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An Improved Method for Evaluating Network-Reliability with Variable Link-Capacities[1)]

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Abstract

This paper presents a method for reliability evaluation of a telecommunication network with variable link-capacities when the simple paths of the network are known. The LP-EM, suggested by Lee and Park (2001), identifies a composite path as a subnetwork and adds only a minimal set of links at each step which gives maximal increase on the maximum capacity flow of the subnetwork. Thereby the LP-EM reduces the possible occurrence of redundant composite paths significantly over other existing methods. Based on the LP-EM, our method further reduces the possible redundancy by identifying such simple paths that could give no increase of maximum capacity flow on the current subnetwork and excluding those simple paths from consideration in the process of constructing composite paths.

Keywords : Simple path; saturated unilateral link; flowless link.

1. Introduction

To develop an efficient method of reliability evaluation for a stochastic network whose links are subject to failure has attracted a great deal of attention in the literature. A network is modeled as a graph $G(V,E)$ where V is a set of nodes and E is a set of links (edges). Each link of the network may have different capacity and the network is required to transmit a specified amount of flow from the source node to the terminal node. Thus successful operation of the network is not necessarily characterized by connectedness only, but by the maximum capacity flow(MFC) that can be transmitted through the network. The network reliability is measured as the probability of transmitting the required amount of flow successfully from the source node to the terminal node.

For the brevity of presentation, we use the following acronyms and notations.

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sp	simple path(s)
cp	composite path(s)
mscp	minimal success *cp*
FSP	set of available failure *sp*
LIST	set of success *cp* found in the process
LP-EM	evaluation method in Lee and Park (2001)
MCF	maximum capacity flow
$W(C)$	MCF of the subnetwork induced by *cp* C
W_{min}	required amount of capacity flow for the network

Recently, a number of algorithms have been proposed to evaluate the capacity related reliability for a network with variable link capacities under the assumption that all *sp* of the network are known. A *sp* is a minimal set of links connecting the source node and the terminal node. In the evaluation process, *cp* are generated in succession as unions of *sp* and then the MCF of corresponding subnetwork is computed for each *cp* generated. Aggarwal et al. (1982) generate *cp* by combining the rows of a path matrix each row of which is a *sp* of the network, and Misra and Prasad (1982) use a list of failure *sp* to generate higher order *cp*. Aggarwal (1988) suggests a method for evaluating the expected MCF of the network and Varshney et al. (1994) suggest a method, based on Aggarwal (1988), to allow the links to have multiple states of capacities with corresponding probabilities. However, these methods have some drawbacks in MCF computation of a *cp*, or do not generate enough *cp* to give correct results. Schanzer (1995) presents a counter example to show that the methods of Aggarwal (1988) and Varshney et al. (1994) fail in some cases. Rai and Soh (1991) also discuss the drawbacks of the preceding results, and propose a method based on the *cp* enumeration technique using the idea of clique and key_cut to reduce the number of redundant *cp* generated in the process. However, the MCF computation needs extra efforts of converting the given simple paths into minimal cuts, and the redundancy is still quite large and can occur repeatedly each time the higher order *cp* are generated. To complement and correct the drawbacks of the preceding methods, Lee and Park (2001) propose a method, the LP-EM, based on the concepts of eligibility and additivity and significantly reduce both the possible occurrence of redundancy and the number of *cp* considered for MCF computation, but do not provide the procedure of MCF determination explicitly. Recently, Lee et al. (2004) present an efficient method for determining the MCF of a given network using signed *sp* where a sign is affixed to each link for the direction of possible flow moving along it.

In this paper, we propose a method, based on the LP-EM, which further reduces the possible redundancy in the process by utilizing the concepts of unilateral links and flowless links. Section 2 describes the basic procedures of the LP-EM and MCF computation. Section 3 provides the detailed descriptions of the methodology and our algorithm. Some numerical examples are also presented to illustrate the use and efficiency of the method in section 4.

2. Basic Procedures

Assumptions

1. The nodes are perfect and each has no capacity limit.
2. The links are independent and either function or fail with known probabilities.
3. All the links are undirected and each link flow is bounded by the link capacity.
4. No flow can be transmitted through a failed link.
5. The *sp* of the network, considering connectivity only, is known.

In the process of reliability evaluation for a stochastic network whose links are subject to failure, *cp* are repeatedly generated to reflect various states of the network and the MCF of each *cp* is to be computed upon generation.

2.1 The LP-EM

Although some of the existing methods lead to correct results, the redundancy is still quite large and may occur repeatedly each time the higher order *cp* are generated. To reduce effectively the possible occurrence of redundant *cp* in the process, the LP-EM introduce the properties: additivity and eligibility. A *sp* P is said to have additivity on C if there is no *sp* P' such that $(P' - C)$ is a proper subset of $(P - C)$. A *sp* P having the additivity assures itself to add only a minimal set of new links to C when C and P combine to generate a new *cp*. For given FSP, a *sp* P is said to have eligibility on C in FSP if the subnetwork induced by $C \cup P$ contains no *sp* other than those in FSP. The eligibility property prevents current *cp* from combining a *sp* which has been checked previously in the process, and is utilized to effectively remove the redundancy which may occur in generating higher order *cp*. Then, a *sp* P is called an *additive sp* if P has the eligibility on C (in FSP) and the additivity on C. In the LP-EM, the basic idea is to add, each time, a choice of a minimal set of links, P, which gives maximal increase on MCF of the subnetwork induced by a failure *cp* C. Such a choice is taken only from additive *sp* among all *sp* in FSP. With P chosen, if $W(C \cup P) \geq$

W_{min}, then record $C \cup P$ as a success *cp* in LIST, otherwise repeat the process with the failure *cp* $C \cup P$. The set of *mscp* is obtained from LIST by removing the possible redundancy.

2.2 MCF computation

Nomenclature

- *Signed sp.* Regarding a *sp* as an edge sequence from the source node to the terminal node, a sign is affixed to each link i in *sp*, i^+ or i^- say. Each *sp* can be converted to its corresponding signed *sp*.
- *Unilateral link.* A link is said to be unilateral, if the signs of the link are the same in all signed *sp* containing it. A unilateral link i is *saturated*, if $f_i = c_i$ where $f_i(c_i)$ is the link flow(capacity) of i.

Lee et al. (2004) present an efficient method for determining the MCF of a given network using signed *sp*. Given a *cp* C, the procedure *CAPACITY*(C) computes $W(C)$, and produces the corresponding flow vector $\boldsymbol{f}$. In the procedure, v is the value of current net flow and, for a signed *sp* Q, δ_Q is the maximum amount of additional flow that can be sent by Q. The procedure is re-stated below:

Procedure: *CAPACITY*(C)

1. Set $v = 0$, $\boldsymbol{f} = (0,0,\cdots,0)$, and S = {all signed *sp* of C}.
   Arrange signed *sp* in S in ascending order of its number of links, and try one by one from the shortest.
2. Given a signed *sp* Q, compute δ_Q, and set $v = v + \delta_Q$.
   For each link i in Q, [adjust f_i. If i is a saturated unilateral link, then delete all signed *sp* containing i from S.]
   If there is no more signed *sp* to try, then *STOP*; else go to 2.

3. Algorithm

Based on the LP-EM, our method further reduces the possible redundancy which may occur during the process by utilizing the concepts of unilateral links and flowless links. At each iteration, we have current *cp* C and an additive *sp* P which gives maximal increase on $W(C)$. The procedure *CAPACITY* computes $W(C \cup P)$, and returns the corresponding flow vector $\boldsymbol{f}$. If a unilateral link i is saturated once, then i remains saturated thereafter and hence, any *sp* containing i could never augment the net flow. A *sul* means a saturated unilateral link throughout this paper. If there exist *sul* in f, then we delete those *sp* containing a *sul* from FSP. The number of iterations in the LP-EM mainly depends on the number of *sp* in FSP and hence, our method reduces the number of iterations necessary for the completion of the process. Note that this results in reducing the number of *cp* for MCF computation and the number of success *cp* generated in the process as well. Further, if a link i is flowless as $f_i = 0$ in f, then the same amount of MCF can be achieved without link i. Now, if $C \cup P$ is a success *cp*, i.e., $W(C \cup P) \geqq W_{\min}$, then we check flowless links in f and record $(C \cup P) - \{$all flowless links$\}$ in LIST, rather than recording $(C \cup P)$ itself as in the LP-EM. Each success *cp* in LIST, hence, contains fewer links than the corresponding one in the LP-EM. The process of checking redundancy in LIST is essentially the same for our method and the LP-EM, but our method deals with a set of fewer *cp*, each of which contains fewer links.

Notation for Algorithm:

k	number of composition needed to generate the current *cp*, $k = 0, 1, \cdots$
$C(k)$	current *cp* at level k
$f(k)$	flow vector at level k
FSP(k)	FSP at level k
N_k	number of additive *sp* on $C(k)$
$P_{(k:n_k)}$	current additive *sp* on $C(k)$, $n_k = 1, \cdots, N_k$
$W_{(k:n_k)}$	$= W(C(k) \cup P_{(k:n_k)})$

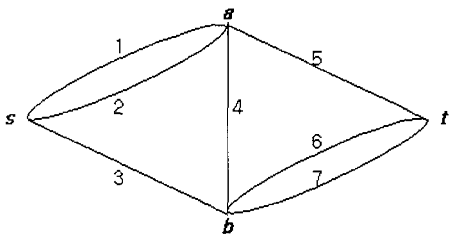
Algorithm:

1. Set $k = 0$, $C(0) = \varnothing$, FSP(0)={set of all *sp*}. Go to 2.
2. Find all additive *sp* in FSP(k).
   Call *CAPACITY*$(C(k) \cup P)$ for each additive *sp* P.
   Arrange all additive *sp* in descending order of $W(C(k) \cup P)$, and record them as $P_{(k:n_k)}$'s with its corresponding $W_{(k:n_k)}$ where $n_k = 1, \cdots, N_k$. Set $n_k = 1$.
3. Set FSP(k) = FSP$(k) - \{P_{(k:n_k)}\}$.
   Utilizing flowless links: If $W_{(k:n_k)} \geq W_{\min}$, [remove flowless links from $C(k) \cup P_{(k:n_k)}$, and record the resulting *cp* in LIST. Go to 5.]
   If FSP$(k) = \varnothing$, then go to 4.
   Set $C(k+1) = C(k) \cup P_{(k:n_k)}$.
   Utilizing sul: Set FSP$(k+1)$ = FSP$(k) - \{P \mid P$ contains a *sul*$\}$.

Set $k=k+1$. Go to 2.
4. Set $k=k-1$. If $k<0$, then *STOP*.
5. If $n_k=N_k$, then go to 4. Set $n_k=n_k+1$. Go to 3.

4. Numerical Examples

Example 1. Consider the network shown in <Figure 1>, and suppose that the capacity vector is given as $c=(1,1,2,2,2,1,1)$ and $W_{min}=4$. We have 9 *sp*: (1,5), (1,4,6), (1,4,7), (2,5), (2,4,6), (2,4,7), (3,4,5), (3,6), (3,7). In the following tables, a *sul* is marked by 's' in flow vectors. The column 'success' is marked with 'Y' when a success *cp* is obtained, and with 'N' otherwise.



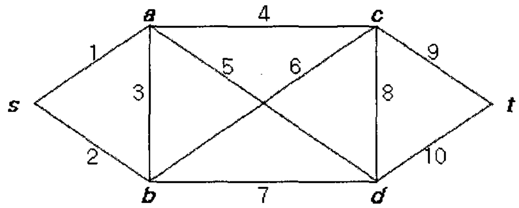
<Figure 1> A 7-Branch Network

At $k=0$, all *sp* are additive and hence, $N_0=9$. The MCF for each *sp* is computed and, since (3,4,5) the largest MCF, we have $P_{(0:1)}$=(3,4,5). Note that link 3 and 5 are marked by 's' as *sul* in $f(0)$. Since (3,4,5) is not a success *cp*, we set $k=1$, $C(1)$=(3,4,5) and FSP(1)={(1,4,6), (1,4,7), (2,4,6), (2,4,7)}. Note that the *sp* (1,5), (2,5), (3,6) and (3,7) are deleted from FSP, because each of them contains a *sul*. All of 4 *sp* in FSP(1) are found additive, and hence we set $N_1=4$. Choose $P_{(1:1)}$=(1,4,6) and delete it from FSP to prevent from choosing the same *sp* when we re-visit level $k=1$ later in the process. Now, since $C(1)\cup P_{(1:1)}$ is not a successive *cp* yet, we advance to level $k=2$ with $C(2)=C(1)\cup P_{(1:1)}$ and FSP(2)={(2,4,7)}. Again, there exist *sul*, link 1 and 6, in $f(1)$, and the *sp* (1,4,7) and (2,4,6) are deleted from FSP. At $k=2$, $C(2)\cup$(2,4,7)=(1,2,3,4,5,6,7) is found to be a success *cp*. Note that link 4 is flowless in $f(2)$ and hence, we record (1,2,3,5,6,7) in LIST by deleting link 4. At this stage, since $n_2=N_2(=1)$, we go back to level $k=1$, and restart the process with $C(1)$=(3,4,5) and $P_{(1:2)}$=(1,4,7). Since $C(1)\cup P_{(1:2)}$ is not a success *cp*, we advance to the next level with $C(2)=C(1)\cup P_{(1:2)}$ and FSP(2)={(2,4,6)}. Note that (2,4,7) is deleted on preparing FSP, since it contains link 7 which is a *sul* in $f(1)$ at the re-visited level $k=1$. In the subsequent process, (1,2,3,5,6,7) is generated as unions of (1,5)∪(2,5)∪(3,6)∪(3,7), which is already in LIST. The LP-EM would record both (1,2,3,4,5,6,7) and (1,2,3,5,6,7) as success *cp*. Note that the number of *cp* in FSP is further reduced in our method at each level. Moreover, the number of *cp* for MCF computation is 18 for our method, while it is 24 for the LP-EM. The network reliability R is computed as $p_1p_2p_3p_5p_6p_7$ and, when p_i = 0.9 for all i, R = 0.53144.

<Table 1> Process for Figure 1

no.	k	$C(k)$	n_k	N_k	$P_{(k:n_k)}^{W_{(k:n_k)}}$	FSP(k)	$f(k)$	success
1	0	∅	-	-	-	∅		
2				9		$(3,4,5)^2,(1,5)^1,(2,5)^1$ $(3,6)^1,(3,7)^1,(1,4,6)^1$ $(1,4,7)^1,(2,4,6)^1,(2,4,7)^1$		
3			1		$(3,4,5)^2$	$(1,5)^1,(2,5)^1$ $(3,6)^1,(3,7)^1,(1,4,6)^1$ $(1,4,7)^1,(2,4,6)^1,(2,4,7)^1$	$(0,0,s,-2,s,0,0)$	N
	1	(3,4,5)	-	-		(1,4,6),(1,4,7) (2,4,6),(2,4,7)		
2				4		$(1,4,6)^3,(1,4,7)^3$ $(2,4,6)^3,(2,4,7)^3$		
3			1		$(1,4,6)^3$	$(1,4,7)^3,(2,4,6)^3,(2,4,7)^3$	$(s,0,s,-1,s,s,0)$	N
	2	(1,3,4,5,6)	-	-	-	(2,4,7)		
2				1		$(2,4,7)^4$		
3			1		$(2,4,7)^4$	∅	$(s,s,s,0,s,s,s)$	Y
4,5	1		1	4				
3		(3,4,5)	2		$(1,4,7)^3$	$(2,4,6)^3,(2,4,7)^3$	$(s,0,s,-1,s,0,s)$	N

Example 2. Consider the network given in <Figure 2> and let the capacity vector $c=(8,8,2,6,2,2,5,2,10,5)$ be given. There are 18 *sp* in the network. Given $W_{min}=15$, the LP-EM produces (1,2,3,4,5,6,7,8,9,10) and (1,2,4,5,6,7,8,9,10) whereas our method records only one success *cp* (1,2,4,5,6,7,8,9,10) and hence, needs not to check LIST for possible redundancy in this case. Further, the number of *cp* for MCF computation is reduced to 91 in our method, whereas it is 150 for the LP-EM.



<Figure 2> A 10-Branch Network

5. Conclusion

Based on the LP-EM, our method further reduces the possible redundancy by utilizing *sul* and flowless links, which can be identified while computing MCF for subnetworks. Our method reduces the number of iterations necessary for the completion of the process and the number of subnetworks for MCF computation. Further, it reduces the number of success *cp* generated and each success *cp* contains fewer links. Future research interests include network models in which links are allowed to have multiple states of capacities.

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