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DEGENERATE PRINCIPAL SERIES FOR EXCEPTIONAL p-ADIC GROUPS OF TYPE G_2

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Abstract. We determine reducibility points of degenerate principal series for exceptional p-adic groups of type G_2 via Jacquet module techniques and Hecke algebra isomorphisms.

1. Introduction

Given a connected reductive group G over a p-adic field F, Jacquet's theorem [5] shows that every irreducible admissible representation π of G is either supercuspidal or $\pi \subset i_M^G(\sigma)$ for a proper parabolic subgroup $P = MU \subset G$ and an irreducible supercuspidal representation σ of M.

To understand the admissible dual of G, we investigate $i_M^G(\sigma)$. In this paper, we focus on degenerate principal series. Degenerate principal series are representations obtained by inducing a one-dimensional representation of a maximal parabolic subgroup. For $GL(n)$ and $SL(n)$, degenerate principal series were studied by Zelevinsky [15] and Tadić [13] in general setting. For symplectic and orthogonal groups, Ban and Jantzen studied reducibility points of degenerate principal series [1], [6], [7], [8].

Among exceptional groups, degenerate principal series for groups of type G_2 were understood by Muić using Plancherel measures [10]. However, his method cannot be applied to other exceptional groups because explicit Plancherel measures for those groups are as yet unknown. Instead of using Plancherel measures, we use Jacquet module techniques and Hecke algebra isomorphisms to determine reducibility points of exceptional p-adic groups of type G_2. This approach provides another way of investigating degenerate principal series for exceptional p-adic groups.

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2. Preliminaries and methodology

Let G be a split connected reductive p-adic group, $P = MU$ a parabolic subgroup of G, Ω an irreducible admissible representation of M. Set $\pi = i_M^G(\Omega)$. Let $s_1, \dots, s_n$ be the simple reflections in W. Let $B = AU_{min}$ be the minimal parabolic, and set $N_i = \langle B, s_i \rangle$, which is the Levi factor of a larger parabolic subgroup P_i. We now recall a theorem of Bernstein-Zelevinsky [3]. Let M, N be Levi factors of standard parabolic subgroups of G. Set $W^{MN} = \{w \in W \mid w(\Phi_M^+) \subset \Phi^+, w^{-1}(\Phi_N^+) \subset \Phi^+\}$.

THEOREM 2.1. *Let Ω be an admissible representation of M. Then, $r_N^G \circ i_M^G(\Omega)$ has a composition series with factors $i_{N'}^N \circ w \circ r_{M'}^M(\Omega)$, $w \in W^{MN}$ where $M' = M \cap w^{-1}(N)$, $N' = w(M) \cap N$.*

We denote by $BZ_N(\pi)$ the collection of representation $i_{N'}^N \circ w \circ \tau$ as τ runs over the components of $r_{M'}^M(\Omega)$ and w runs over W^{MN}. The following theorem is the main tool of determining reducibility points in regular cases. First, we want Ω to be irreducible. Second, we want $r_A^M(\Omega) \neq 0$. Third, we require a regularity condition on Ω. If ψ is a character in $r_A^M(\Omega)$, we require that ψ be regular with respect to W. Jantzen [7] used a technique of Tadić [14] involving Jacquet modules to prove the following theorem.

THEOREM 2.2. *Under the three conditions above, the followings are equivalent :*

1. π is irreducible

2. σ is irreducible for any i and $\sigma \in BZ_{N_i}(\pi)$, where the s_i's are reflections associated to simple roots and $N_i = \langle B, s_i \rangle$.

This provides a criterion to determine the reducibility points in regular cases. We now establish a framework of studying non-regular cases with Hecke algebra isomorphisms.

First, we establish an equivalence between the category of representations of a group and the category of representations of its Hecke algebra. We have a pair (J, ρ), where J is a compact open subgroup of G and ρ is an irreducible smooth representation of J. Let $\mathcal{H}(G, \rho) = \{f \in C_c^\infty(G) \mid f(j_1 g j_2) = \rho^{-1}(j_1) f(g) \rho^{-1}(j_2) \text{ for all } j_1, j_2 \in J, g \in G\}$. We denote by $\mathcal{R}_\rho(G)$ the full subcategory of $\mathcal{R}(G)$, the category of smooth representations of G, consisting of all $(\pi, V) \in \mathcal{R}(G)$ such that $V = \mathcal{H}(G)V^\rho$ where $\mathcal{H}(G) = \mathcal{H}(G, 1_I)$ with an Iwahori subgroup I of G and V^ρ is the ρ-isotypic subspace of V. We denote by $\mathcal{H}(G, \rho)$-Mod the category of representations of $\mathcal{H}(G,\rho)$ such that $V = \mathcal{H}(G)V^{\rho}$. In general,

$$\mathcal{R}_{\rho}(G) \to \mathcal{H}(G,\rho)\text{-}Mod \ : \ V \mapsto V^{\rho}$$

is not an equivalence of categories. However, if (J,ρ) is a type in the sense of Bushnell-Kutzko [4], $\mathcal{R}_{\rho}(G) \to \mathcal{H}(G,\rho)\text{-}Mod \ : \ V \mapsto V^{\rho}$ is an equivalence of categories.

We now discuss Roche's results [12]. Fix a character $\chi : A \cap K \to \mathbb{C}^{\times}$ where $\mathcal{O}$ is the ring of integers in F and $K = G(\mathcal{O})$. To the character χ, he associates an open compact subgroup J and a character $\rho : J \to \mathbb{C}^{\times}$ with $\rho|A \cap K = \chi$. The pair (J,ρ) is a type in the sense of Bushnell-Kutzko.

Roche constructs a split connected reductive group H and a finite abelian group C_{χ}, which acts on H, such that

$$\mathcal{H}(G,\rho) \cong \mathcal{H}(H, 1_I) \tilde{\otimes} \mathbb{C}[C_{\chi}].$$

He also constructs a disconnected group $\tilde{H}$ such that $\mathcal{H}(H,1_I) \tilde{\otimes} \mathbb{C}[C_{\chi}] \cong \mathcal{H}(\tilde{H}, 1_I)$. And we have support-preserving isomorphisms

$$\Psi_G : \mathcal{H}(G,\rho) \to \mathcal{H}(H,1_I) \tilde{\otimes} \mathbb{C}[C_{\chi}] = \mathcal{H}(\tilde{H}, 1_I)$$

$$\Psi_M : \mathcal{H}(M,\rho_M) \to \mathcal{H}(L,1_{I_L}) \tilde{\otimes} \mathbb{C}[D_{\chi}] = \mathcal{H}(\tilde{L}, 1_{I_L})$$

where M is a standard Levi of G.

THEOREM 2.3 (Jantzen and Kim [9]). *We may take* $\Psi_M = \Psi_G|_{\mathcal{H}(M,\rho_M)}$ *to get a support-preserving isomorphism*

$$\Psi_M : \mathcal{H}(M,\rho_M) \to \mathcal{H}(L,1_{I_L}) \tilde{\otimes} \mathbb{C}[D_{\chi}].$$

By combining Roche's results with Theorem 2.3, we obtain the following commuting diagram

$$\begin{array}{ccccccc}
\mathcal{R}_{\rho}(G) & \xrightarrow{\cong} & \mathcal{H}(G,\rho)\text{-}Mod & \xrightarrow{\Psi_G} & \mathcal{H}(H,1_I)\text{-}Mod & \xrightarrow{\cong} & \mathcal{R}_1(H) \\
\uparrow & & \uparrow & & \uparrow & & \uparrow \\
\mathcal{R}_{\rho}(H) & \xrightarrow{\cong} & \mathcal{H}(M,\rho)\text{-}Mod & \xrightarrow{\Psi_M} & \mathcal{H}(L,1_I)\text{-}Mod & \xrightarrow{\cong} & \mathcal{R}_1(L).
\end{array}$$

In this commuting diagram, $i_M^G(\Omega)$ corresponds to $i_L^H(\Omega_u)$ where Ω_u is an unramified character of L.

Roche [12] describes how to find a smaller group whose Hecke algebra is isomorphic to the Hecke algebra of G. Via Hecke algebra isomorphisms, the degenerate principal series of G are transferred to a representation of the smaller group. In section 5, the degenerate principal series of exceptional groups are transferred to the representations of classical groups. Using the known results for the classical groups, we can determine reducibility points of non-regular cases of exceptional groups.

3. Main results

Let F be a p-adic field of characteristic zero with $\mathrm{char}(k_F) \neq 2, 3$. Let G be a split group of type G_2 defined over F. We set $\Delta = \{\alpha_1, \alpha_2\}$ where α_1 is a short root and α_2 is a long root. Since G is simply connected, every element $h \in A$ can be written uniquely as

$$h(t_1, t_2) = \alpha_1^\vee(t_1)\alpha_2^\vee(t_2)$$

where $\alpha_i^\vee$ is the coroot associated with α_i. There are two maximal standard Levi subgroups in G. Set $M_1 = \langle B, s_2\rangle$ and $M_2 = \langle B, s_1\rangle$, where the s_i's are reflections associated to simple roots. Note that $M_1 = N_2$ and $M_2 = N_1$.

We use the following result for $G_0 = GL(2, F)$ in [15]: $i_{A_0}^{G_0}(\chi_0)$ is reducible if and if $\chi_0/\chi_0^{s_0}(\mathrm{diag}(a, b)) = \nu^{\pm 1}(a/b)$ where $\mathrm{diag}(a, b) \in A_0$ and s_0 is a simple reflection in G_0. To use this result for $GL(2, F)$, we need to describe $\mathrm{diag}(p, q)$ inside N_1 and N_2 which are isomorphic to $GL(2, F)$. Using the seven dimensional representation of the Lie algebra of type G_2 in [11], we have $\alpha_1^\vee(t) = \mathrm{diag}(t, 1/t, t^2, 1, 1/t^2, t, 1/t)$ and $\alpha_2^\vee(t) = \mathrm{diag}(1, t, 1/t, 1, t, 1/t, 1)$. From this seven dimensional representation, $\mathrm{diag}(p, q)$ in N_1 is expressed as $h(p, pq)$ and $\mathrm{diag}(p, q)$ in N_2 is expressed as $h(pq, p^2q)$.

For any $\phi \in \Phi$,

$$\phi^\vee(t) = \alpha_1^\vee(t^{\omega_1(\phi^\vee)})\alpha_2^\vee(t^{\omega_2(\phi^\vee)}) = h(t^{\omega_1(\phi^\vee)}, t^{\omega_2(\phi^\vee)}),$$

where ω_1, ω_2 are fundamental weights such that $\omega_i(\alpha_j^\vee) = \delta_{ij}$. In the following description, we denote the coroot of $p\alpha_1 + q\alpha_2$ by h_{pq} :

$$h_{10} = h(t, 1), h_{01} = h(1, t), h_{11} = h(t, t^3)$$

$$h_{21} = h(t^2, t^3), h_{31} = h(t, t), h_{32} = h(t, t^2).$$

Muić [10] studied the degenerate principal series of type G_2 using the Plancherel measure. We obtain the same result in the following sections using Jacquet module techniques and Hecke algebra isomorphisms.

THEOREM 3.1. *Let G be a split connected reductive group of type G_2 and Ω be a character of the Levi factor M of a maximal parabolic subgroup of G, which is defined by a character χ of $F^\times$ such that $\chi = \nu^s\chi_0$ where $\nu = |\cdot|$ and $s \in \mathbb{R}$ and χ_0 is a unitary character.*

(1) *M is the Levi subgroup corresponding to the long root and Ω is defined by $\Omega(h(t_1,t_2)) = \chi(t_1)$. Then, $i_M^G(\Omega)$ is reducible if and only if $s = \pm 5/2$, $\chi_0 = 1$ or $s = \pm 1/2$, $\chi_0^2 = 1$.*

(2) *M is the Levi subgroup corresponding to the short root and Ω is defined by $\Omega(h(t_1,t_2)) = \chi(t_2)$. Then, $i_M^G(\Omega)$ is reducible if and only if $s = \pm 3/2$, $\chi_0 = 1$ or $s = \pm 1/2$, $\chi_0^2 = 1$ or $s = \pm 1/2$, $\chi_0^3 = 1$.*

4. Regular cases

For each maximal standard Levi subgroup of G, we determine the regularity conditions on $r_A^M(\Omega)$ and the reducibility points of $BZ_{N_i}(\pi)$ (i=1,2). Then we use Theorem 2.2 to find the reducibility points of regular cases.

Case 1 : $M = M_1$

Let χ be a character of $F^\times$ such that $\Omega(h(t_1,t_2)) = \chi(t_1)$.

Lemma 4.1. *$r_A^M(\Omega)$ is nonregular precisely when $\chi = \nu^{\pm 3/2, \pm 1/2}$ or $\chi^2 = 1$.*

Proof. Set $\psi(h(t_1,t_2)) = |t_1|^{3/2}\chi(t_1) \otimes |t_2|^{-1}$. Then, $\psi \in r_A^M(\Omega)$. For $w = s_1$, $\psi^w(h(t_1,t_2)) = \psi(h(t_1^{-1}t_2, t_2))$ because $h_{10}^w(t) = h_{10}(t^{-1})$ and $h_{01}^w(t) = h_{31}(t)$. $\psi = \psi^w$ implies that $\chi = \nu^{-3/2}$. Similarly, we check the regularity of ψ and obtain the regularity condition on $r_A^M(\Omega)$. □

Lemma 4.2. *All the BZ composition factors of $r_{N_i}^G(\pi)$ $(i = 1,2)$ are irreducible except when $\chi = \nu^{\pm 5/2, \pm 3/2, \pm 1/2}$ or $\chi^2 = \nu^{\pm 1}$.*

Proof. For $N = N_1$, $W^{MN} = \{1,\ s_1s_2,\ s_1s_2s_1s_2\}$ and $\mathrm{diag}(p,q) = h(p,pq)$. For $w = 1$, $i_A^N \circ w \circ \psi$ is reducible if and only if $\psi/\psi^{s_0}(\mathrm{diag}(p,q)) = \psi/\psi^{s_0}(h(p,pq)) = \nu^{\pm 1}(p/q)$ (i.e., $\chi = \nu^{-1/2,-5/2}$). Similarly, for $w = s_1s_2$, $i_{N'}^N \circ w \circ \psi$ is reducible if and only if $\chi = \nu^{5/2,1/2}$; for $w = s_1s_2s_1s_2$, $i_{N'}^N \circ w \circ \psi$ is reducible if and only if $\chi^2 = \nu^{\pm 1}$.

For $N = N_2$, $W^{MN} = \{1,\ s_1,\ s_1s_2s_1,\ s_1s_2s_1s_2s_1\}$ and $\mathrm{diag}(p,q) = h(pq, p^2q)$. For $w = 1$ or $w = s_1s_2s_1s_2s_1$, $i_{N'}^N \circ w \circ \psi$ is irreducible because $N' = N$ and $M' = M$. For $w = s_1$, $i_{N'}^N \circ w \circ \psi$ is reducible if and only if $(\psi^w)/(\psi^w)^{s_0}(\mathrm{diag}(p,q)) = (\psi^w)/(\psi^w)^{s_0}(h(pq,p^2q)) = \nu^{\pm 1}(p/q)$ (i.e., $\chi = \nu^{1/2,-3/2}$). Similarly, for $w = s_1s_2s_1$, $i_{N'}^N \circ w \circ \psi$ is reducible if and only if $\chi = \nu^{3/2,-1/2}$. □

Corollary 4.3. *Under the regularity condition on $r_A^M(\Omega)$, $\pi = i_M^G(\Omega)$ is reducible if and only if $\chi = \nu^{\pm 5/2}$ or $\chi^2 = \nu^{\pm 1}$.*

Proof. It follows from two preceding lemmas and Theorem 2.2. □

Case 2 : $M = M_2$

Let χ be a character of $F^\times$ such that $\Omega(h(t_1,t_2)) = \chi(t_2)$.

LEMMA 4.4. *$r_A^M(\Omega)$ is nonregular precisely when $\chi = \nu^{\pm 1/2}$* or $\chi^2 = 1$, $\chi^3 = \nu^{\pm 1/2}$.

Proof. Set $\psi(h(t_1,t_2)) = |t_1|^{-1} \otimes |t_2|^{1/2}\chi(t_2)$. Then, $\psi \in r_A^M(\Omega)$. By checking the regularity of ψ, we obtain the regularity condition on $r_A^M(\Omega)$. □

LEMMA 4.5. *All the BZ composition factors of $r_{N_i}^G(\pi)$ (i=1,2) are irreducible except when $\chi = \nu^{\pm 3/2, \pm 1/2}$ or $\chi^2 = \nu^{\pm 1}$ or $\chi^3 = \nu^{\pm 3/2, \pm 1/2}$.*

Proof. For $N = N_1$, $W^{MN} = \{1, s_2s_1, s_2s_1s_2s_1\}$. For $w = 1$, $i_{N'}^N \circ w \circ \psi$ is reducible if and only if $\chi = \nu^{-1/2, 3/2}$. For $w = s_2s_1$, $i_{N'}^N \circ w \circ \psi$ is reducible if and only if $\chi = \nu^{3/2, -1/2}$. For $w = s_2s_1s_2s_1$, $i_{N'}^N \circ w \circ \psi$ is reducible if and only if $\chi^2 = \nu^{\pm 1}$.

For $N = N_2$, $W^{MN} = \{1, s_2, s_2s_1s_2, s_2s_1s_2s_1s_2\}$. For $w = 1$ or $w = s_2s_1s_2s_1s_2$, $i_{N'}^N \circ w \circ \psi$ is irreducible because $N' = N$ and $M' = M$. For $w = s_2$, $i_{N'}^N \circ w \circ \psi$ is reducible if and only if $\chi^3 = \nu^{1/2, -3/2}$. For $w = s_2s_1s_2$, $i_{N'}^N \circ w \circ \psi$ is reducible if and only if $\chi^3 = \nu^{3/2, -1/2}$. □

COROLLARY 4.6. *Under the regularity condition on $r_A^M(\Omega)$, $\pi = i_M^G(\Omega)$ is reducible if and only if $\chi = \nu^{\pm 3/2}$ or $\chi^2 = \nu^{\pm 1}$ or $\chi^3 = \nu^{\pm 3/2}$.*

Proof. It follows from two preceding lemmas and Theorem 2.2. □

5. Nonregular cases

Roche [12] constructs a split connected reductive group H and a finite abelian group C_χ, which acts on H, such that

$$\mathcal{H}(G,\rho) \cong \mathcal{H}(H, 1_I) \tilde{\otimes} \mathbb{C}[C_\chi].$$

Set $\Phi_\chi = \{\alpha \in \Phi \ : \ \chi \circ \alpha^\vee | \mathcal{O}^\times = 1\}$. Then the quadruple $\Psi_\chi = (X, \Phi_\chi, Y, \Phi_\chi^\vee)$ is a root datum. Hence there exists a connected reductive $\mathcal{O}$-group H with $\mathcal{O}$-split maximal torus A such that the root datum $\Psi(\mathsf{H}, \mathsf{A})$ equals Ψ_χ. We denote by H and A the corresponding groups of F-rational points.

Roche shows that if G has a connected center, then $C_\chi = \{id\}$. Since G has a trivial center, $C_\chi = \{id\}$ and $\mathcal{H}(G,\rho) \cong \mathcal{H}(H, 1_I)$.

Case 1 : $M = M_1$

For the non-regular and unramified case $\chi = \nu^{\pm 1/2}$, π is irreducible from Barbasch's computation [2]. Thus, we just consider the non-regular and ramified case $\chi^2 = 1$.

For $\chi^2 = 1$, $\Phi_\chi^+ = \{01, 21\}$. As shown in the following diagram, Φ_χ is of type $A_1 \times A_1$. From the root datum $\Psi_\chi = (X, \Phi_\chi, Y, \Phi_\chi^\vee)$, H is isomorphic to $SO(4)$. More precisely, let $\mathrm{diag}(a, b, 1/b, 1/a)$ be the torus and γ_1, γ_2 be two positive roots of $SO(4) = \{X \in SL(4) \mid {}^T X J X = J\}$ where $J = (a_{ij})$ is a 4 by 4 matrix with four nonzero entries $a_{14} = a_{23} = a_{32} = a_{41} = 1$. By sending (01) to γ_1 and (21) to γ_2, we identify $\mathrm{diag}(a, b, 1/b, 1/a)$ with $h(ab, a^2 b)$.

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Thus we have a Hecke algebra isomorphism $\phi : \mathcal{H}_1 = \mathcal{H}(G, \rho) \to \mathcal{H}_2 = \mathcal{H}(H, 1_I)$.

By the argument following Hecke algebra isomorphisms in section 2, $\pi = i_M^G(\Omega)$ is transferred to $i_L^H(\Omega_u)$ where L is the Levi factor of a parabolic subgroup of H generated by γ_1 and Ω_u is a character of L such that $\Omega_u(L) = \chi(\det(L))$. Jantzen [6] shows that $i_L^H(\Omega_u)$ is reducible if and only if $\chi^2 = \nu^{\pm 1}$. Hence $i_L^H(\Omega_u)$ is irreducible and so π is irreducible.

Case 2 : $M = M_2$

For the non-regular and unramified case $\chi = \nu^{\pm 1/2}$, π is irreducible from Barbasch's computation [2]. Thus, we just consider the non-regular and ramified cases $\chi^2 = 1, \chi^3 = \nu^{\pm 1/2}$.

Subcase 2-1 : $\chi^2 = 1$

We have $\Phi_\chi^- = \{10, 32\}$. Similarly as in Case 1, H is isomorphic to $SO(4)$. More precisely, let $\mathrm{diag}(a, b, 1/b, 1/a)$ be the torus and γ_1, γ_2 be two positive roots of $SO(4)$. By sending (10) to γ_1 and (32) to γ_2, we identify $\mathrm{diag}(a, b, 1/b, 1/a)$ with $h(a, ab)$.

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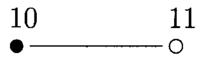
Also, $\pi = i_M^G(\Omega)$ is transferred to $i_L^H(\Omega_u)$ where L is the Levi subgroup of H generated by γ_1 and Ω_u is a character of L such that $\Omega_u(L) = \chi(\det(L))$. As in Case 1, $i_L^H(\Omega_u)$ is irreducible and so π is irreducible.

Subcase 2-2 : $\chi^3 = \nu^{\pm 1/2}$

We have $\Phi_\chi^+ = \{10, 11, 21\}$. As shown in the following diagram, Φ_χ is of type A_2. From the root datum $\Psi_\chi = (X, \Phi_\chi, Y, \Phi_\chi^\vee)$, H is isomorphic to $SL(3)/\mathcal{Z}(SL(3))$, where $\mathcal{Z}(SL(3))$ is the center of $SL(3)$. Let γ_1, γ_2

be two positive roots of $SL(3)$. We may send (10) to γ_1 and (11) to γ_2. Let L be the Levi subgroup of H generated by (10) and Ω_u be the character of L such that $\Omega_u = \Omega|_L$.



LEMMA 5.1. *Let N be a subgroup of the center of H contained in the Iwahori subgroup I of H and set $H_0 = H/N$. Then $\mathcal{H}(H, 1_I)$ is isomorphic to $\mathcal{H}(H_0, 1_{I_0})$.*

Proof. Note that $\mathcal{H}(H, 1_I) = \{f \in C_c^\infty(H) \mid f(i_1 h i_2) = f(h)$ for all $i_1, i_2 \in I, h \in H\}$. Since the group N is in the Iwahori subgroup I, we can establish an isomorphism between $\mathcal{H}(H, 1_I)$ and $\mathcal{H}(H_0, 1_{I_0})$ via the quotient map $q : H \to H_0$ defined by $q(a) = aN$. □

By Lemma 5.1, we may assume H is isomorphic to $SL(3)$. Then Ω_u is a character of L such that $\Omega_u(h_{10}) = 1$ and $\Omega_u(h_{11}) = \nu^{\pm 1/2}$. In Tadić's paper [13], it is shown that $i_L^H(\Omega_u)$ is irreducible. Hence, π is irreducible.

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