International Journal of Automotive Technology, Vol. 7, No. 4, pp. 477−483 (2006)

1229−9138/2006/028−11

DEVELOPMENT OF A VIBRATION MODEL OF A HELICAL GEAR PAIR FOR VEHICLE TRANSMISSION

W. S. KO[1], H. W. LEE[2)*] and N. G. PARK[3]

[1)]Hyosung Company, 454-2 Nae-dong, Changwon-si, Gyeongnam 641-712, Korea
[2)]Research Institute of Mechanical Technology, Pusan National University, Busan 609-735, Korea
[3)]Department of Mechanical Engineering, Pusan National University, Busan 609-735, Korea

(Received 27 September 2005; Revised 9 January 2006)

ABSTRACT−A vibration model of a helical gear pair for vehicle transmission is developed by considering the elastic deformation of the active teeth and the body to be a rigid. The main source of vibration in a helical gear system which caused by the mass unbalances of rotors and the transmission errors of gearings is mathematically formulated and applied to the analysis of vibration characteristics of geared systems. As an example, a simple geared system containing a helical gearing is considered. The critical speeds are found by the Campbell diagram and compared with the experimental results. We expect this developed program to contribute to the reduction of the vibration and noise on vehicle a transmission in the field of both design and manufacturing. In addition, this program can be used as a basic program for CAD/CAM of low-noised gear teeth.

KEY WORDS : Helical gear, Vehicle transmission, Transmission error, Critical speeds, Vibration

1. INTRODUCTION

Recent consumers' propensities for vehicles are intended for both eminent performance and high quality. This includes agreeable driving, comfort and safety, power performance, stability of the steering system and fuel economy. The transmission, a main part of the vehicles, is developed to satisfy the more strict requirements of high capacity, high endurance, compact size and lowered vibration/noise. The vehicle transmission has a very complex helical gear system and the vibration and noise problems of these systems according to the requirement of high speed, high precision and high power are increasing now (Bierman, 2005). The helical gear system has the merits of noise reduction but has complicated flexible characteristics compared with a spur gear system. Six degrees of freedom should be considered - three translational motions and three rotational motions - to analyze the helical gear system excited by bending, torsional and axial force.

In the study of the analysis of helical gear system (Umezawa *et al.*, 1986, 1988) calculated the rotational motion of a helical gear pair with small face width by the numerical method to reduce the transmission error. Neriya *et al.* (1988; 1989) calculated the response of a helical gear pair which is flexed with bending, torsional and axial forces and excited with the static transmission error and achieved a stable region by Floquet theory. Also, Lee *et al.* (1998) formulated the vane passing frequencies of the impeller in the air type turbo-compressor by the perturbation method which was excited with the mass unbalance of rotors, the misalignment of shafts, the transmission error of the gear pairs, the backlash and bearing clearance, and the periodic variation of gear contact coefficients.

In this paper, we developed the generalized vibrational model of the helical gear pair under the bending moment, the torsion and the axial force of the shaft. This helical gear system model enables to understand the complicated helical gear system which drives the vehicle transmission, turbo compressor, gas turbine or driveline of the craft. The single step helical gear system was manufactured to verify the vibrational model. The critical speed was calculated by analytic method and measured in test with recognizing of the sources of the vibration such as the unbalance of the rotating rotor, the gear transmission error, the misalignment of the shaft, the gear backlash, the clearance of bearing and the alternation of the exciting frequency which is caused by periodical changes of the stiffness coefficient on the contact of the gear teeth. Lastly, the critical speeds are found by the Campbell diagram and compared with the experimental results.

**Corresponding author.* e-mail: leehwoo@gmail.com

2. MATHEMATICAL MODEL OF A SIMPLE HELICAL GEAR SYSTEM

2.1. Dynamic of Helical Gear System

The mathematical model of a simple helical gear system, which is composed of one gear pair of mating gears, two shafts and four bearings, is developed by the assumption of a lumped parameter system. This is assuming the existence of mating helical gear teeth with elastic deformation, that the bodies of gears are rigid rotors with gyroscopic effect, the bearings to be a linear spring, and the shaft to be an Euler beam with the elastic and inertia effect.

The equations of motion for helical gear system can be written in matrix from as

$$[M]\{\ddot{w}\}+[G]\{\dot{w}\}+[K]\{w\}=\{0\} \tag{1}$$

where the generalized displacement vector $\{w\}$ consists of the three displacement vectors, x, y, z with the corresponding lateral θ_x, θ_y, and torsional θ_z rotational vectors as

$$\{w\}=\begin{pmatrix} x \\ y \\ z \\ \theta_x \\ \theta_y \\ \theta_z \end{pmatrix} \tag{2}$$

The equation of motion shown in equation (1) includes the effects of inertia, [*M*], gyroscopic forces [*G*], stiffness, [*K*].

2.2. Vibration Model of a Helical Gear Pair

The process of the vibration model of a teeth-contact region is as follows:

(1) Calculated the equivalent mesh stiffness considering the elastic deformation of mating gear teeth.
(2) Neglected the friction force in the distributed transmitted force spread over face width of mating gear teeth which can be defined by the average concentrated force at pitch point and average Couple Force.
(3) Neglecting Couple Force, considering lead crowning of gear tooth surface can define the transmitted force of mating gear teeth as the average concentrated force at pitch point as shown in Figure 1.
(4) Considered the elastic deformation of a gear tooth only and not the body of a gear.
(5) As shown in Figure 2, assumed the mating gear teeth as two separate compressed linear springs, P-G1 and P-G2. Here, the direction of the springs is vertical to the teeth contact line, $\overline{AB}$.
(6) The equivalent spring coefficients, K1 and K2, can be calculated with the assumption of equivalent spur gear transformation of a helical gear and Cornell's method (1980) which regards a gear tooth as a cantilever beam considering bending and shear deformations and gear contact deformation derived from Hertz's contact theory.

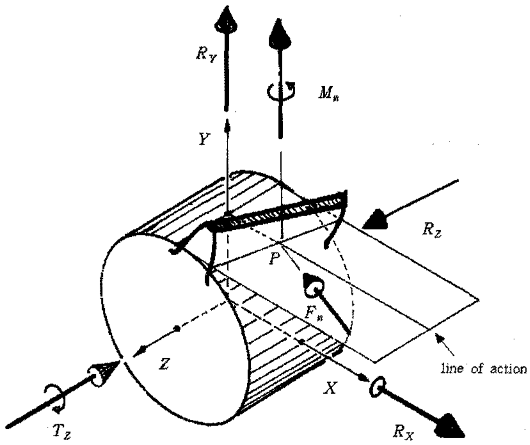

Figure 1. Vibration model of face contact of helical gear tooth.

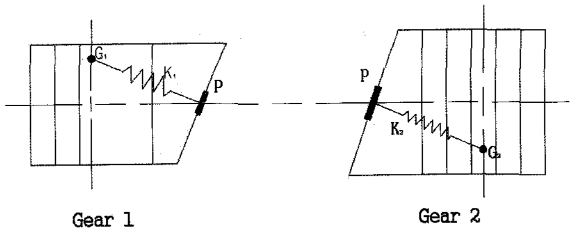

Figure 2. Model of helical gear pair.

The mathematical model of a helical gear pair is shown in Figure 3. Let the center of drive gear be the origin of coordinates, the radial horizontal direction be x-

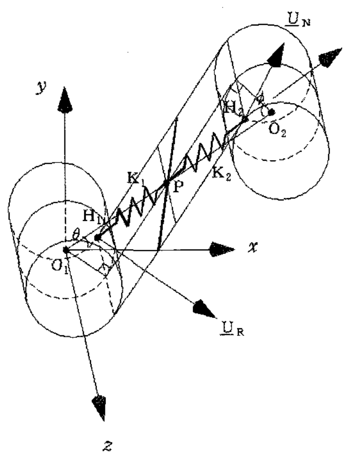

Figure 3. Schematic of a mathematical model on a pair of helical gear.

axis, the radial vertical direction be y-axis and the positive rotational direction be z-axis.

Put the nodes at the center of the helical gears. The generalized displacement vector which is composed of translational displacement vector, $\underline{u}$ and rotational displacement vector $\underline{\theta}$ at both modes is defined as

$$\underline{q}=\begin{pmatrix}\underline{u}\\ \underline{\theta}\end{pmatrix} \tag{3}$$

where

$$\underline{u}=\begin{pmatrix}x\\ y\\ z\end{pmatrix} \quad \underline{\theta}=\begin{pmatrix}\theta_x\\ \theta_y\\ \theta_z\end{pmatrix}$$

The generalized force vector at both nodes, $\underline{f}$ is defined as

$$\underline{f}=\begin{pmatrix}\underline{F}\\ \underline{M}\end{pmatrix} \tag{4}$$

where

$$\underline{F}=\begin{pmatrix}F_x\\ F_y\\ F_z\end{pmatrix} \quad \underline{M}=\begin{pmatrix}M_x\\ M_y\\ M_z\end{pmatrix}$$

The direction vector of tooth contact force, $\underline{\eta}_t$ is defined as

$$\underline{\eta}_t=\begin{pmatrix}\cos\alpha\cos\varphi\\ \sin\alpha\cos\varphi\\ \sin\varphi\end{pmatrix} \tag{5}$$

where ϕ is the helix angle of base circle and the angle between the center of drive and driven gear is θ. As the rotational direction of the drive gear is counterclockwise in Figure 3, the angle of line of action, α is expressed as

$$\alpha=\frac{\pi}{2}-\phi+\theta \tag{6}$$

where ϕ is the transverse running pressure angle.

The potential energy of gear contact tooth is defined as

$$V_{th}=\frac{1}{2}K_{th}\Delta^2 \tag{7}$$

where, K_{th} is the equivalent tooth stiffness and Δ, is the amount of compressed deformation.

$$\Delta=\underline{\eta}_t^T(\underline{u}_R-\underline{u}_N) \tag{8}$$

where, $\underline{u}_R$, u_N are the displacement vector of the tooth contact of drive and driven gear and can be described by the generalized displacement $\underline{u}_R$, u_N at the center of gears.

$$\underline{u}_R=D_1\underline{q}_1 \tag{9}$$

$$\underline{u}_N=D_2\underline{q}_2 \tag{10}$$

where, D_1, D_2 are the proportional matrix calculated from linear correlation of rigid body motion between the displacement of tooth contact and the center of gears.

$$D_1=\begin{vmatrix}1&0&0&0&L_{z1}&-L_{y1}\\ 0&1&0&-L_{z1}&0&L_{x1}\\ 0&0&1&L_{y1}&-L_{x1}&0\end{vmatrix}$$

$$D_2=\begin{vmatrix}1&0&0&0&L_{z2}&-L_{y2}\\ 0&1&0&-L_{z2}&0&L_{x2}\\ 0&0&1&L_{y2}&-L_{x2}&0\end{vmatrix}$$

The potential energy of a helical gear tooth is derived by the equations of (7) with the equations of (8), (9), (10).

$$V_{th}=\frac{1}{2}\begin{pmatrix}\underline{q}_1\\ \underline{q}_2\end{pmatrix}^T\begin{pmatrix}K_{11}&K_{12}\\ K_{21}&K_{22}\end{pmatrix}\begin{pmatrix}\underline{q}_1\\ \underline{q}_2\end{pmatrix} \tag{11}$$

where

$K_{11}=K_{th}D_1^T\underline{\eta}_t\underline{\eta}_t^TD_1$

$K_{12}=-K_{th}D_1^T\underline{\eta}_t\underline{\eta}_t^TD_2$

$K_{21}=-K_{th}D_2^T\underline{\eta}_t\underline{\eta}_t^TD_1$

$K_{22}=K_{th}D_2^T\underline{\eta}_t\underline{\eta}_t^TD_2$

The element stiffness matrix between two nodes can be calculated by equation (11), the equation of potential energy, which is described with generalized displacement vector at the center of both mating gears with the assumptions of a lumped parameter system.

2.3. Exciting Source of a Helical Gear System

A helical gear system is excited by the exciting source which is classified as mass unbalance, assembling errors of bearings and shafts, tooth profile and lead errors of gears, the clearance and non-linear deformation of rolling bearings and periodic variation of gear tooth stiffness. The exciting frequency of mass unbalance is the same as the speed of rotation, ω The exciting frequency of a tooth

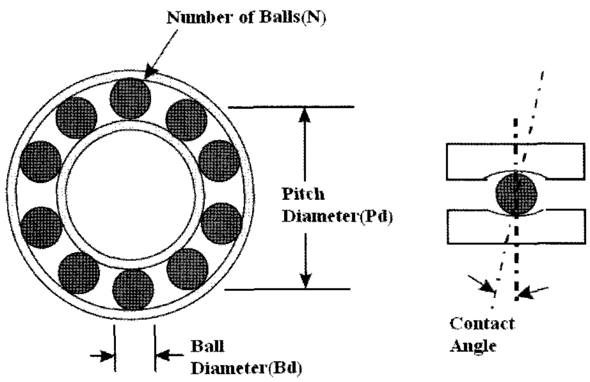

Figure 4. Parameters for calculating bearing frequencies.

Table 1. Self exciting vibration of gear system.

Self exciting frequencies	Contents	Equation	Remark
The rotational velocity of input shaft (ω_1)	The mass unbalance of gear on input shaft	1X	ω_1
The rotational velocity of output shaft (ω_2)	The mass unbalance of gear on output shaft	1.33X	ω_2
The self exciting frequencies of bearing clearances (η)	$FTF=\frac{rps}{2}\left[1-\frac{B_d}{P_d}\cos\phi\right]$	0.4X	η_1
	$BS=\frac{p_d}{2B_d}(rps)\left[1-\left(\frac{B_d}{P_d}\right)^2\cos^2\phi\right]$	2.8X	η_2
	$OR=N(FTF)$	7.2X	η_3
	$IR=N(rps-FTF)$	9.8X	η_4
Tooth passing frequency (Ω)	Self-exciting force caused by tooth errors	43X	Ω
Side band frequencies composed of tooth passing frequency and rotational velocity of shaft ($k\Omega \pm l\omega$, $k\Omega \pm m\Gamma\omega$)	Self-exciting force caused by periodic variation of tooth stiffness coefficient	42X	$\Omega-\omega_1$
		44X	$\Omega+\omega_1$
		42.7X	$\Omega-\omega_2$
		44.3X	$\Omega+\omega_2$
The integer multiples of tooth passing frequency ($k\Omega$, $k=2, 3, \ldots$)	Self-exciting force caused by tooth errors	86X	2Ω
Side band frequencies composed of tooth passing frequency and bearing clearances and rotational velocity of shaft	Self-exciting force caused by periodic variation of tooth stiffness coefficient	71.6X	$2\Omega\text{-}2\eta_3$
		98.4X	$2\Omega+2\eta_3$
		112.4X	$3\Omega-3\eta_3+3\omega_2+\omega$

profile and lead errors and pitch error of gears is the integer multiples of tooth passing frequency, Ω, which is defined as the multiple of the speed of rotation and the number of gear teeth. The exciting frequency of bearings, FTF (fundamental train frequency), BS (ball spin frequency), OR (outer race frequency), IR (inner race frequency) can be calculated by Work (1991).

Parameters for calculating bearing frequencies are shown in Figure 4. The equations of the exciting frequency of bearings are defined as

$$FTF=\frac{rps}{2}\left[1-\frac{B_d}{P_d}\cos\phi\right] \tag{12}$$

$$BS=\frac{p_d}{2B_d}(rps)\left[1-\left(\frac{B_d}{P_d}\right)^2\cos^2\phi\right] \tag{13}$$

$$OR=N(FTF) \tag{14}$$

$$IR=N(rps-FTF) \tag{15}$$

where

rps : a speed of rotation

B_d : ball or roller diameter

P_d : pitch diameter

ϕ : contact angle

N : the number of ball or roller

As the gear tooth stiffness varies periodically according to the tooth contact position, this self exciting force generates exciting frequencies; i.e., side bands frequencies $k\Omega \pm \omega$, $k = 1, 2, 3$ etc) which is described as the composition of the integer multiples of the speed of rotation, omega and tooth passing frequency, Ω. The exciting frequencies caused by the various exciting sources of a helical gear system are shown in Table 1.

2.4. Analysis Results of a Helical Gear System

The specifications of helical gears, shafts and bearings are shown in Tables 2, 3, 4. The stiffness of bearings is calculated by Rotor Bearing, Technology & Software (1998). The coefficient of average tooth stiffness in mating helical gears is calculated by Park's (1987) program and the result is 0.25×10^9 N/m.

A Campbell diagram to analyze the critical speeds

Table 2. Helical gear specifications.

Description	Gear (G #1)	Pinion (G #2)
Number of teeth	43	33
Module (mm)	2.0	2.0
Pressure angle (deg)	15	15
Helix angle (deg)	25	25
Tooth width (mm)	17	17
Center distance	86	86
I_d (kg·m^2)	1.3023×10^{-3}	5.963×10^{-3}
I_p (kg·m^2)	2.7840×10^{-3}	1.4605×10^{-3}
Mass (kg)	2.21896	1.8023

Table 3. Shaft specifications.

Number	Length (mm)	Diameter (mm)	Young's modulus	Poisson's ratio
S1	70.4	30	2.0×10^{11}	0.28
S2	69	30	2.0×10^{11}	0.28
S3	70.4	30	2.0×10^{11}	0.28
S4	69	30	2.0×20^{11}	0.28

Table 4. Bearing specifications.

Description	B #1	B #2	B #3	B #4
Roller number	17	17	17	17
Roller diameter (mm)	6.771	6.771	6.771	6.771
Pitch diameter (mm)	38.4232	38.4232	38.4232	38.4232
Effective length of roller (mm)	9.8	9.8	9.8	9.8
Contact angle (deg)	28.42	28.42	28.42	28.42
K_{11} ($\times 10^8$ N/m)	3.189	3.282	2.802	2.855
K_{22} ($\times 10^8$ N/m)	3.189	3.282	2.802	2.855
K_{33} ($\times 10^8$ N/m)	1.909	2.012	1.571	1.593
K_{44} ($\times 10^4$ N/m)	7.813	8.017	7.530	7.607
K_{55} ($\times 10^4$ N/m)	7.813	8.017	7.530	7.607
K_{66} ($\times 10^4$ N/m)	0	0	0	0

based on the exciting frequencies which are described in Table 1, is shown in Figure 5. This diagram demonstrated the change of the natural frequencies and the exciting frequencies, 43X, 86X, 71.6X, 98.4X and 112.4X, in the range of the input velocity of 1000–3200 rpm, and the exciting frequency under 5000 Hz.

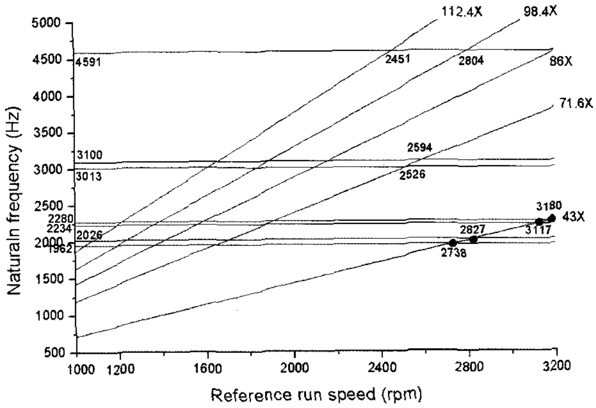

Figure 5. Campbell diagram.

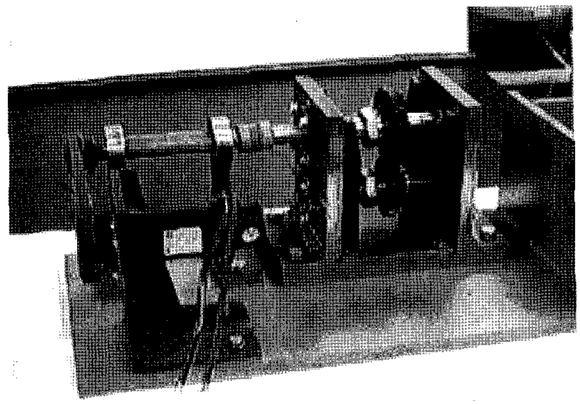

Figure 6. Test rig of the helical gear system.

3. EXPERIMENTAL EVALUATION OF THE MODEL OF A HELICAL GEAR SYSTEM

3.1. Experimental Setup

As shown in Figure 6, the test rig of a simple helical gear system was composed of two helical gears, two shafts, four taper roller bearings, one drive motor and one flexible coupling. This flexible coupling was used to avoid the external excitation caused by the drive motor.

To measure and analyze the vibration signal, Polytec OFV-352 laser vibrometer, Polytec OFV-2601 controller and SA 390 FFT were used. The schematic of experiment system is shown in Figure 7.

3.2. Results and Discussions

For experiment, the drive motor was operated from 1000 to 3200 rpm. As a result of the experiment, the waterfall diagram was shown in Figure 8. In Figure 8, the exciting frequencies caused by the mass unbalance of input shaft (1X), the mass unbalance of output shaft (1.33X) and the rolling bearings (0.4X, 2.8X, 7.2X, 9.8X) hardly appear. However the exciting frequencies caused by the tooth

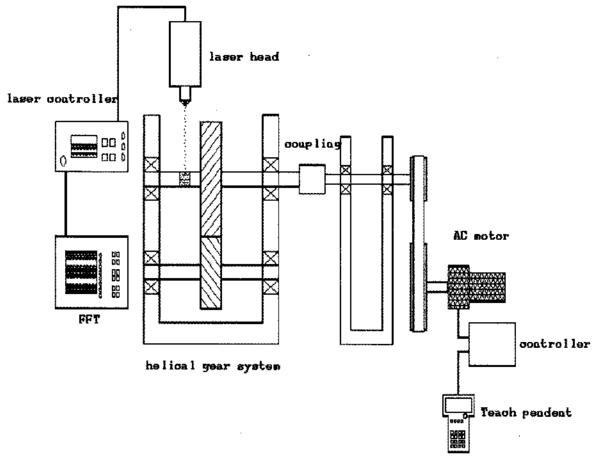

Figure 7. Schematic of experimental system.

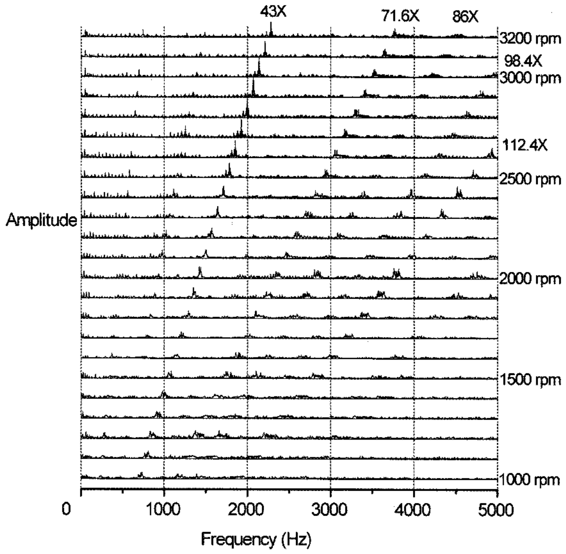

Figure 8. Waterfall diagram.

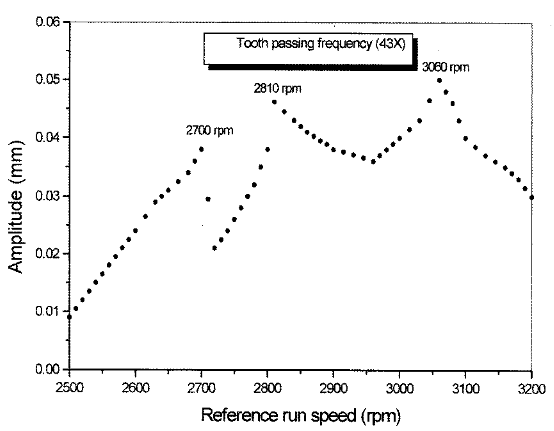

Figure 9. Bode Plot at Tooth-passing frequency (43X).

Table 5. Comparison for analysis and experiment results.

Critical speed (rpm)		Difference (%)
Analysis	Experiment	
2738	2700	1.4
2827	2810	1.0
3117	3060	1.8
3180	–	–

passing frequency (43X), double of tooth passing frequency (86X) and the periodic variation of tooth stiffness coefficient (71.6X, 98.4X, 112.4X) appear apparently. These exciting frequencies were described in Table 1.

In the case of the tooth passing frequency, 43X, the best apparent exciting frequency, there are high amplitude signals between 2500 rpm and 3200 rpm and the Bode diagram of these is shown in Figure 9. We can appreciate the experimental results of critical speeds of this helical gear system are 2700, 2810 and 3060 rpm, respectively.

The analytical and experimental results on the critical speeds in the case of the tooth passing frequency, 43X, are compared in Table 5. We can appreciate that the possibility of error between the analysis and experiments is within 2%.

4. CONCLUSIONS

The mathematical model of a simple helical gear system is developed and verified by experiments to study on the vibration characteristics of a helical gear system.

(1) The results show that the exciting frequencies caused by the mass unbalance of input shaft, the mass unbalance of output shaft and the rolling bearings hardly appear. However the exciting frequencies caused by the tooth passing frequency, double of the tooth passing frequency and the periodic variation of tooth stiffness coefficient appear apparently.
(2) From experimental results, we can find the critical speeds of this simple helical gear system are 2700, 2810 and 3060 rpm. The results obtained from the proposed methods are in good agreement with those from the experiment results within 2%.
(3) We can appreciate the validity of a lumped parameter method on the modeling of a helical gear system.
(4) We expect this program to contribute to the reduction of the vibration/noise on vehicle transmissions in the field of both design and manufacturing. As well, this program can be used as a basic sub-program for CAD/CAM's of low-noised gear teeth.

REFERENCES

Biermann, J. W. and Reitz, A. (2005). Possibilities to improve transient gear shift noise (shift clonk) in a passenger car. *Int. J. Automotive Technology* **6**, **1**, 22–28.

Cornell, R. W. (1980). Compliance and stress sensitivity of spur gear teeth. *American Society of Mechanical Engineers Paper No.* 80-C2/DET-24.

Lee, H. W., Lee, D. W. and Park, N. G. (1998). An analytical investigation on vibration characteristics of turbo compressor. *J. Korean Society for Noise and Vibration Engineering* **7**, **4**, 1069–1077.

Neriya, S. V., Bhat, R. B. and Sankar, T. S. (1988). On the dynamic response of a helical geared system subjected to a static transmission error in the form of deterministic and filtered white noise input. *American Society of Mechanical Engineers J. Vibration, Acoustics, Stress, and Reliability in Design*, **110**, 501–506.

Neriya, S. V., Bhat, R. B. and Sankar, T. S. (1989). Stability analysis of force coupled in helical geared rotor systems. *Proc. 12th Biennial Ameriacn Society of Mechanicla Engineers Conf. Mechanical Vibration and Noise*, Montreal, Canada, Sept. 17–21, 225–229.

Park, N. G. (1987). *An Analysis Investication of Geared System Dynamics Containing Spur and Helical Gears*. Ph. D. Dissertation. North Calorina State University. Raleigh.

Rotor Bearing, Technology & Software (1998). *Advanced Rotating Machinery Dynamics*. COBRA. USA.

Umezawa, K., Suzuki, T. and Sato, T. (1986). Vibration of power transmission helical gears (Approximate equation of tooth stiffness). *Bulletine of Japan Society of Mechaniclal Engineers* **29**, **251**, 1605–1611.

Umezawa, K., Suzuki, K. and Houjoh, H. (1988). Estimation of vibration of power transmission helical gears by means of performance diagrams on vibration. *Japan Society of Mechanical Engineers Int. J. Series III* **31**, **3**, 598–605.

Work, V. (1991). *Machinery Vibration: Measurement and Analysis*. McGraw-Hill, Inc.. New York. 148–160.