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# Fractional Integration in the Context of Periodicity: A Monte Carlo Experiment and an Empirical Study[1)]

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Abstract

Recent results in applied statistics have shown that the presence of periodicities in time series may influence the estimation and testing of the fractional differencing parameter. In this article, we provide further evidence on the issue by using several procedures of fractional integration. The results show that in the presence of periodicities, the order of integration can be erroneously detected. An empirical application in the context of seasonal data is also carried out at the end of the article.

*Keywords* : Fractional integration; seasonality; long memory; JEL classification; C15.

## 1. Introduction

Time series data occur commonly in the natural and engineering sciences, economics and many other fields of enquiry. A typical feature of such data is their apparent dependence across time, for example, sometimes records close together in time are strongly correlated. This paper focuses on which is usually called 'long range dependence', so-named because of the strong association between observations widely separated in time. Then, the autocorrelations decay very slowly at a hyperbolic rate. Recent theoretical results in probability and statistics have directed applied scientists to develop new methods for detecting the presence of long range dependence in time series. This characteristic has been found to be present by many authors in hydrology (e.g., Hurst, 1951; Montanari et al, 1996); economics (Diebold and Rudebusch, 1989; Baillie, 1996); high speed networks (Beran et al., 1995; Willinger et al., 1995) and other areas. A useful model to describe this type of behaviour is the fractionally integrated model. We say that a time series $\{x_t , t = 1, 2, \cdots\}$ is integrated of order $d$, (and denoted by

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$x_t \sim I(d)$) if

$$(1-L)^d x_t = u_t, \; t = 1, 2, \cdots, \quad (1)$$
$$x_t = 0, \; t \leq 0$$

where $u_t$ is an $I(0)$ process, defined for the purpose of the present paper as a covariance stationary process with spectral density function that is positive and finite at the zero frequency. Clearly, if $d=0$ in (1), $x_t = u_t$ and a 'weakly autocorrelated' (e.g., ARMA) $x_t$ is allowed for. However, if $d > 0$, $x_t$ is said to be a long memory process, also called 'strongly autocorrelated', and, as d increases beyond 0.5 and through 1, $x_t$ can be viewed as becoming "more nonstationary", in the sense, for example, that the variance of partial sums increases in magnitude.-Models with d ranging between -0.5 and 0 are short memory and have been addressed as anti-persistent by Mandelbrot (1977) because the spectral density function is dominated by high frequency components-. These processes were initially introduced by Granger (1980, 1981), Granger and Joyeux (1980) and Hosking (1981), and were theoretically justified in terms of aggregation of ARMA processes with randomly varying coefficients by Robinson (1978) and Granger (1980).

The fractional differencing parameter $d$ plays a crucial role in describing the intensity of the association between the observations. Thus, if $d \in (0, 0.5)$, $x_t$ is covariance stationary and mean-reverting; if $d \in [0.5, 1)$, $x_t$ is no longer covariance stationary but it is still mean-reverting, with the effect of the shocks dying away in the long run; finally, if $d \geq 1$, $x_t$ is nonstationary and non-mean-reverting.

There exist many ways of estimating and testing the fractional differencing parameter. Many of the estimators are graphical in nature (heuristic estimators) while some involve numerical minimization of a likelihood-type function (e.g., Fox and Taqqu, 1986; Dahlhaus, 1989; Sowell, 1992; Smith et al., 1997; Hauser, 1999; etc.). However, several papers conducted by Montanari, Rosso and Taqqu (1995, 1996, 1997) in a hydrological context, showed that the presence of periodicities might influence the reliability of the estimators. Analyzing the series of the monthly flows of the Nile River at Aswan, these authors found that many heuristic estimators gave a positive value for $d$, indicating long memory where none was present. In another recent paper, Montanari, Taqqu and Teverowsky (1999) performed an extensive Monte Carlo investigation in order to find out how reliable the estimators of long memory are in the presence of periodicities, and they concluded that the best results were those obtained using likelihood-type methods.

In this paper, we further examine the above-mentioned issue by means of procedures for estimating and testing the fractional differencing parameter. We tried first with various classical methods (Lo's, 1991, modified R/S statistic; Sowell's, 1992, maximum likelihood, and Geweke and Porter-Hudak, GPH, 1983) and the results were very similar to those reported across the paper. We concentrate on two methods proposed by P.M. Robinson. The first is a parametric testing procedure (Robinson, 1994a), which is supposed to be the most efficient one when directed against the appropriate (fractional) alternatives. The second is semiparametric and is a Whittle estimator in the frequency domain (Robinson, 1995a).

The outline of the paper is as follows. In section 2 we briefly describe the procedures for estimation and testing of long memory processes. Section 3 contains the Monte Carlo experiments. An empirical application based on seasonal monthly data is carried out in section 4, while section 5 contains some concluding comments.

## 2. Procedures for long memory

There exist many approaches for estimating and testing the fractional differencing parameter $d$. Earlier studies tested the long memory hypothesis using the rescaled-range (R/S) method, suggested by Hurst (1951), and defined as

$$R\diagdown S = \frac{\max_{1 \le j \le T} \sum_{t=1}^{j} (x_t - \bar{x}) - \min_{1 \le j \le T} \sum_{t=1}^{j} (x_t - \bar{x})}{\left( \frac{1}{T} \sum_{t=1}^{T} (x_t - \bar{x})^2 \right)^{\frac{1}{2}}}$$

where $\bar{x}$ is the sample mean of the process $x_t$. The specific estimate of $d$ (Mandelbrot and Wallis (1968)) is given by:

$$\hat{d} = \frac{\log(R\diagdown S)}{\log T} - \frac{1}{2}.$$

Its properties were analyzed in Mandelbrot and Wallis (1969), Mandelbrot (1972, 1975) and Mandelbrot and Taqqu (1979). Beran (1994) provides a neat explanation of how to implement the R/S procedure. A problem with this statistic is that the distribution of its test statistic is not well defined and is sensitive to short-term dependence and heterogeneities of the underlying data generating process. Lo (1991) developed a modified R/S method which addresses these drawbacks of the classical R/S method.

Another method, widely used in the empirical work is the one proposed by Geweke and Porter-Hudak (GPH, 1983), which is a semiparametric procedure to obtain an estimate of the fractional differencing parameter $d$ based on the slope of the spectrum around the zero frequency. This method, however, has some potential problems. First, it is too sensitive to the choice of the bandwidth parameter numbers, and, in the presence of short range dependence, such as autoregressive or moving average terms, the GPH estimator is known to be biased in small samples (see, e.g. Agiakloglou et al., 1992).

In the context of parametric approaches, Sowell (1992) analyzed the exact maximum likelihood estimates of the parameters of the fractionally ARIMA $(p, d, q)$ model

$$\phi(L)(1-L)^d x_t = \theta(L)\epsilon_t, \qquad t = 1, 2, \cdots, \tag{2}$$

where $\phi(L)$ and $\theta(L)$ are polynomials of orders $p$ and $q$ respectively, with all zeroes of $\phi(L)$ and $\theta(L)$ outside the unit circle, and $\epsilon_t$ is white noise. He uses a recursive procedure that allows quick evaluation of the likelihood function in the time domain, which is given by

$$(2\pi)^{-T/2}|\Sigma|^{-1/2}\exp\left(-\frac{1}{2}X_T'\Sigma^{-1}X_T\right)$$

where $X_T = (x_1, x_2, \cdots, x_T)'$ and $X_T \sim N(0, \Sigma)$.

Other parametric methods of estimating d based on the frequency domain were proposed among others by Fox and Taqqu (1986) and Dahlhaus (1989). Small sample properties of these and other estimates were examined in Smith et al. (1997) and Hauser (1999). In the first of these articles, they compare several semi-parametric procedures with the maximum likelihood estimation method of Sowell (1992), finding that Sowell's (1992) procedure outperforms the semipara-metric ones in terms of the bias and the mean square errors. Hauser (1999) also compares Sowell's (1992) procedure with others based on the exact and the Whittle likelihood function in the time and in the frequency domain and shows that Sowell's (1992) dominates the others in case of fractionally integrated models.

In this article we use both parametric and semiparametric methods. First, we present a parametric testing procedure due to Robinson (1994a) that permits us to test $I(d)$ statistical models in raw time series.

### 2.1 A parametric testing procedure

Robinson (1994a) proposed a Lagrange Multiplier (LM) test of the null hypothesis:

$$H_0: d = d_0 \tag{3}$$

in a model given by (1) and (2) for any real value $d_0$. Specifically, the test statistic is given by:

$$\hat{R} = \frac{T}{\hat{\sigma}^4} \hat{a}' \hat{A}^{-1} \hat{a} \tag{4}$$

where $T$ is the sample size and

$$\hat{a} = \frac{-2\pi}{T} \sum_{j=1}^{T-1} \psi(\lambda_j) g(\lambda_j; \hat{\tau})^{-1} I(\lambda_j); \quad \hat{\sigma}^2 = \frac{2\pi}{T} \sum_{j=1}^{T-1} g(\lambda_j; \hat{\tau})^{-1} I(\lambda_j);$$

$$\hat{A} = \frac{2}{T} \left( \sum_{j=1}^{T-1} \psi(\lambda_j)^2 - \sum_{j=1}^{T-1} \psi(\lambda_j) \hat{\epsilon}(\lambda_j)' \times \left( \sum_{j=1}^{T-1} \hat{\epsilon}(\lambda_j) \hat{\epsilon}(\lambda_j)' \right)^{-1} \times \sum_{j=1}^{T-1} \hat{\epsilon}(\lambda_j) \psi(\lambda_j) \right)$$

$$\psi(\lambda_j) = \log \left| 2 \sin \frac{\lambda_j}{2} \right|; \ \hat{\epsilon}(\lambda_j) = \frac{\partial}{\partial \tau} \log g(\lambda; \hat{\tau}); \ \lambda_j = \frac{2\pi j}{T}; \ \hat{\tau} = \operatorname{argmin} \sigma^2(\tau)$$

$I(j)$ is the periodogram of $u_t$ evaluated under the null, i.e., $\hat{u}_t = (1-L)^{d_0} x_t$ and $g$ above is a known function of the spectral density of $u_t$,

$$f(\lambda; \sigma^2; \tau) = \frac{\sigma^2}{2\pi} g(\lambda; \tau), \ -\pi < \lambda < \pi.$$

These tests are purely parametric and therefore, they require specific modeling assumptions regarding the short memory specification of $u_t$. Thus, if $u_t$ is white noise, $g \equiv 1$, and if $u_t$ is an AR process of form $\phi(L) u_t = \epsilon_t$, then, $g = |\phi(e^{i\lambda})|^{-2}$, with $\sigma^2 = V(\epsilon_t)$, so that the AR coefficients are a function of $\tau$.

Based on $H_0$ (3), Robinson (1994a) established that under certain regularity conditions:

$$\hat{R} \rightarrow_d \chi_1^2 \quad as \ T \rightarrow \infty. \tag{5}$$

Thus, unlike other procedures, we are in a classical large-sample testing situation by reasons described in Robinson (1994a), who also showed that the tests are efficient in the Pitman sense against local departures from the null. A test of (3) will reject $H_0$ against the alternative $H_a$: $d \neq d_0$ if $\hat{R} > \chi_{1,\alpha}^2$

where $P(\chi_{1,\alpha}^2 > \chi_1^2) = \alpha$.

## 2.2 A semiparametric estimation procedure

There exist several methods for estimating the fractional differencing parameter in a semiparametric way. Examples are the log-periodogram regression estimate (LPE), initially proposed by Geweke and Porter-Hudak (1983) and modified later by Künsch (1986) and Robinson (1995b), the average periodogram estimate of Robinson (APE, 1994b) and a Whittle estimator (Robinson, 1995a) which we are now to describe.

The semiparametric method of Robinson (1995a) is basically a 'Whittle estimate' in the frequency domain, considering a band of frequencies that degenerates to zero. The estimate is implicitly defined by:

$$\hat{d} = \arg\min_d \left( \log \overline{C(d)} - 2d \frac{1}{m} \sum_{j=1}^{m} \log \lambda_j \right) \tag{6}$$

$$\overline{C(d)} = \frac{1}{m} \sum_{j=1}^{m} I(\lambda_j) \lambda_j^{2d}, \quad \lambda_j = \frac{2\pi j}{T}, \quad \frac{m}{T} \to 0$$

where $I(\lambda_j)$ is the periodogram of the raw time series, $x_t$, given by:

$$I(\lambda_j) = \frac{1}{2\pi T} \left| \sum_{t=1}^{T} x_t e^{t} \right|^2$$

and $d \in (-0.5, 0.5)^2$.. Under finiteness of the fourth moment and other mild conditions, Robinson (1995a) proved that:

$$\sqrt{m}\,(\hat{d} - d_0) \to_d N(0, 1/4) \quad as \quad T \to \infty .$$

where $d_0$ is the true value of $d$ and the additional requirement that $m \to \infty$ slower than $T$.

## 3. A Monte Carlo experiment

In this section we want to examine if the estimation of the fractional differencing parameter $d$ is affected by the presence of periodicities in the data. For this purpose, we simulate a seasonal process defined as the solution of:

$$(1 - \phi L^s) x_t = \epsilon_t \tag{7}$$

where $L^s$ is the seasonal lag-operator $(L^s x_t = x_{t-s})$; $\phi$ is the seasonal autoregressive coefficient that takes values 0.5, 0.7 and 0.9; $s$ is the period of the seasonality ($s$ = 4, 12, 16 and 24) and $\epsilon_t$ is white noise (i.e., uncorrelated and zero-mean noise). We generate Gaussian series using the routines GASDEV and RAN3 of Press et al. (1986), with 5,000 replications of each case. Clearly, for this type of series, $d$ should be equal to 0 since long-range effects are not present.

First, we examine the performance of the parametric procedure of Robinson (1994a). Thus, the alternatives are of form as in model (1) with $d$ = 0.1, (0.1), 1 and the disturbances are initially white noise. In this case, the functional form of the test statistic greatly simplifies, adopting the form:

$$\tilde{R} = \frac{T}{\tilde{\sigma}^4} \tilde{a}' \tilde{A}^{-1} \tilde{a} \tag{8}$$

$$\tilde{a} = \frac{-2\pi}{T} \sum_{j=1}^{T-1} \psi(\lambda_j) I(\lambda_j); \; \tilde{\sigma}^2 = \frac{2\pi}{T} \sum_{j=1}^{T-1} I(\lambda_j); \; \tilde{A} = \frac{2}{T} \left( \sum_{j=1}^{T-1} \psi(\lambda_j)^2 \right)$$

The results based on $\tilde{R}$ in (8) are given in <Table 1>. We observe that if $T$ is small, (e.g., $T$ = 120), the rejection frequencies are very small with $d$ = 0.1, ranging between 0.292 (with $\phi = 0.5$ and $s = 4$) and 0.516 (with $\phi = 0.9$ and $s = 4$). If $d = 0.2$, the values never exceed 0.800. However, if $d \geq 0.5$, they are

<Table 1> Rejection frequencies of Robinson's (1994a) tests with white noise disturbances. (5,000 replications were used in each case)

| $T$ = 120 | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\phi$ | $S$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
| 0.5 | 4 | 0.292 | 0.683 | 0.961 | 0.999 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 12 | 0.388 | 0.724 | 0.935 | 0.987 | 0.999 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 16 | 0.388 | 0.721 | 0.937 | 0.990 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 24 | 0.367 | 0.726 | 0.931 | 0.989 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.7 | 4 | 0.399 | 0.714 | 0.961 | 0.999 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 12 | 0.470 | 0.734 | 0.919 | 0.970 | 0.989 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 16 | 0.483 | 0.739 | 0.916 | 0.974 | 0.996 | 0.999 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 24 | 0.427 | 0.722 | 0.908 | 0.975 | 0.997 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.9 | 4 | 0.516 | 0.729 | 0.971 | 0.997 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 12 | 0.511 | 0.737 | 0.870 | 0.929 | 0.969 | 0.981 | 0.990 | 0.997 | 0.999 | 1.000 |
| | 16 | 0.510 | 0.738 | 0.865 | 0.947 | 0.981 | 0.993 | 0.999 | 1.000 | 1.000 | 1.000 |
| | 24 | 0.472 | 0.699 | 0.870 | 0.951 | 0.986 | 0.998 | 0.999 | 1.000 | 1.000 | 1.000 |
| $T$ = 240 | | | | | | | | | | | |
| $\phi$ | $S$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
| 0.5 | 4 | 0.319 | 0.881 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 12 | 0.502 | 0.945 | 0.999 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 16 | 0.517 | 0.938 | 0.999 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 24 | 0.546 | 0.939 | 0.998 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.7 | 4 | 0.385 | 0.840 | 0.997 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 12 | 0.572 | 0.911 | 0.995 | 0.999 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 16 | 0.599 | 0.913 | 0.993 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 24 | 0.597 | 0.915 | 0.990 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.9 | 4 | 0.604 | 0.857 | 0.990 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 12 | 0.680 | 0.865 | 0.956 | 0.989 | 0.998 | 0.998 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 16 | 0.649 | 0.866 | 0.963 | 0.992 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 24 | 0.649 | 0.866 | 0.963 | 0.992 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| $T$ = 360 | | | | | | | | | | | |
| | $S$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
| 0.5 | 4 | 0.342 | 0.954 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 12 | 0.602 | 0.990 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 16 | 0.653 | 0.987 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 24 | 0.645 | 0.982 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.7 | 4 | 0.383 | 0.919 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 12 | 0.628 | 0.976 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 16 | 0.675 | 0.976 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 24 | 0.673 | 0.959 | 0.998 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.9 | 4 | 0.591 | 0.900 | 0.998 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 12 | 0.699 | 0.925 | 0.990 | 0.997 | 1.000 | 0.998 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 16 | 0.741 | 0.923 | 0.984 | 0.996 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 24 | 0.715 | 0.911 | 0.986 | 0.998 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

<Table 2> Rejection frequencies of Robinson's (1994a) tests with AR(1) disturbances. (5,000 replications were used in each case)

| $T = 120$ | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\phi$ | $S$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
| 0.5 | 4 | 0.217 | 0.323 | 0.604 | 0.847 | 0.957 | 0.989 | 0.999 | 1.000 | 1.000 | 1.000 |
| | 12 | 0.322 | 0.535 | 0.740 | 0.891 | 0.955 | 0.983 | 0.992 | 0.999 | 0.999 | 1.000 |
| | 16 | 0.302 | 0.525 | 0.735 | 0.877 | 0.943 | 0.976 | 0.991 | 0.997 | 1.000 | 1.000 |
| | 24 | 0.312 | 0.487 | 0.708 | 0.858 | 0.935 | 0.975 | 0.989 | 0.999 | 0.999 | 1.000 |
| 0.7 | 4 | 0.354 | 0.414 | 0.655 | 0.855 | 0.962 | 0.990 | 0.999 | 1.000 | 1.000 | 1.000 |
| | 12 | 0.445 | 0.627 | 0.778 | 0.883 | 0.938 | 0.969 | 0.985 | 0.991 | 0.995 | 1.000 |
| | 16 | 0.442 | 0.602 | 0.767 | 0.859 | 0.929 | 0.958 | 0.977 | 0.991 | 0.995 | 1.000 |
| | 24 | 0.387 | 0.554 | 0.710 | 0.835 | 0.911 | 0.964 | 0.982 | 0.995 | 0.999 | 1.000 |
| 0.9 | 4 | 0.567 | 0.647 | 0.794 | 0.901 | 0.963 | 0.997 | 0.999 | 0.999 | 0.999 | 1.000 |
| | 12 | 0.623 | 0.750 | 0.820 | 0.875 | 0.911 | 0.933 | 0.951 | 0.969 | 0.983 | 0.991 |
| | 16 | 0.562 | 0.702 | 0.790 | 0.847 | 0.901 | 0.932 | 0.961 | 0.975 | 0.987 | 0.995 |
| | 24 | 0.481 | 0.603 | 0.719 | 0.814 | 0.886 | 0.942 | 0.967 | 0.986 | 0.993 | 0.998 |
| $T = 240$ | | | | | | | | | | | |
| $\phi$ | $S$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
| 0.5 | 4 | 0.236 | 0.354 | 0.750 | 0.960 | 0.996 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 12 | 0.328 | 0.691 | 0.937 | 0.996 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 16 | 0.350 | 0.691 | 0.923 | 0.984 | 0.998 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 24 | 0.362 | 0.706 | 0.915 | 0.985 | 0.998 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.7 | 4 | 0.462 | 0.415 | 0.733 | 0.953 | 0.995 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 12 | 0.413 | 0.752 | 0.937 | 0.991 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 16 | 0.457 | 0.735 | 0.910 | 0.975 | 0.992 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 24 | 0.470 | 0.727 | 0.896 | 0.970 | 0.997 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.9 | 4 | 0.604 | 0.636 | 0.832 | 0.960 | 0.995 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 12 | 0.644 | 0.839 | 0.935 | 0.964 | 0.982 | 0.995 | 0.996 | 0.998 | 0.998 | 0.999 |
| | 16 | 0.625 | 0.793 | 0.896 | 0.946 | 0.975 | 0.987 | 0.998 | 1.000 | 1.000 | 1.000 |
| | 24 | 0.587 | 0.755 | 0.872 | 0.930 | 0.975 | 0.992 | 0.998 | 1.000 | 1.000 | 1.000 |
| $T = 360$ | | | | | | | | | | | |
| $\phi$ | $S$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
| 0.5 | 4 | 0.264 | 0.377 | 0.880 | 0.998 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 12 | 0.326 | 0.804 | 0.990 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 16 | 0.352 | 0.820 | 0.992 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 24 | 0.406 | 0.832 | 0.976 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.7 | 4 | 0.531 | 0.387 | 0.819 | 0.993 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 12 | 0.427 | 0.816 | 0.984 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 16 | 0.459 | 0.833 | 0.976 | 0.998 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 24 | 0.513 | 0.834 | 0.958 | 0.995 | 0.999 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.9 | 4 | 0.672 | 0.629 | 0.863 | 0.982 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 12 | 0.624 | 0.872 | 0.962 | 0.987 | 0.995 | 0.999 | 0.999 | 1.000 | 1.000 | 1.000 |
| | 16 | 0.655 | 0.876 | 0.940 | 0.982 | 0.992 | 0.997 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 24 | 0.663 | 0.825 | 0.923 | 0.963 | 0.988 | 0.997 | 1.000 | 1.000 | 1.000 | 1.000 |

<Table 3> Rejection frequencies of Robinson's (1994a) with seasonal AR(1) disturbances.(5,000 replications were used in each case)

| $T=120$ | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\phi$ | $S$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
| 0.5 | 4 | 0.289 | 0.697 | 0.952 | 0.999 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 12 | 0.278 | 0.692 | 0.938 | 0.999 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 16 | 0.296 | 0.682 | 0.936 | 0.997 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 24 | 0.284 | 0.684 | 0.930 | 0.998 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.7 | 4 | 0.280 | 0.702 | 0.951 | 0.999 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 12 | 0.284 | 0.689 | 0.933 | 0.993 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 16 | 0.312 | 0.667 | 0.925 | 0.991 | 0.999 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 24 | 0.293 | 0.676 | 0.917 | 0.992 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.9 | 4 | 0.298 | 0.714 | 0.942 | 0.993 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 12 | 0.327 | 0.671 | 0.894 | 0.973 | 0.993 | 0.998 | 0.999 | 0.999 | 1.000 | 1.000 |
| | 16 | 0.355 | 0.657 | 0.874 | 0.961 | 0.995 | 0.999 | 0.999 | 1.000 | 1.000 | 1.000 |
| | 24 | 0.304 | 0.661 | 0.894 | 0.976 | 0.998 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| $T=240$ | | | | | | | | | | | |
| $\phi$ | $S$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
| 0.5 | 4 | 0.451 | 0.953 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 12 | 0.456 | 0.945 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 16 | 0.439 | 0.948 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 24 | 0.452 | 0.943 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.7 | 4 | 0.460 | 0.954 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 12 | 0.469 | 0.940 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 16 | 0.453 | 0.939 | 0.999 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 24 | 0.467 | 0.930 | 0.999 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.9 | 4 | 0.487 | 0.947 | 0.999 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 12 | 0.489 | 0.925 | 0.995 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 16 | 0.487 | 0.903 | 0.993 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 24 | 0.490 | 0.904 | 0.995 | 0.999 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| $T=360$ | | | | | | | | | | | |
| $\phi$ | $S$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
| 0.5 | 4 | 0.632 | 0.994 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 12 | 0.626 | 0.995 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 16 | 0.623 | 0.994 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 24 | 0.627 | 0.994 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.7 | 4 | 0.632 | 0.994 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 12 | 0.622 | 0.993 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 16 | 0.627 | 0.989 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 24 | 0.613 | 0.990 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.9 | 4 | 0.645 | 0.996 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 12 | 0.617 | 0.980 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 16 | 0.640 | 9.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 24 | 0.607 | 0.969 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

higher than 0.950 in all cases. Thus, using this version of the tests of Robinson (1994a), we observe that the tests will not reject the long memory hypotheses in some cases although this characteristic is not present in the data. Increasing the sample size, the rejection probabilities also increase. Thus, for example, if $T = 240$, the values corresponding to $d = 0.1$ range between 0.319 and 0.680 and, if $T = 360$, they are between 0.342 and 0.741. In the latter case, however, the rejection probabilities are practically 1 if $d \geq 0.3$.

Next, in <Table 2>, we perform the same experiment but assuming that the disturbances follow a non-seasonal AR(1) process. Here, the values are smaller than in the previous case and the rejection frequencies are smaller than 0.900 in some cases with $T = 120$ even for $d = 0.4$. Similarly to <Table 1>, the values improve with $T$. However, values strictly smaller than 1 are obtained in some cases even for values of d higher than 0.5. Therefore, using this version of the tests, the results also tend to produce non-rejections of long memory when this hypothesis is not present in the data. Finally, we correctly assume that the disturbances are seasonally AR, and the results are displayed in <Table 3>. As expected, the values are now higher than in the previous cases, though we still observe low values when $d$ is relatively low, especially for small sample sizes. Thus, if we test $d = 0.1$ with $T = 120$, the rejection probabilities never exceed 0.400. If $T = 240$, they do not exceed 0.500 and even with $T = 360$, the largest probability is 0.640 corresponding to $\phi = 0.9$ and s = 16. In general, we do not observe large differences across $\phi$ and s, though, even in the case of correctly assuming the type of $I(0)$ disturbances underlying the process, the possibility of long memory is still present in the context of Robinson's (1994a) parametric tests.

<Table 4> displays the mean and variance of estimates of d based on the Whittle method of Robinson (1995a) assuming that the true model is given by (7). We see that if $T = 120$, the estimated values of $d$ are positive in all cases, ranging between 0.100 ($\phi$ = 0.7 and s = 4) and 0.210 ($\phi$ = 0.5 and s = 24). Increasing $T$, the values are smaller though still positive in most of the cases with $T = 240$. Thus, for example, if $\phi$ = 0.7 and s = 24, the estimated value of $d$ is 0.106. However, increasing the sample size to 360 observations, the values are close to 0, sometimes positive, sometimes negative, ranging from -0.078 ($\phi$ = 0.9 and s = 4) to 0.080 ($\phi$ = 0.5, s = 24).

## 4. An empirical application

The time series data analysed in this section correspond to the number of foreign visitors (NVISIT) and the number of occupations in hotels (NOCCUP) in Spain, monthly, for the time period April 1965 - April 2002, obtained from the Bank of Spain database.

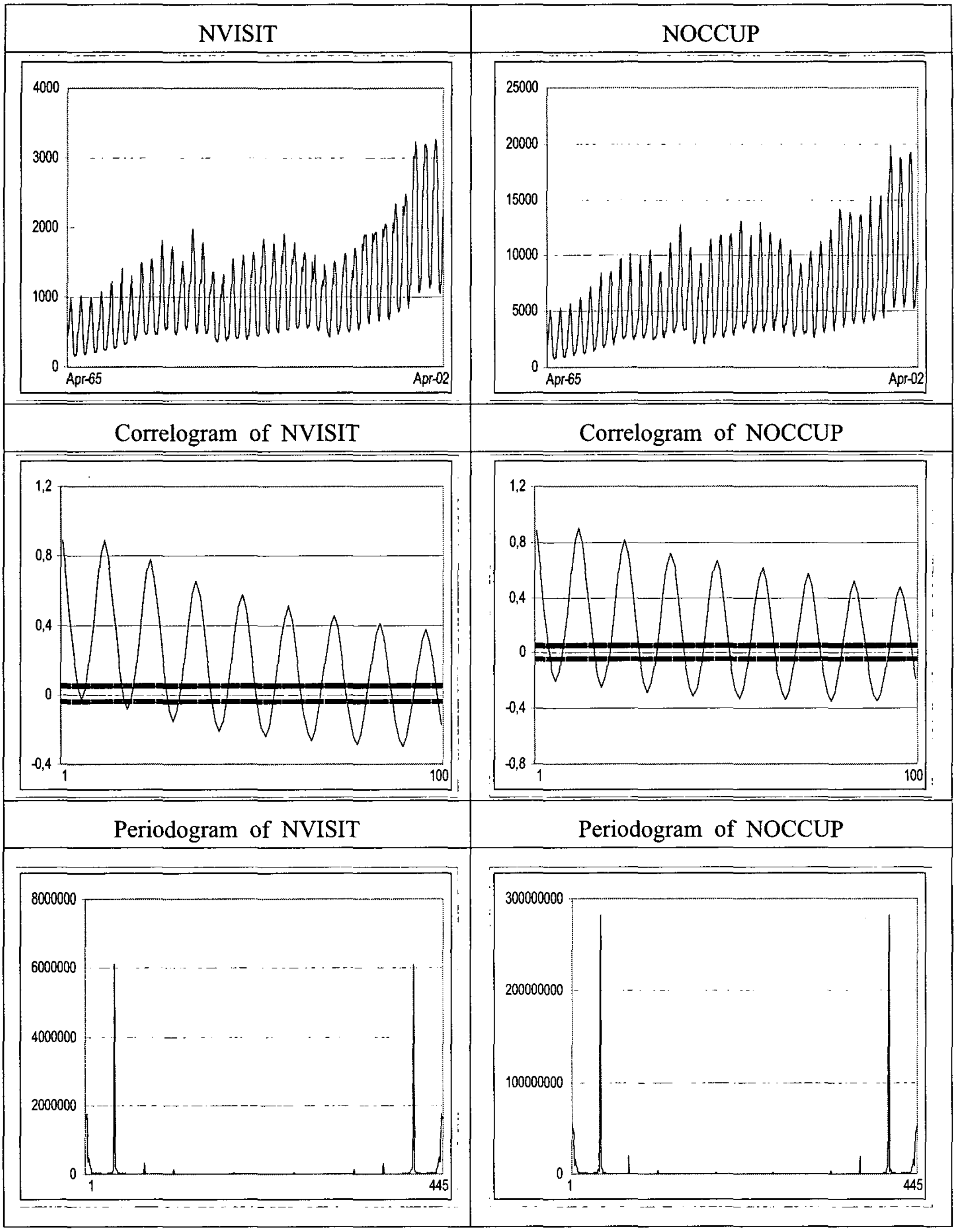
<Table 4> Mean and variance of the Whittle estimate (Robinson, 1995a) in a model given by (7). (5,000 replications were used)

| | | $T=120$ | | $T=240$ | | $T=360$ | |
|---|---|---|---|---|---|---|---|
| $\phi$ | $S$ | Mean | Mean | Mean | Variance | Mean | Variance |
| 0.5 | 4 | 0.145 | 0.018 | 0.083 | 0.013 | 0.049 | 0.009 |
| | 12 | 0.185 | 0.017 | 0.103 | 0.014 | 0.064 | 0.009 |
| | 16 | 0.194 | 0.016 | 0.110 | 0.013 | 0.069 | 0.011 |
| | 24 | 0.210 | 0.017 | 0.123 | 0.013 | 0.080 | 0.012 |
| 0.7 | 4 | 0.100 | 0.019 | 0.032 | 0.011 | 0.001 | 0.010 |
| | 12 | 0.165 | 0.017 | 0.072 | 0.014 | 0.028 | 0.007 |
| | 16 | 0.180 | 0.017 | 0.085 | 0.014 | 0.039 | 0.008 |
| | 24 | 0.203 | 0.017 | 0.106 | 0.011 | 0.057 | 0.008 |
| 0.9 | 4 | 0.135 | 0.021 | -0.046 | 0.013 | -0.078 | 0.011 |
| | 12 | 0.141 | 0.019 | 0.030 | 0.017 | -0.022 | 0.012 |
| | 16 | 0.166 | 0.018 | 0.053 | 0.014 | -0.002 | 0.009 |
| | 24 | 0.195 | 0.018 | 0.085 | 0.014 | 0.025 | 0.010 |

<Figure 1> displays plots of the original series along with their corresponding correlograms and periodograms. We see that both series are nonstationary with a clear changing seasonal pattern. The correlograms explicitly show the seasonal structure and the periodograms show peaks, not only at zero but also at the seasonal frequencies. Similar plots based on the first differenced data are given in <Figure 2>. The correlograms and the periodograms of the differenced data show that the seasonal components are still present.

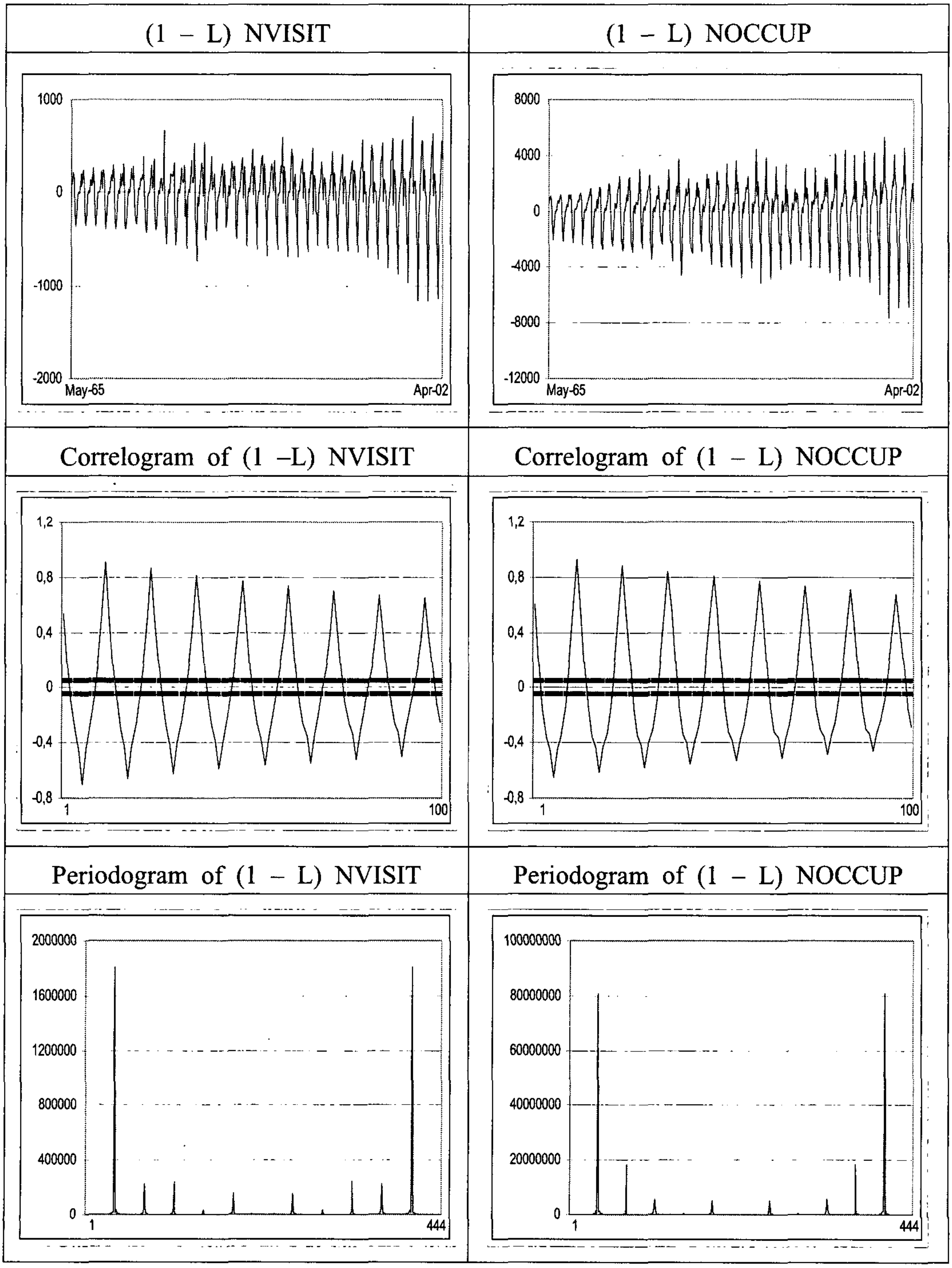
Denoting any of the series by $x_t$, the first thing we do is to perform the parametric procedure of Robinson (1994a), testing the null hypothesis given by (3) in model (1) using $\hat{R}$ given by (4). We take values $d_0$ from 0 to 2, with increments of 0.25, and perform the tests assuming that $u_t$ is white noise, AR(1) and seasonal AR(1). The results are displayed in <Table 5>. The last column of the table reports the confidence intervals of those values of $d_0$ where $H_0$ (3) cannot be rejected at the 95% significance level. Starting with the case of white noise $u_t$, (in Table 5(i)), we observe that the unit root null hypothesis is rejected in both series. In fact, the only non-rejection value of $d_0$ takes place at $d=1.25$ for the NVISIT series and at $d=1.75$ for the NOCCUP. Thus, we observe a higher order of integration in the number of occupations compared with the number of visitors. In fact, the confidence intervals are respectively [1.39 - 1.75] and [1.53 - 1.91]. In the light of the results reported in this table, we could conclude that the first differenced series still present a component of long memory

<Figure 1> Plots of the original time series with their corresponding correlograms and periodograms




The large sample standard error under the null hypothesis of no autocorrelation is $1/\sqrt{T}$ or roughly 0.047.

<Figure 2> Plots of the first differenced data with their corresponding correlograms and periodograms




The large sample standard error under the null hypothesis of no autocorrelation is $1/\sqrt{T}$ or roughly 0.047.

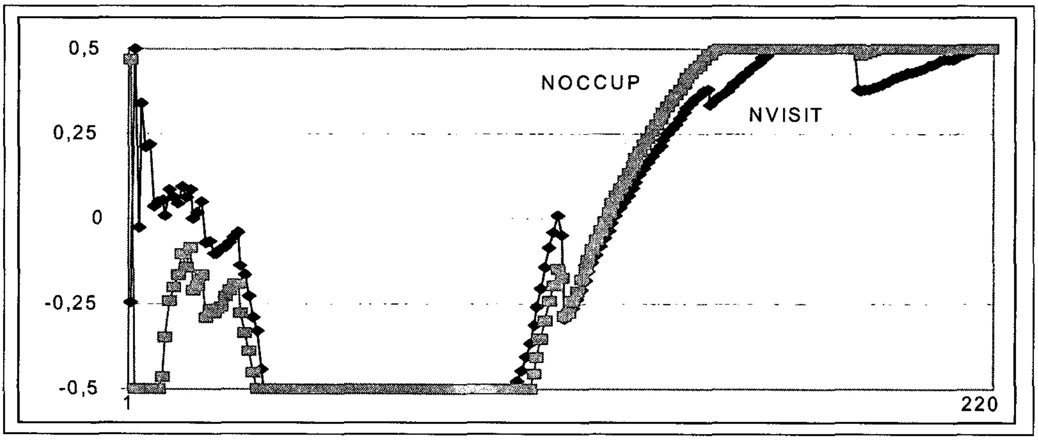
behavior. However, we have seen in the previous section that in the presence of periodicities, the tests tend to favor long memory even if it is not present. In <Tables 5 (ii) and (iii)>, we assume that $u_t$ is AR(1) and seasonal AR(1) respectively. Imposing non-seasonal disturbances, $H_0$ (3) cannot be rejected with $d_0$ = 0.75 for NVISIT and for $d_0$ = 1 for NOCCUP. Thus, the results are completely different to those obtained with white noise disturbances, though still the order of integration at NOCCUP is higher than the one corresponding to the number of visitors. However, the results in section 3 and a visual inspection at the correlograms and periodograms in <Figure 1> and <Figure 2> suggest that the most reliable results should be those based on seasonal AR disturbances. (Table 5(iii)). Here, the unit root null (i.e., $d$ = 1) cannot be rejected for any of the series and the confidence intervals are [0.81 - 1.09] for the number of visitors, and [0.95 - 1.25] for the occupations. In this context of seasonal $I(0)$ disturbances, the tests were again performed for values of $d_0$ with 0.01 increments, and the lowest statistics were obtained at $d_0$ = 0.94 in the case of NVISIT and at $d_0$ = 1.08 for NOCCUP.

<Table 5> Testing $H_0$ (3) in (1) with the tests of Robinson (1994a)

| i) with white noise disturbances | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Series | 0.00 | 0.25 | 0.50 | 0.75 | 1.00 | 1.25 | 1.50 | 1.75 | 2.00 | Conf. Interval |
| NVISIT | 1017.18 | 188.85 | 91.79 | 58.45 | 32.28 | ***11.15*** | 0.51 | 3.60 | 17.55 | [1.39 - 1.75] |
| NOCCUP | 753.73 | 141.29 | 89.65 | 65.19 | 41.65 | 19.77 | 4.42 | ***0.13*** | 7.38 | [1.53 - 1.91] |
| ii) with AR(1) disturbances | | | | | | | | | | |
| Series | 0.00 | 0.25 | 0.50 | 0.75 | 1.00 | 1.25 | 1.50 | 1.75 | 2.00 | Conf. Interval |
| NVISIT | 144.16 | 186.67 | 63.06 | ***0.31*** | 19.12 | 26.87 | 20.91 | 17.19 | 18.38 | [0.71 - 0.85] |
| NOCCUP | 242.63 | 254.71 | 127.62 | 23.39 | ***3.31*** | 25.14 | 27.80 | 22.99 | 20.50 | [0.86 - 1.00] |
| iii) with seasonal AR(1) disturbances | | | | | | | | | | |
| Series | 0.00 | 0.25 | 0.50 | 0.75 | 1.00 | 1.25 | 1.50 | 1.75 | 2.00 | Conf. Interval |
| NVISIT | 929.71 | 256.34 | 53.09 | 7.80 | ***0.62*** | 13.04 | 33.25 | 54.20 | 72.24 | [0.81 - 1.09] |
| NOCCUP | 914.08 | 287.08 | 93.97 | 25.60 | ***1.30*** | ***3.85*** | 18.71 | 37.05 | 54.46 | [0.95 - 1.25] |

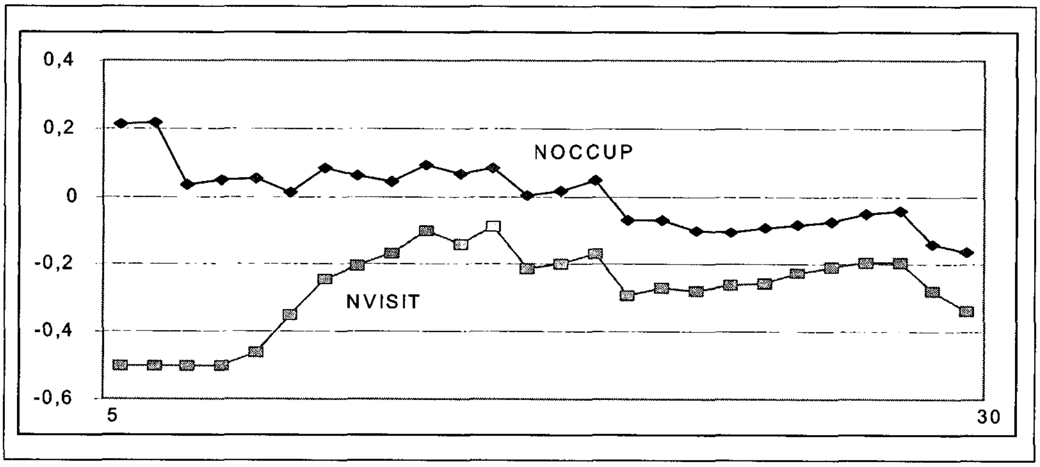
In bold-italic, the non-rejection values of the null hypothesis at the 95% significance level.

<Figure 3> Whittle estimates of Robinson (1995a) for a range of values of $m=1,220$




The horizontal axe refers to the bandwidth parameter number $m$, while the vertical one corresponds to the estimated values of $d$.

<Figure 4> Whittle estimates of Robinson (1995a) for a shorter range of values of $m$




The horizontal axe refers to the bandwidth parameter number $m$, while the vertical one corresponds to the estimated values of $d$.

<Figure 3> displays the estimates of $d$ based on the Whittle estimate of Robinson (1995a), i.e., $\hat{d}$ given by equation (6), for a range of values of the bandwidth number m from 1 to $T/2$. Since the time series are clearly nonstationary, the analysis was carried out based on the first differenced data, adding then 1 to the estimated values of $d$ to obtain the proper orders of

integration of the series. We see that the estimates are very similar in both series and they behave erratically from $-0.5$ to 0.5. The most stable behavior seems to be obtained when $m$ is constrained between 5 and 30. <Figure 4> displays the estimates for that range of values of m. We see that the estimated values of $d$ are in some cases slightly higher than 0 for NOCCUP and they are strictly below 0 for NVISIT, which is completely in line with the previous results based on the parametric procedure of Robinson (1994a) with seasonal AR disturbances. We can then conclude by saying that both series present some degree of long memory behavior with $d$ higher than 1 for the number of occupations and below 1 and thus showing mean reversion for the number of visitors.

## 5. Concluding comments

In this article we have examined if the presence of periodicities in raw time series may influence the estimation and testing of the fractional differencing parameter. For this purpose we have used a parametric testing procedure and a semiparametric estimation method proposed by Robinson (1994a, 1995a). Several Monte Carlo experiments conducted across the paper show that the tests of Robinson (1994a) tend to accept the null hypothesis of long memory (i.e., $d > 0$) in the context of periodicity if we misspecify the $I(0)$ disturbance term. However, even if we correctly assume the periodicity in the disturbances, the results show that the tests still may have low power in finite samples. The Whittle method of Robinson (1995a) also presents a bias in the estimation in favor of long memory if $T$ is small. However, if $T$ is relatively large (e.g., $T = 360$), the performance of the procedure seems to be adequate. These two methods were then applied to the monthly series of the number of visitors and number of occupations in hotels in Spain for the time period 1965m4-2002m4. The results show that in spite of the seasonality, long memory is present in both series, with the order of integration slightly above 1 for the number of occupations and slightly smaller than 1 for the number of visitors.

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