Influence of Moving Mass on Dynamic Behavior of Simply Supported Timoshenko Beam with Crack

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In this paper, the effect of open crack on the dynamic behavior of simply supported Timoshenko beam with a moving mass was studied. The influences of the depth and the position of the crack on the beam were studied on the dynamic behavior of the simply supported beam system by numerical methods. The equation of motion is derived by using Lagrange's equation. The crack section is represented by a local flexibility matrix connecting two undamaged beam segments. The crack is modeled as a rotational spring. This flexibility matrix defines the relationship between the displacements and forces on the crack section and is derived by applying fundamental fracture mechanics theory. As the depth of the crack increases, the mid-span deflection of the Timoshenko beam with a moving mass is increased.

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1. Introduction

The detection and control of damage in mechanical structures are important concerns of engineering communities. When a structure is subjected to damage, its dynamic response is varied due to the change of its mechanical characteristics. The coupling effects of an open crack and a moving mass on the structures and the machines are an important problem both in the field of transportation and design of machining processes. In order to study the effects of a crack on dynamic behavior of the structures, researchers recently use the analysis method of using the flexibility matrix.[1-2] Lee[3] studied the dynamic response of a clamped-clamped beam acted upon by a moving mass. He analyzed the problem of the moving mass separated from the beam by monitoring the contact forces between them. A lot of studies about the dynamic behavior of the beam structure under the moving load and mass were reported.[4,5] Zheng et al.[6] reported that when a crack is present in structure, the better results are obtained by applying the Timoshenko beam theory than using the Euler-Bernoulli beam theory. Based on this result, studies about the dynamic behavior of the cracked Timoshenko beam were investigated.[7-10] Recently, Mahmoud[11] used an equivalent static load approach to determine the stress intensity factors for a single or double-edge crack in a beam subjected to a moving load. Chondros and Dimarogonas[12] studied the effect of the crack depth on the dynamic behavior of a cantilevered beam. They showed that increasing the crack depth reduced the natural frequency of the beam.

In this study, the effects of a moving mass on the dynamic behavior of the cracked simply supported beam are investigated. That is, the influences of the crack depth, the position of crack and the moving mass have been studied on the dynamic behavior of a simply supported beam. The simply supported beam has a circular cross-section. The crack is assumed to be always open during the vibrations.

2. Theory and Formulations

A uniform beam of length L applying the Timoshenko beam theory was considered. Figure 1 represents the simply supported beam acted upon by the moving mass m_m with a constant velocity v along the beam. Where L is the total length of the beam, x_c is the position of a crack. Figure 2 shows a circular cross-section of the cracked section. a_c and $2b$ are the maximum depth of a crack and the length of the crack, respectively. Two equations of motion are derived for the two parts of the beam located on the left and on the right of the cracked section. In this study, the dimensionless moving mass is 0.3 and the velocity of the moving mass is selected two values (0.8 m/s, 1.6 m/s).

2.1 Energy of beam and moving mass

By using the assumed mode method, the lateral displacement $y(x, t)$ of a simply supported beam and the rotation $\Theta(x, t)$ in xy plane respectively can be assumed to be as

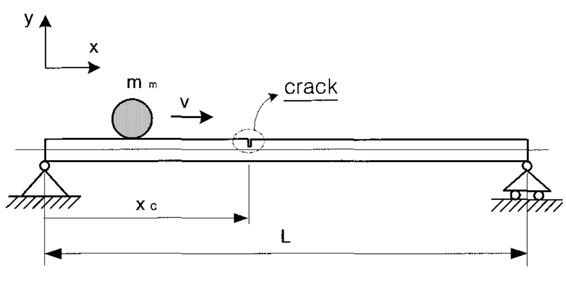

Fig. 1 Geometry of the cracked simply supported beam with a moving mass

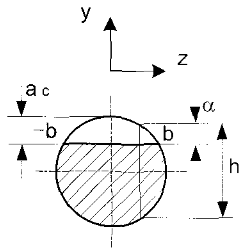

Fig. 2 Cross-section of a cracked section

$$y(x,t) = \sum_{i=1}^{\mu} \Phi_i(x)\, q_i(t) \tag{1}$$

$$\Theta(x,t) = \sum_{i=1}^{\mu} \phi_i(x)\, d_i(t) \tag{2}$$

where $q_i(t)$ and $d_i(t)$ are the generalized coordinates, which are time dependent, and μ is the total number of the generalized coordinates. $\Phi_i(x)$ and $\phi_i(x)$ are the spatial mode functions of a simply supported beam when there is no moving mass. The functions are

$$\Phi_i(x) = B_i \sin\left(\frac{i\pi x}{L}\right)$$
$$\phi_i(x) = \cos\left(\frac{i\pi x}{L}\right) \tag{3}$$

where

$$B_i = \frac{i\pi L}{(i\pi)^2 - b_n^2 s^2},\quad s^2 = \frac{EI}{\kappa AGL^2},\quad r^2 = \frac{I}{AL^2},$$

$$b_i^2 = \frac{1+(i\pi)^2(r^2+s^2)}{2r^2s^2} - \frac{\sqrt{(1+(i\pi)^2(r^2+s^2)^2)^2 - 4(i\pi)^4 r^2 s^2)^2}}{2r^2s^2}$$

where E is the Young's modulus, A is the cross-sectional area, G is the shear modulus of the beam material, and I is the moment of the inertia of the beam cross-section. In addition, $\kappa = \frac{6(1+\nu_p)}{(7+6\nu_p)}$ is a shearing coefficient[13] of the circular cross-section, and ν_p is the Poisson's ratio. In Fig. 1, the kinetic energy T_p of the beam is given by

$$T_p = \frac{1}{2}\sum_{j=1}^{2}\int_0^{L_j}\left\{\begin{bmatrix}\frac{\partial y_j}{\partial t}\\ \frac{\partial \Theta_j}{\partial t}\end{bmatrix}^T \begin{bmatrix}\rho A & 0\\ 0 & \rho I\end{bmatrix}\begin{bmatrix}\frac{\partial y_j}{\partial t}\\ \frac{\partial \Theta_j}{\partial t}\end{bmatrix}\right\}dx \tag{4}$$

where ρ is the mass density of the material. The strain energy of the cracked beam can be written as

$$V_p = \frac{1}{2}\sum_{j=1}^{2}\int_0^{L_j}\left\{\begin{bmatrix}\frac{\partial \Theta_j}{\partial x}\\ \frac{\partial y_j}{\partial x} - \Theta_j\end{bmatrix}^T \begin{bmatrix}EI & 0\\ 0 & \kappa GA\end{bmatrix} \times \begin{bmatrix}\frac{\partial \Theta_j}{\partial x}\\ \frac{\partial y_j}{\partial x} - \Theta_j\end{bmatrix}\right\}dx + \frac{1}{2}K_R(\Delta y'_c)^2 \tag{5}$$

where EI and κGA mean the bending stiffness and the shear stiffness, respectively. j is the number of the segments of the simply supported beam, K_R stands for the additional coefficient of a rotating spring due to the crack. In equation (5), the quantity

$$\Delta y'_c = \left.\frac{dy_2}{dx}\right|_{x_2=0} - \left.\frac{dy_1}{dx}\right|_{x_1=x_c} \tag{6}$$

represents the jumps in the rotation. The kinetic energy of the moving mass can be expressed as

$$T_m = \frac{1}{2} m_m \sum_{i=1}^{\mu}\sum_{j=1}^{2}\{v^2 q_i^2(t)\Phi_{ij}'^2[f(t)] + 2v q_i(t)\dot{q}_i(t)\Phi_{ij}[f(t)]\Phi_{ij}'[f(t)] + \dot{q}_i^2(t)\Phi_{ij}^2[f(t)] + v^2\} \tag{7}$$

where $(\cdot)$ denotes $\frac{\partial}{\partial t}$, and $(\,'\,)$ represents $\frac{\partial}{\partial x}$. Since the horizontal velocity of the moving mass is v, the horizontal displacement of the moving mass is

$$f(t) = x_m = \begin{cases}\int_0^t v\, dt & (0 \le x_m \le L)\\ 0 & (x_m > L)\end{cases} \tag{8}$$

2.2 Crack modeling

Consider the bending vibration of a uniform Timoshenko beam in the plane, which is assumed to be a plane of symmetry for any cross-section. The additional strain energy due to the crack leads to flexibility coefficients expressed by the stress intensity factors. In addition, the crack produces a local additional displacement u_i between the right and left section of the crack. According to Castigliano's theorem in the linear elastic range, these direction displacements u_i under the action of the force P_i are given by the following expression,

$$u_i = \frac{\partial}{\partial P_i}\int_{-b}^{b}\int_0^{a_c} J(\alpha)\, d\alpha\, dz \tag{9}$$

The local flexibility in the presence of the width $2b$ of a crack is defined by

$$C_{ij} = \frac{\partial u_i}{\partial P_j} = \frac{\partial^2}{\partial P_i \partial P_j}\left[\int_{-b}^{b}\int_0^{a_c} J(\alpha)\, d\alpha\, dz\right] \tag{10}$$

where $J(\alpha)$ is the strain energy density function. The function is

$$J = \frac{1}{E^*}(K_{IM} + K_{IP})^2 \tag{11}$$

where $E^* = E/(1-\nu_p^2)$ for the plane strain and K_I is the stress intensity factor for the fracture mode I. The stress intensity factors are given by

$$K_{IP} = \frac{P}{\pi R^2}\sqrt{R^2 - z^2}\sqrt{\pi\alpha}\, F_I\left(\frac{\alpha}{h}\right)$$
$$K_{IM} = \frac{4M}{\pi R^4}\sqrt{R^2 - z^2}\sqrt{\pi\alpha}\, F_{II}\left(\frac{\alpha}{h}\right) \tag{12}$$

where K_{IP} denotes the opening-type mode by the shear force, K_{IM} represents the opening-type mode by the bending moment. R is the radius of the beam and

$$F_I\left(\frac{a}{h}\right)=\sqrt{\frac{\tan(\zeta)}{\zeta}}\,\frac{[0.752+2.02(\frac{a}{h})+0.37(1-\sin\zeta)^3]}{\cos\zeta}$$

$$F_{II}\left(\frac{a}{h}\right)=\sqrt{\frac{\tan(\zeta)}{\zeta}}\,\frac{[0.923+0.199(1-\sin\zeta)^4]}{\cos\zeta} \tag{13}$$

where $\zeta=\frac{\pi a}{2h}$. Substituting the equations (11)~(13) into the equation (10), the flexibility matrix due to the crack can be obtained. And the boundary conditions of this cracked simply supported beam are

$$\begin{aligned}&\Phi_{i1}(0)=\frac{\partial^2\Phi_{i1}(0)}{\partial x^2}=0,\quad \Phi_{i2}(L)=\frac{\partial^2\Phi_{i2}(L)}{\partial x^2}=0\\&\Phi_{i1}(x_c)=\Phi_{i2}(x_c),\quad \frac{\partial^2\Phi_{i1}(x_c)}{\partial x^2}=\frac{\partial^2\Phi_{i2}(x_c)}{\partial x^2},\\&\frac{\partial^3\Phi_{i1}(x_c)}{\partial x^3}=\frac{\partial^3\Phi_{i2}(x_c)}{\partial x^3},\\&\phi_{i2}(x_c)-\phi_{i1}(x_c)=\frac{EI}{K_R}\frac{\partial^2\Phi_{i2}(x_c)}{\partial x^2}\end{aligned} \tag{14}$$

where

$$\Phi_{ij}(x)=\begin{cases}\Phi_{i1}(x) & : (0\le x\le x_c)\\ \Phi_{i2}(x) & : (x_c\le x\le L)\end{cases}$$

$$\phi_{ij}(x)=\begin{cases}\phi_{i1}(x) & : (0\le x\le x_c)\\ \phi_{i2}(x) & : (x_c\le x\le L)\end{cases}$$

2.3 Equation of motion

From the Lagrange's equation using the above energy functions, the equation of motion of the system is obtained. The following dimensionless parameters are introduced :

$$\begin{aligned}&\xi=\frac{x}{L},\quad \tau=\frac{t}{L^2}\sqrt{\frac{EI}{m}},\quad \xi_c=\frac{x_c}{L},\\&\xi_j=\frac{x_j}{L}\ (j=1,2),\quad \beta=\frac{m_m L}{\sqrt{mEI}}\bar{v},\\&\gamma=\frac{m_m L^3}{EI}\bar{v}^2,\quad M_m=\frac{m_m}{mL},\\&K_R^*=\frac{K_R L}{EI},\quad \xi_m=\bar{v}L^2\sqrt{\frac{m}{EI}}\tau,\\&Q=\frac{\kappa GAL^2}{EI},\quad T=\frac{mL^2}{\rho I},\quad w=\frac{q}{L}\end{aligned} \tag{15}$$

where $\bar{v}$ is v/L.

Therefore, the dimensionless equations of motion in the matrix form using the equation (15) are obtained as follows :

$$\begin{aligned}&\mathbf{M}_b\ddot{\mathbf{w}}+\mathbf{C}_b\dot{\mathbf{w}}+\mathbf{K}_b\mathbf{w}=\mathbf{F}_b\mathbf{d}\\&\mathbf{M}_r\ddot{\mathbf{d}}+\mathbf{K}_r\mathbf{d}=\mathbf{F}_r\mathbf{w}\end{aligned} \tag{16}$$

where $(\cdot)$ denotes $\frac{\partial}{\partial\tau}$, and the matrices of the equation (16) can be writt en as follows :

$$\mathbf{M}_b=\sum_{i=1}^{\mu}\sum_{j=1}^{2}\left\{\int_0^{\xi_j}\Phi^2_{ij}(\xi)\,d\xi+M_m\Phi^2_{ij}(\xi_m)\right\} \tag{17-1}$$

$$\mathbf{C}_b=\sum_{i=1}^{\mu}\sum_{j=1}^{2}\left\{M_m\frac{d}{d\tau}[\Phi^2_{ij}(\xi_m)]\right\} \tag{17-2}$$

$$\begin{aligned}\mathbf{K}_b=&\sum_{i=1}^{\mu}\sum_{j=1}^{2}\Big[Q\int_0^{\xi_j}\Big\{\frac{d\Phi_{ij}(\xi)}{d\xi}\Big\}^2 d\xi\\&+\beta\frac{d^2\{\Phi_{ij}(\xi_m)\}}{d\tau\,d\xi}\Phi_{ij}(\xi_m)-\gamma\Big\{\frac{d\Phi_{ij}(\xi_m)}{d\xi}\Big\}^2\\&+\beta\frac{d}{d\tau}\{\Phi_{ij}(\xi_m)\}\frac{d}{d\xi}\{\Phi_{ij}(\xi_m)\}\\&+K_R^*\Big\{\frac{d\Phi_{i2}}{d\xi}\Big|_{\xi_2=0}-\frac{d\Phi_{i1}}{d\xi}\Big|_{\xi_1=\xi_c}\Big\}^2\Big]\end{aligned} \tag{17-3}$$

$$\mathbf{F}_b=Q\sum_{i=1}^{\mu}\sum_{j=1}^{2}\int_0^{\xi_j}\left\{\frac{d\Phi_{ij}(\xi)}{d\xi}\phi_{ij}(\xi)\right\}d\xi \tag{17-4}$$

$$\mathbf{M}_r=\sum_{i=1}^{\mu}\sum_{j=1}^{2}\int_0^{\xi_j}\phi^2_{ij}(\xi)\,d\xi \tag{18-1}$$

$$\mathbf{K}_r=T\sum_{i=1}^{\mu}\sum_{j=1}^{2}\int_0^{\xi_j}\left(\left\{\frac{d\phi_{ij}(\xi)}{d\xi}\right\}^2+Q\phi^2_{ij}(\xi)\right)d\xi \tag{18-2}$$

$$\mathbf{F}_r=QT\sum_{i=1}^{\mu}\sum_{j=1}^{2}\int_0^{\xi_j}\left\{\frac{d\Phi_{ij}(\xi)}{d\xi}\phi_{ij}(\xi)\right\}d\xi \tag{18-3}$$

2.4 Modal formulation

The equation (16) can be transformed into the following equation :

$$\{\dot{\eta}\}=[\mathbf{M}^*]\{\eta\} \tag{19}$$

where

$$[\mathbf{M}^*]=\begin{bmatrix}[\mathbf{M}]\begin{bmatrix}\mathbf{C}_b & 0\\ 0 & 0\end{bmatrix} & [\mathbf{M}]\begin{bmatrix}\mathbf{K}_b & \mathbf{F}_b\\ \mathbf{F}_r & \mathbf{K}_r\end{bmatrix}\\ [\mathbf{I}] & [0]\end{bmatrix} \tag{20}$$

$$[\mathbf{M}]=\begin{bmatrix}\mathbf{M}_b & 0\\ 0 & \mathbf{M}_r\end{bmatrix}$$

$$\{\eta\}=[\,\dot{\mathbf{w}}\ \ \dot{\mathbf{d}}\ \ \mathbf{w}\ \ \mathbf{d}\,]^T \tag{21}$$

where $\mathbf{I}$ represents a unit matrix. For the complex modal analysis, it is assumed that η is a harmonic function of τ expressed as

$$\eta=e^{\lambda\tau}\Theta \tag{22}$$

where λ is the eigenvalue, and Θ is the corresponding mode shape. From the eigenvalues obtained form the Eqs. (19)~(22), the frequencies of the beam can be obtained.

3. Numerical Results and Discussion

In this study, the dynamic behaviors of the cracked simply supported beam influenced by the moving mass, the crack severity, and the position of a crack were computed by the forth order Runge-Kutta method.

The simply supported beam under analysis had the following properties : the length of the beam $L = 0.8$ m, radius of the beam $R = 0.1$ m, Poisson's ratio $v_p = 0.3$, Young's modulus $E = 2.1\times1011$ Pa and material density $\rho = 7860$ kg/m3. The numerical results for the mid-span deflection and frequencies of the beam were obtained for the first mode of vibration.

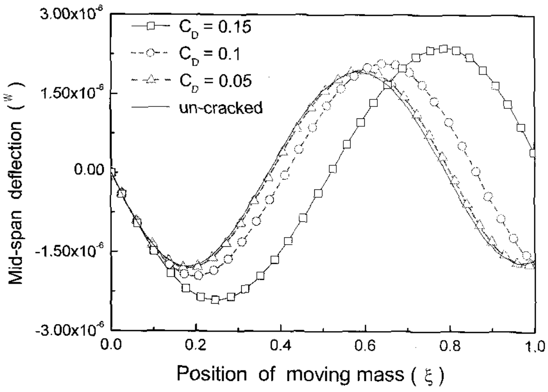

Fig. 3(a) Mid-span deflection of the cracked beam with moving mass (M_m= 0.3, ξ_c = 3/8, v = 0.8 m/s)

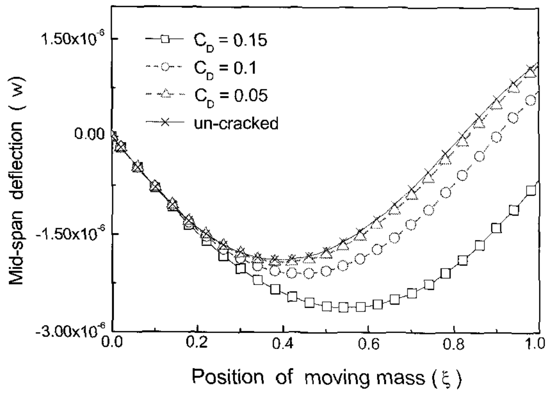

Fig. 3(b) Mid-span deflection of the cracked beam with moving mass(M_m = 0.3, ξ_c = 3/8, v = 1.6 m/s)

3.1 Results for mid-span deflection

Fig. 3 shows the dimensionless mid-span deflection of a cracked beam with moving mass M_m = 0.3 and the crack position ξ_c = 3/8. In Fig. 3, the horizontal axis scale was the position of the moving mass and the axis of the ordinates were the scale of the mid-span deflection of the beam. Figs. 3(a) and (b) show the mid-span deflection of a cracked beam when the velocity of a moving mass v was in the two cases of 0.8 m/s and 1.6 m/s, respectively. Generally, the mid-span deflection of a simply supported beam is proportional to the crack depth. As the crack depth increases, the position of the moving mass that makes the maximum mid-span deflection of the simply supported beam was moved to the rear bound of the beam. In Fig. 3(a), the difference of maximum mid-span deflection of the beam in the two cases of crack depth C_D = 0.05 and C_D = 0.10 was about 6.12 %. In Fig. 3(b), the difference of maximum mid-span deflection of the beam in the two case of crack depth C_D = 0.05 and C_D = 0.10 was about 8.37 %.

Fig. 4 represents the variation of the mid-span deflection of a cracked beam with a moving mass according to the crack positions for v = 0.8 m/s and C_D = 0.1. These results mean that when the crack position was 0.5 its effect was the largest on the mid-span deflection of the beam.

Fig. 5 makes a comparison between mid-span deflection of Euler-Bernoulli beam and Timoshenko beam for the effect of a moving mass.

3.2 Results for frequency

Fig. 6 and Fig. 7 show the frequencies of the cracked beam with a moving mass. In Fig. 6, the crack position ξ_c was 3/8 and the velocity of a moving mass v was 0.8 m/s. As shown in this figure, the frequencies of the simply supported beam were in inverse proportion to the

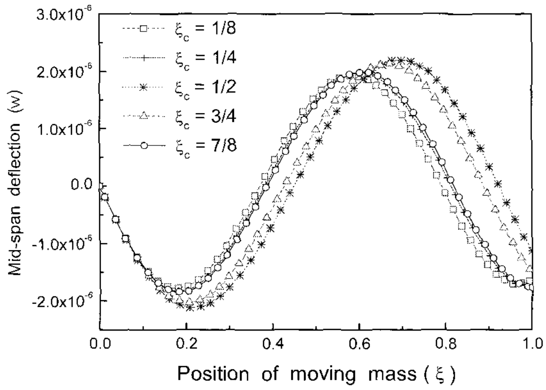

Fig. 4 Mid-span deflection of the cracked beam according to the crack position (C_D = 0.1)

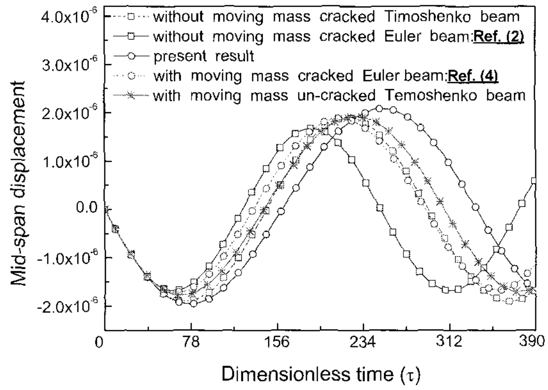

Fig. 5 Comparison of the mid-span deflection of the beams

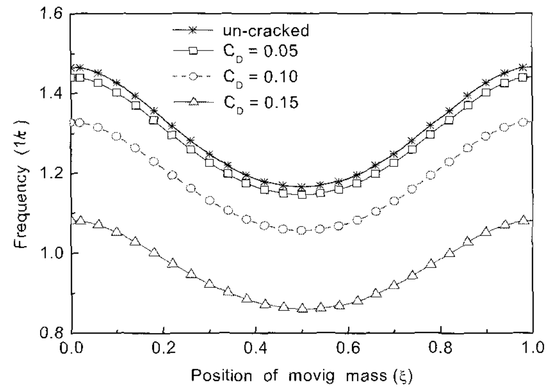

Fig. 6 Frequency vs. position of moving mass ; variation of crack depth (M_m= 0.3, ξ_c = 3/8 : 1st mode)

crack depth. Fig. 7 represents the frequencies of the cracked beam with a moving mass according to the variation of the crack position. When the crack position existed in the center of the simply supported beam, the frequency had the smallest value. The difference of frequencies of the cracked beam in the two cases of ξ_c = 1/8 and ξ_c = 7/8 was about 4.12 %.

Fig. 8 compares the frequencies of Euler-Bernoulli beam and Timoshenko beam for the first mode and the second mode. When the beam had no moving mass, the frequency was the natural frequency of the beam. When the Timoshenko beam had the crack and the moving mass, the frequency had the smallest value.

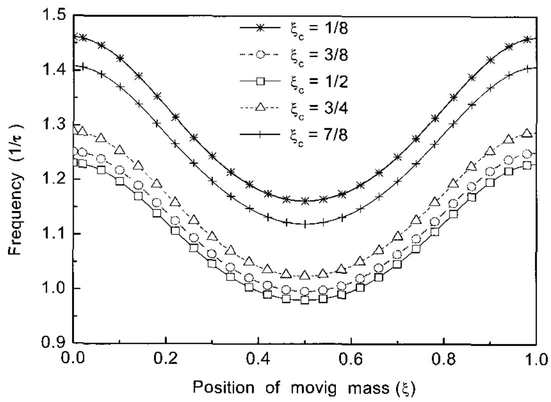

Fig. 7 Frequency vs. position of moving mass ; variation of crack position (M_m= 0.3, C_D = 0.1 : 1st mode)

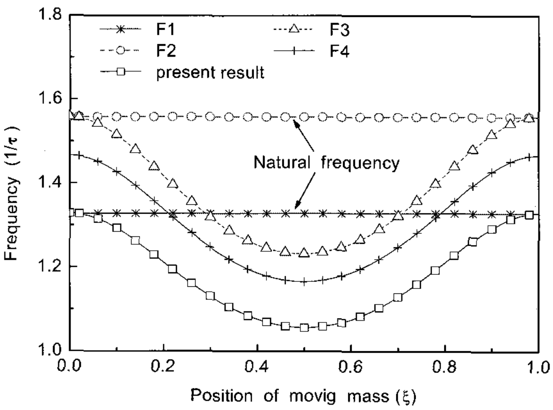

Fig. 8(a) Frequency vs. position of moving mass : (F1 ; without moving mass cracked Timoshenko beam, F2 (Ref. 2) ; without moving mass cracked Euler beam, F3 (Ref. 4) ; with moving mass cracked Euler beam, F4 ; with moving mass un-cracked Timoshenko beam) : 1st mode

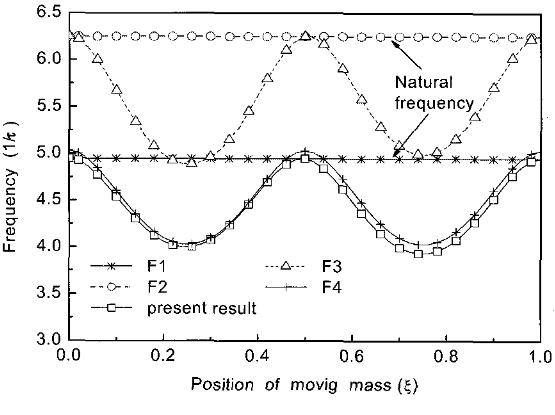

Fig. 8(b) Frequency vs. position of moving mass : (F1 ; without moving mass cracked Timoshenko beam, F2 (Ref. 2) ; without moving mass cracked Euler beam, F3 (Ref. 4) ; with moving mass cracked Euler beam, F4 ; with moving mass un-cracked Timoshenko beam) : 2nd mode

As shown in this study, the dynamic behavior of the simply supported cracked beam according to the effect of a crack had not a striking change.[14] But, these data of the Table 1 will contribute to the stability estimation of the structures of a cracked beam with a moving mass.

Table 1 The natural frequencies of the cracked Timoshenko beam

Case No.	Crack depth (C_D)	Crack position (ξ_c)	Natural frequencies (1/τ) 1st	2nd	3rd
1	0.05	1/8	1.464	5.021	9.520
		3/8	1.428	4.988	9.482
		1/2	1.420	**5.025**	9.513
		3/4	1.439	4.926	9.450
2	0.1	1/8	1.461	4.997	9.518
		3/8	1.251	4.868	9.441
		1/2	1.230	**5.022**	9.521
		3/4	1.288	4.542	9.389
3	0.15	1/8	1.439	5.002	9.511
		3/8	0.7980	4.812	9.435
		1/2	0.5769	**5.023**	9.414
		3/4	1.047	4.540	9.356
4	Uncracked beam		1.466	**5.027**	9.623

In Table 1, we will identify that the natural frequencies of the cracked beam are almost equal values in the second mode at the crack position ξ_c = 1/2. The result is the influence of the mode shape of the simply supported beam.

4. Conclusions

In this paper, the influences of the crack severity and moving mass have been studied on the dynamic behavior of the cracked simply supported beam by the numerical method. The cracked beam was treated as two undamaged segments connected by a rotational elastic spring at the crack section. When the velocity of the moving mass was constant, the influences of the moving mass, the crack severity, the position of the crack, and the coupling of these factors on the frequencies and mid-span deflection of the simply supported Timoshenko beam were depicted. The main conclusions are the following.

(1) When the moving mass is constant, the mid-span deflection of cracked beam is proportional to the crack depth.

(2) As the crack depth increases, the position of the moving mass that makes the maximum mid-span deflection of the beam is moved to the rear bound of the beam.

(3) When the crack position is 0.5, its effect is the largest on the mid-span deflection of the cracked simply supported beam.

(4) When the crack position exists in the center of the beam, its frequency has the smallest value. And totally, the frequencies of the beam are in inverse proportion to the crack depth.

(5) When the crack position exists at the node of the each mode of vibration, the characteristics of the first mode of vibration is very important to estimate the stability of a crack beam.

REFERENCES

1. Ruotolo, R., Surace, C., P. and Storer, D., "Harmonic Analysis of the Vibration of a Cantilevered Beam with a Closing Crack," Computers & Structures, Vol. 61, No. 6, pp. 1057-1074, 1996.
2. Ghondros, T. G., Dimarogonas, A. D. and Yao, J., "A Continuous Cracked Beam Vibration Theory," Journal of Sound and Vibration, Vol. 215, No. 1, pp. 17-34, 1998.

3. Lee, H. P., "The Dynamic Response of a Timoshenko Beam Subjected to a Moving Mass," Journal of Sound and Vibration, Vol. 198, No. 2, pp. 249-256, 1996.

4. Yoon, H. I., Lee, Y. W. and Son, I. S., "Influence of Crack on Dynamic Behavior of Simply Supported Beam with Moving Mass," Transactions of the KSNVE, Vol. 13, No. 9, pp. 720-729, 2003.

5. Bamnios, Y., Douka, E. and Trochidis, A., "Crack Identification in Beam Structures Using Mechanical Impedance," Journal of Sound and Vibration, Vol. 256, No. 2, pp. 284-297, 2002.

6. Zheng, D. Y. and Fan, S. C., "Natural Frequency Changes of a Cracked Timoshenko Beam by Modified Fourier Serics," Journal of Sound and Vibration, Vol. 246, No. 2, pp. 297-317, 2001.

7. Hong, S. W., Kim, M. D. and Lee, J. W., "Dynamic Modeling and Analysis of Beam Structures with Cracks," Journal of the Korean Society of Precision Engineering, Vol. 20, No. 6, pp. 197-204, 2003.

8. Kim, K. H. and Kim, J. H., "Effect of a Crack on the Dynamic Stability of a Free-free Beam Subjected to a Follower Force," Journal of Sound and Vibration, Vol. 233, No. 1, pp. 119-135, 2000.

9. Viola, E., Federici, L. and Nobile, L., "Detection of Crack Location Using Cracked Beam Element Method for Structural Analysis," Theoretical and Applied Fracture Mechanics, Vol. 36, pp. 23-35, 2001.

10. Tsai, T. C. and Wang, Y. Z., "The Vibration of a Multi-Crack Rotor," Int. Journal of Mech. Sci., Vol. 36, pp. 1037-1053, 1997.

11. Mahmoud, M. A. and Abou Zaid, M. A., "Dynamic Response of a Beam with a Crack Subject to a Moving Mass," Journal of Sound and Vibration, Vol. 256, No. 4, pp. 591-603, 2002.

12. Chondros, T. G. and Dimarogonas, A. D., "Identification of Crack in Welded Joints of Complex Structures," Journal of Sound and Vibration, Vol. 150, pp. 191-201, 1980.

13. Kgor, A. K. and Olga, I. L., "Formulas for Structural Dynamics," McGraw-Hill, 2001.

14. Krawczuk, M., Palacz, M. and Ostachowicz, W., "The Dynamic Analysis of a Cracked Timoshenko Beam by the Spectral Element Method," Journal of Sound and Vibration, Vol. 264, pp. 1139-1153, 2003.