive their scales of priorities. Pairwise comparisons enable to improve consistency by using as much as information as possible (Saaty, 1980).

DEA's strength is in simultaneously considering multiple inputs and multiple outputs without any need for a priori assignment of weights. However in view of the possibility of data entry errors, DEA has been questioned regarding the validity and stability of its results (Schmidt, 1985).

The original DEA does not perform full ranking, as it merely provides classification into two groups: efficient and inefficient. There have been a few limited attempts to fully rank units in the context DEA. Yang and Kuo (2003) used the subjective AHP results in DEA for facilities layout design problem. However, this approach has the limitations of both methods: the subjectivity of AHP and the pareto solution of DEA. Sinuany-Stern (2000) developed the two-stage ranking model AHP/DEA without sub-criteria priority weights and overall priority weights.

In this study, the use of the AHP's sub-criteria and overall priority weights and DEA techniques for unifying the tasks of resource allocation and target sets in multi-unit, multi-level planning are examined. Specifically, the two techniques are incorporated into a model that supports a link between resource allocation and performance targets of individual DMUs and that incorporates decision makers' preferences. The new weighted priority weights in DEA (WPDEA) approach integrates subjective and objective data into a combined approach with sub-criteria priority weights and overall priority weights to the problem. The results of WPDEA are more realistic and qualified for each DMU efficiency when the DEA objective values and the AHP subjective values are combined simultaneously. In addition, by reducing the number of efficient units, the WPDEA method provides better discrimination than the DEA method.

## 2. Methodology

### 2.1 Charnes, Cooper and Rhodes's DEA Formulation

The original DEA model for $\mathrm{DMU}_k$ with $s$ inputs and $m$ outputs is given as follows;

Maximize

$$Z_k = \sum_{r=1}^{s} u_{rk} Y_{rk} / \sum_{i=1}^{m} v_{ik} X_{ik} \quad \cdots\cdots (1)$$

Subject to

$$\sum_{r=1}^{s} u_{rk} Y_{rj} / \sum_{i=1}^{m} v_{ik} X_{ij} \leq 1, \qquad \text{for } j = 1,..,n$$

$$u_{rk} > 0, \qquad \text{for } r = 1,..,s$$

$$v_{ik} > 0, \qquad \text{for } i = 1,..,m$$

Here, $u_{rk}$ is the weight given to the $r$th output of the $k$th DMU, $v_{ik}$ is the weight given to the $i$th input of the $k$th DMU. $u_{rk}$ and $v_{ik}$ are decision variables in formula $P_k$. $Y_{rj}$ is the amount of the $r$th output of the $j$th DMU, and $X_{ij}$ is the amount of the $i$th input of the $j$th DMU. It is assumed that all $Y_{rj}$ and $X_{ij}$ are positive.

### 2.2 The Weighted Priority weights DEA (WPDEA) Primal Formulation

Incorporated with the AHP method, the Weighted Priority Weights DEA (WPDEA) primal formulation is designed to evaluate each DMU's efficiency with better discrimination. The objective function of this model maximizes the efficiency rating $Z_k$ for $\mathrm{DMU}_k$.

Maximize

$$Z_k = \sum_{r=1}^{s} U_{rk} \theta_k Y_{rk} \quad \cdots\cdots (2)$$

Subject to

$$\sum_{i=1}^{m} V_{ik} \theta_k X_{ij} = 1$$

$$\sum_{r=1}^{s} U_{rk} e_i^{(k)} Y_{rj} + \sum_{i=1}^{m} V_{ik} e_i^{(k)} X_{ij} \geq 0, \qquad \text{for } j = 1,..,n$$

$$U_{rk} > 0, \qquad \text{for } r = 1,..,s$$

$$V_{ik} > 0, \qquad \text{for } i = 1,..,m$$

where $U_{rk}$ is the variable weight assigned to the output and $\theta_k Y_{rk}$ is the value assigned to the output by valuing each unit of output at $\theta_k$, the priority weight assigned by the AHP method. Also, $V_{ik}$ is the variable weight assigned to the input and $\theta_k X_{ik}$ is the value assigned to the input by valuing each unit of input at $\theta_k$. the priority weight assigned by the AHP method. $e_i^{(k)}$ is sub-criteria priority weights of the $k$th input or $k$th output of $i$th criterion. $\theta_k$ is the overall priority weight of the $k$th criterion.

The WPDEA primal formulation integrates the subjective factors of the AHP's priority weights and objective data of DEA into a combined approach to the problems. The WPDEA primal formulation constraints multiply the weighted sum of the inputs by the input overall priority weights requiring this sum to be equal to 1 and maximizes the sum of the outputs multiplied by the output overall priority weights that can be obtained. In the other constraints the weighted outputs are multiplied by the output sub-criteria priority weight and the sum cannot exceed the weighted inputs multiplied by the input sub-criteria priority weight for any DMU.

## 2.3 The Weighted Priority weights DEA (WPDEA) Dual Formulation

The WPDEA Dual formulation is designed to measure the WPDEA efficiency of a specific $DMU_k(E_k)$.

Minimize $E_k$ ·································· (3)

Subject to

$$\sum_{j=1}^{n} D_j = 1,$$

$$\sum_{j=1}^{n} Y_{rj} e_{kr} D_{kj} \geq Y_{rk}\theta_k, \qquad \text{for } r = 1,..,s$$

$$-X_{ik}\theta_k E + \sum_{j=1}^{n} X_{ij} e_{kr} D_{kj} \leq 0, \qquad \text{for } i = 1,..,m$$

$$D_{kj} > 0\ , \qquad \text{for } j = 1,..,n$$

where the WPDEA dual model will require the sum of the weights to be equal to 1. There is a set of $n$ DMUs to be analyzed, each of which uses $m$ inputs and $s$ outputs. Let objective function $E_k$ denote the $DMU_k$ ($k = 1,..,n$) whose relative efficiency is to be minimized. $e_{kr}$ is sub-criteria priority weights of the $r$th input or $r$th output of $k$th DMU in AHP. $\theta_k$ is the overall priority weight of the $k$th criterion in AHP. $X_{ij}$ is the amount of the $i$th input to the $j$th DMU, and $Y_{rj}$ is the amount of the $r$th output to the $j$th DMU. $Y_{rk}$ is the amount of the $r$th output to the $k$th DMU, $X_{ik}$ is the amount of the $i$th input to the $k$th DMU. $D_k$ is the weight applied to inputs and outputs for $DMU_k$.

## 2.4 The Steps of the WPDEA Approach

This section provides an illustrated data example model of WPDEA. Note that three DMUs with two inputs and one output are assumed for the illustration.

***Step 1.*** Use the original DEA hypothetical data and to find out the efficiency of each DMU. <Table 1> shows the results of DEA hypothetical data set.

<Table 1> The Hypothetical Model of DEA Data Set and Efficiency

| | $X_1$ | $X_2$ | Y | Efficiency |
|---|---|---|---|---|
| $DMU_1$ | 4 | 2.8 | 1 | 0.7143 |
| $DMU_2$ | 2 | 2 | 1 | 1.0000 |
| $DMU_3$ | 2.3 | 3.5 | 1 | 0.8685 |

note: $X_1$= Input 1 , $X_2$ = Input 2 , Y = Output

***Step 2.*** Make a pair-wise comparison matrix of the relative importance of each input and output (qualitative data) for the AHP model. Then, calculate priority weights of all sub-criteria and overall alternatives by normalized comparison matrix in AHP. <Table 2> shows all priority weights of the given hypothetical AHP's example. There are unique properties in priority weights:

$$\sum_{r=1}^{s} e_{kr} = 1, \text{ and } \sum_{k=1}^{n} \theta_k = 1.$$

<Table 2> The Priority Weights of the Given Hypothetical AHP's Example

| Priority Weights | $X_1$ ($e_{kr}$) | $X_2$ ($e_{kr}$) | Y ($e_{kr}$) |
|---|---|---|---|
| $DMU_1$ | 0.1263 | 0.5437 | 0.3333 |
| $DMU_2$ | 0.4576 | 0.3459 | 0.3333 |
| $DMU_3$ | 0.4160 | 0.1103 | 0.3333 |
| Overall ($\theta_k$) | 0.6234 | 0.2395 | 0.1373 |

***Step 3.*** Return to the WPDEA formulation and determine the sub-criteria priority weights ($e_{kr}$) for all output into the 2nd constraint of Equation (3). Also, apply the overall priority weights ($\theta_k$) for all output into the 2nd constraint of

Equation (3).

$$\sum Y_{rj}\, e_{kr}\, D_{kj} \geq Y_{rk}\, \theta_k \quad \textit{(for all output)}$$

Determine the sub-criteria priority weights ($e_{kr}$) for all input into the 3rd constraint of Equation (3). Also, apply the overall priority weights ( $\theta_k$) for all input into the 3rd constraint of Equation (3). And then, move the terms involving $E$ to the left-hand side of the all input constraints because $E$ is a decision variable.

$$-X_{ik}\, \theta_k\, E + \sum X_{ij}\, e_{kr}\, D_{kj} \leq 0 \qquad \textit{(for all input)}$$

***Step 4.*** Solve the Equation (3).

In developing a linear programming model for $DMU_1$, the weights used to construct the composite DMU are the variables. They are defined as follows:

$D_i$ = weight applied to inputs and outputs for $DMU_i$, i=1, 2, 3.

The weights will be used to determine the inputs and outputs for the hypothetical DMU. The input/output relationships that will be included in the model will have the following general form:

*Input/output of the composite DMU*
= (*input/output at* $DMU_1$)$D_1$
+ (*input/output at* $DMU_2$)$D_2$
+ (*input/output at* $DMU_3$)$D_3$ ………… (4)

<Table 3> The Hypothetical Data of Input Resources for Three DMUs.

| Input Measure | $DMU_1$ | $DMU_2$ | $DMU_3$ |
|---|---|---|---|
| $X_1$ | 4 | 2 | 2.3 |
| $X_2$ | 2.8 | 2 | 3.5 |

<Table 4> The Hypothetical Data of Output Resources for Three DMUs.

| Output Measure | $DMU_1$ | $DMU_2$ | $DMU_3$ |
|---|---|---|---|
| Y | 1 | 1 | 1 |

In formulating the WPDEA dual model, use Equation (4) to develop a constraint for each input and output measure. In computing the weights for the three DMUs forming the composite DMU, the WPDEA dual model will require the sum of the weights to be equal to 1. Thus, for the example, the first constraint will be as follows:

$$D_1 + D_2 + D_3 = 1$$

Using the general input/output relationship for the one output measure, the constraint for the WPDEA dual model is written as follows:

$$(1*0.3333)D_1 + (1*0.3333)D_2 + (1*0.3333)D_3 \geq (1*0.1373)$$

The above constraint requires the linear programming solution to provide weights such that all outputs for the composite DMU will be greater than or equal to the one output of $DMU_1$. If a solution satisfying the output constraint can be found, then the composite DMU produces at least as much of each output as $DMU_1$.

Next, the two input measures must be considered. The WPDEA dual model contains the variable $E$, which determines the fraction of $DMU_1$'s input required by the composite DMU. The use of $E$, which is referred to as the efficiency index, is shown on the next step.

<Table 5> The Composite Data for Inputs

| Input Measure | Used by $DMU_1$ | Available to Composite DMU |
|---|---|---|
| $X_1$ | (4*0.6234) | (4*0.6234)E |
| $X_2$ | (2.8*0.2395) | (2.8*0.2395)E |

As shown above, the resources available to the composite DMU are simply a multiple of the resources used at $DMU_1$. If $E$ = 1, the resources available to the composite DMU are the same as those used by $DMU_1$. If $E$ is greater than 1, the composite DMU would have available proportionally less resources. The WPDEA dual model constraints for the two input measures are written as follows:

Input1 :

$$(4*0.1263)D_1 + (2*0.4576)D_2 + (2.3*0.4160)D_3 \leq (4*0.6234)E$$

Input2 :

$$(2.8*0.5437)D_1 + (2*0.3459)D_2 + (3.5*0.1103)D_3 \leq (2.8*0.2395)E$$

If a solution with $E$ < 1 can be found, the composite DMU does not need as many resources as $DMU_1$ needs.

The objective function for the WPDEA dual model is to minimize the values of $E$, which is equivalent to minimizing the input resources available to the composite DMU. Thus, the objective function is written as

Minimize $E$

The DEA efficiency conclusion is based on the optimal objective function value for $E$. The decision rule is as follows:

If $E = 1$, the composite DMU requires as much input as $DMU_1$ does. If $E < 1$, the composite DMU requires less input to obtain the output achieved by $DMU_1$.

<Table 6> The New Efficiency Results by the WPDEA

| | $X_1$ | $X_2$ | Y | Efficiency |
|---|---|---|---|---|
| $DMU_1$ | 4 | 2.8 | 1 | 0.5757 |
| $DMU_2$ | 2 | 2 | 1 | 0.8060 |
| $DMU_3$ | 2.3 | 3.5 | 1 | 0.6283 |

The original DEA data efficiency may now be compared with the traditional AHP normalized efficiency and newly applied AHP's priority weights to DEA model efficiency. The resulting WPDEA dual model solution is presented in <Table 7>. Based on the WPDEA dual model solution the organization should select $DMU_2$ also.

<Table 7> The Efficiency Results of DMUs by Different Approaches

| | AHP | DEA | WPDEA |
|---|---|---|---|
| $DMU_1$ | 0.6154 | 0.7143 | 0.5757 (0.7142) |
| $DMU_2$ | 1.0000 | 1.0000 | 0.8060 (1.0000) |
| $DMU_3$ | 0.8008 | 0.8685 | 0.6283 (0.7795) |

Note: The values in the brackets are normalized values.

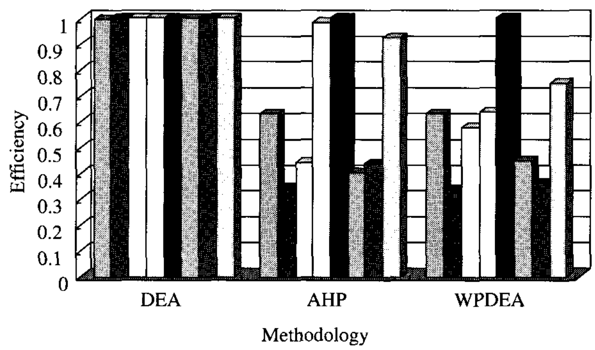
## 3. Applications of WPDEA

### 3.1 Bank Branches Evaluation

To further demonstrate the usefulness of the WPDEA approach, an application example is introduced in this section. This example used 8 branches of a regional division of the Commercial Bank of Greece (Giokas, 1991). Only those inputs which concern bank branches directly were used, ignoring bank overheads, since the objective of this analysis was to evaluate the use of inputs consumed directly at the branch. Inputs employed are the following: (1) Labor hours, (2) Operating expenses, and (3) Square meters of branch space. Outputs employed are the following: (1) Weighted number of transactions performed by section of deposits and capital transfers, (2) Weighted number of transactions performed by section of credit, (3) Weighted number of transactions performed by section of foreign receipts, and (4) Total weighted number of transactions performed by each branch (Giokas, 1991). The DEA, AHP, and WPDEA results obtained from the data sets are presented in <Table 8> and <Figure 1>.

<Table 8> The Efficiency Results of Bank Branches

| DMU | DEA | AHP | WPDEA |
|---|---|---|---|
| 1 | 0.9978 | 0.6299 | 0.6318 |
| 2 | 1.0000 | 0.3432 | 0.3412 |
| 3 | 1.0000 | 0.4424 | 0.5765 |
| 4 | 1.0000 | 0.9832 | 0.6378 |
| 5 | 1.0000 | 1.0000 | 1.0000 |
| 6 | 1.0000 | 0.4056 | 0.4512 |
| 7 | 1.0000 | 0.4404 | 0.3619 |
| 8 | 1.0000 | 0.9254 | 0.7492 |




<Figure 1> The Efficiency of Bank Branches by Different Approaches

The results of DEA approach indicate that 7 out of 8 branches are relatively efficient, that is, they have an efficiency rating of equal 1. It is not clear that which bank branch is the most efficient branch. The results of the AHP approach indicate that $DMU_5$ is the most efficient branch and $DMU_4$ is the second most efficient bank branch in terms of customer evaluation. The WPDEA approach integrates subjective and objective data into a combined approach to the bank branch problem. The results of the WPDEA approach indicate that $DMU_5$ is the most efficient bank branch. $DMU_5$ is a more efficient and qualified bank branch when the DEA objective values and the AHP subjective values are combined simultaneously.

## 4. Conclusion

The Weighted Priority weights in Data Envelopment Analysis (WPDEA) study compares and combines the DEA and AHP techniques in the evaluation of DMUs. The new method integrates subjective and objective data into a combined approach to the problem. Hence, the results of WPDEA are more realistic and qualified for determining each DMU's efficiency. Moreover, the evaluation results would allow a decision maker to see his input used in the model and have confidence in the logic which was involved in the analysis.

The objective of the WPDEA research is to design a decision support process for managers to incorporate the positive aspects of the DEA's absolute numerical evaluations and AHP's human preference structure values. It is believed that a pragmatic manager will be more receptive to the results that include subjective opinions incorporated into the evaluation of the efficiency of each DMU when the AHP and DEA are combined simultaneously. Further, by reducing the number of efficient units, the WPDEA method provides better discrimination than the DEA method.

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