Application of Collaborative Optimization Using Genetic Algorithm and Response Surface Method to an Aircraft Wing Design

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Collaborative optimization (*CO*) is a multi-level decomposed methodology for a large-scale multidisciplinary design optimization (*MDO*). *CO* is known to have computational and organizational advantages. Its decomposed architecture removes a necessity of direct communication among disciplines, guaranteeing their autonomy. However, *CO* has several problems at convergence characteristics and computation time. In this study, such features are discussed and some suggestions are made to improve the performance of *CO*. Only for the system level optimization, genetic algorithm is used and gradient-based method is used for subspace optimizers. Moreover, response surface models are replaced as analyses in subspaces. In this manner, *CO* is applied to aero-structural design problems of the aircraft wing and its results are compared with the multidisciplinary feasible (*MDF*) method and the original *CO*. Through these results, it is verified that the suggested approach improves convergence characteristics and offers a proper solution.

Key Words : Multidisciplinary Design Optimization, Collaborative Optimization, Genetic Algorithm, Response Surface Method, Wing Design

Nomenclature

Roman Symbols

- c : Subspace constraints
- F : System objective function
- g : Subspace objective function
- L : Lift
- L/D : Lift to drag ratio
- M : Number of subspace
- N : Number of design variable
- SFC : Specific fuel consumption
- V : Cruise velocity
- x : Subspace design variables
- $\bar{x}$: Domain-specific design variables
- y : Subspace responses
- z : Design variables of the system level

Subscript Symbols

- C_D : Drag coefficient
- C_L : Lift coefficient
- d_{tip} : Displacement at wing tip
- m_j : Number of constraints in analysis-block j
- n_j : Number of design variables in analysis-block j
- W_f : Aircraft weight after finishing its mission
- W_{fuel} : Fuel weight
- W_i : Initial aircraft weight
- W_{wing} : Semi-wing weight

Superscript Symbols

- g^* : Interdisciplinary compatibility constraints
- h' : Number of interdisciplinary inputs
- h'' : Number of interdisciplinary outputs

1. Introduction

Recently, the multidisciplinary design optimi-

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zation (MDO) has been received great attention as a method of the system optimization and the integration in many areas of industry as well as aerospace field. At the early stage of MDO development, there were many problems in MDO itself because of its computational and organizational difficulties. Even though MDO has gained large potentials as computer technology and many methodologies for MDO have been developed and researched, it is difficult to apply them to practical design problems composed of several disciplines or many design variables such as large-scale aircraft design.

The reason of this fact is a large amount of data communication required by disciplines during the optimization process. If high fidelity analyses like Euler equations or finite element method (FEM) are used to guarantee accuracy, the cost of analyses and data communication is increased. And then, the computational cost for the optimization design would be very expensive. Furthermore, MDO methods with multidisciplinary analysis (MDA) reduce the degree of freedom that each disciplinary expert has in the design process. It deteriorates the performance of outcomes designed.

To lessen such problems, this research deals with one of multi-level optimization : collaborative optimization (CO), which is a tool for a large-scale MDO. As each discipline improves performances independently without much data communication with other disciplines, this optimization tool has strong points at both of computational and organizational aspects. However, some problems regarding convergence and computation time have been reported (Alexandrov et al., 2000 ; Braun, 1996 ; Braun et al., 1996a ; Kroo and Manning, 2000).

First of all, the multi-level structure of CO disturbs the optimizer of the system level in finding a feasible solution. Because non-linear constraints of the system level work tightly during the optimization process, it is often difficult to obtain a feasible solution along an initial condition. Therefore, researches have progressed about the use of genetic algorithm that is not dependent on an initial condition (Ghim et al., 2002 ; Ghim, 2003).

Secondly, since the optimization is performed in the system level and the subspace level, the number of design variables and constraints is more than the conventional MDO. They interfere with not only finding a proper solution, but also converging efficiently. If the optimization is performed with high fidelity analyses, this disadvantage grows heavier. So, it has been researched that response surface models are replaced as subspaces including optimization and analyses (Sobieski et al., 2000 ; Jeon, 2001 ; Jun et al., 2003 ; 2004 ; Jeon et al., 2004 ; Jang et al., 2005).

Accordingly, this study will be tough upon the characteristics, the architecture and defects of CO and then discuss the reason to use genetic algorithm only for the optimizer of the system level and to replace response surface models as analyses in subspaces. The suggested CO like this will be applied to MDO problems of aircraft wing and its results will be compared with the conventional MDO method. Through examples, we will investigate that the suggested approach enhances convergence characteristics with offering a proper solution and that it is possible to apply it to the real problem.

2. Collaborative Optimization

2.1 Characteristics and architecture

CO is designed to tackle the large-scale, distributed-analysis applications often found in industry. It is a two-level hierarchical scheme, made up of upper system level and lower subspace (Kodiyalam, 1998).

Since CO is owing to its bi-level and distributed structure, it has computational and organizational advantages. These can be maximized when CO is applied to a large-scale design optimization consisting of a great number of design variables and disciplines. However, several problems like inefficient convergence result from the interdisciplinary compatibility constraints and the bi-level structure of CO.

On the other hand, CO is formulated to remove direct communication among disciplines so as to guarantee disciplinary autonomy. Dissatisfaction among each discipline is minimized by an introduction of system target values that have to be matched through the subspace optimizations. Therefore, CO cannot only solve an optimization problem without requiring direct communication among disciplines, but also provide autonomy to them (Braun, 1996; Braun et al, 1996b). Computationally, the decrease of direct communication among disciplines makes possible the reduction of the computational cost, especially in large-scale problems that deal with large amount of data. Design freedom can be also achieved in each discipline, as each subspace makes the domain-specific decisions through the subspace optimization. In addition, the analysis of each discipline can be directly integrated with a specific optimization algorithm without much modification. Organizationally, the architecture of CO provides a natural fit to the current disciplinary expertise structure found in most design organizations and used by most project teams. It also provides a coarse-grained modularity such that individual groups may alter a piece of the system without necessarily invalidating domain-specific decisions that other groups have already made.

As sketched in Fig. 1, CO is posed in a two-level hierarchic structure. The top level is a system optimizer that finds the multidisciplinary variables (the system level targets, z) to satisfy the interdisciplinary compatibility constraints (g^*) while the system objective (F) is minimized. The system level optimization is represented as follows.

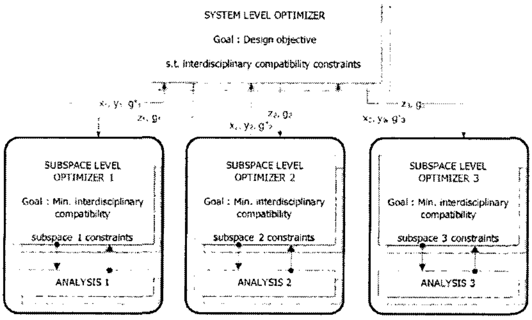

Fig. 1 Collaborative optimization architecture

$$
\begin{aligned}
&\min \quad F(z) \\
&\text{s.t.} \quad g_j^*(z) = 0 \quad j=1, M \\
&\qquad z_{\min_i} \leq z_i \leq z_{\max_i}
\end{aligned} \tag{1}
$$

The system level constraint (g^*) is obtained from the optimal solution of a set of M subspace problems. This interdisciplinary compatibility constraint is designed to drive the discrepancy among the disciplinary inputs and outputs to zero.

In the meantime, each subspace optimizer minimizes the subspace objective function (g) that is the interdisciplinary compatibility constraint in the system level during satisfying the subspace constraints (c). The system level targets (z) may be split into M non-mutually disjoint segments of length (h_j). These elements may then be partitioned into the interdisciplinary inputs of analysis-block j (h'_j) and interdisciplinary outputs computed in analysis-block j (h''_j). Each subspace optimization problem may be expressed as,

$$
\begin{aligned}
&\min \ g_j(\bar{x}, x) = \sum_{i=1}^{h'_j} (x_{ij} - z_i)^2 + \sum_{i=1+h}^{n_j} (y_{ij} - z_i)^2 \\
&\text{s.t.} \quad c_i(\bar{x}, x) \geq 0 \qquad i=1, m_j \\
&\qquad x_{\min_k} \leq x_k \leq x_{\max_k} \quad k=1, h'_j \\
&\qquad \bar{x}_{\min_l} \leq \bar{x}_l \leq \bar{x}_{\max_l}
\end{aligned} \tag{2}
$$

where,

g : Subspace objective function
z : Fixed parameter vector specified by system level optimization, length h_j
x : Interdisciplinary subset of subspace design variable vector, length h'_j
$\bar{x}$: Disciplinary subset of subspace design variable vector, length $n_j - h'_j$
c : Subspace nonlinear constraint vector, length m_j
y : Subspace interdisciplinary output vector, $y(\bar{x}, x)$, length h''_j

Only a subset of the subspace design variables is represented in the subspace objective function. As a result, analyses with less interdisciplinary coupling have increased freedom in satisfying the analysis-block constraints. Additionally, the system level targets appear as parameters in the subspace optimization problem. Hence, as in the original problem statement, the analysis-block constraints are explicitly dependent on the subspace design variables, $(\bar{x}, x)$, only.

The collaborative solution process begins with an initial set of the system level design variables (z_0). Here, the subscript refers to the zeroth system level iteration. These variables are sent to the subspace optimization problems and treated as a set of fixed parameters referred to as the system level targets. The subspace optimization problem is then solved in which the corresponding subspace interdisciplinary design variables (x_0) and interdisciplinary outputs (y_0) move as close as possible to their respective targets (z_0) while satisfying the sub-problem constraints (c). Optimum values of the subspace objective functions (g_0^*) return to the system level where a new set of system level design variables (z_1) is selected. This process is repeated until z reaches the optimum (Braun, 1996).

2.2 Defects

CO often leads to inefficient convergence, especially when gradient-based method is used for the system level optimization. This problem of *CO* occurs mainly due to an initial condition and the interdisciplinary compatibility constraints at the system level. Specially, these constraints take quadratic forms and make changes in the system targets near the solution have little effect on the constraint values (Kroo and Manning, 2000).

The system level compatibility constraints are equality ones and this fact also leads to poor convergence, especially for *SQP* method. *SQP* method uses a linear approximation of the constraint, and a maximum step size is chosen as the line search, which will not exceed the bound of the linearized constraint. The step can be too short to meet the true quadratic constraint (Sobieski et al., 2000). Therefore, these equality constraints are generally changed into inequality constraints of $g_j^* \leq 0.0001$, and they are used in this research.

CO requires large amount of computation time because extra design variables are required for the interdisciplinary inputs and outputs as well as the target values when *CO* is formulated. *CO* also requires many times of system iteration to get the optimum. If a great deal of function calls are required at the subspace level, the total computation cost can be very large.

In addition to these defects, *CO* results can sometimes be inaccurate. Because the system level objective is treated like the other interdisciplinary variables, it looses some of its influence in the optimization process. Although this loss of influence may be mitigated with a scaling procedure that forces more stringent compatibility of the objective, finding a proper scaling factor will not be an easy job.

3. Complements to CO Performance

Generally, when it is optimized in one subspace, it is required for the function call $(O(N))$ to decide the step-size and the sensitivity of performances with respect to design variables. Since *CO* has at least two or over subspaces, as many as the number of subspaces (M), the function call exponentially increase $(O(N^M))$. Therefore, if analyses are performed as many as the function call, the analyses cost increases as the order of $O(N^M)$.

On the other hand, the approximated model such as response surface models can reduces this cost because analyses are replaced with the formulation of simple functions. Since response surface models often represent as a quadratic polynomial function, their coefficients are decided by the numerical experiments at the minimum $(N+1)(N+2)/2$. If the design of experiment theory like central-composite design and D-optimal experimental design is used, then the cost of analysis becomes the order of $O(N^2)$.

Though the order of the function call is same for above two cases, the order of the analysis cost is that using response surface model is smaller than not using. Hence, if analyses in subspaces are substituted with response surface models, it is possible to drop the computational cost from $O(N^M)$ to $O(N^2)$. Also, it can be expected the effect that an optimum is found quickly by eliminating noise during the optimization process.

To make the design problem more robust, genetic algorithm (GA) is used in place of gradient-based method. As GA does not depend on a gradient information and an initial condition, problems of CO caused by gradient-dependency may be solved. But this will increase computation time, as many design points have to be selected for GA operation. Moreover, a great number of function calls in the subspace level and system iteration will drastically increase the computation time. These facts require GA to be used in a limited way. First, in the system level, it is good to use GA since the objective function and constraints are non-linear. In the subspace level, however, the use of GA is meaningless because analyses are replaced with response surface models that are convex or concave. Consequently, GA is assigned only for the system level optimizer whereas SQP is still used at the subspace level.

To speed up the optimization at the system level, the optimization problem is reformulated by drawing on penalty functions instead of the constrained optimization problem, and the penalty factor changes from small value to large one. Also warm-start, suggested by Braun (1996), is used to reach the optimum more easily and quickly. At the last of each iteration level, optimum points are used for starting points of the subspace optimization. It makes the problem converge more quickly.

4. Transport Wing Design

4.1 Definition

The wing of aircraft is one of the most important components that have it fly in the air and hold out severely structural load. The planform of the wing has dominant influence on performances of the airplane. Therefore, the wing design is not only the kernel of the aircraft design but also the part required much effort. In this research, CO is applied to the wing design for a commercial aircraft of DC-9, considering aerodynamic and structural disciplines.

The objective is range maximization :

$$Range = \frac{V}{SFC} \cdot \frac{L}{D} \cdot \ln\left(\frac{W_i}{W_f}\right) \qquad (3)$$

Eq. (3), Brequet range equation, includes lift to drag ratio (L/D) represented the aerodynamic performance and weights $(W_i,\ W_f)$ estimated from the structure analysis. Because cruise velocity (V), specific fuel consumption (SFC) and the initial aircraft weight (W_i) are constant, L/D has to be increased and the aircraft weight after finishing its mission (W_f) must be decreased to maximize range. By the way, the reduction of W_f means that the portion of fuel weight (W_{fuel}) grows larger in the aircraft weight and the wing weight (W_{wing}) becomes smaller relatively. This fact may result in structural failure. For that reason, we choose constraints as follows.

- Lift coefficient (C_L) must be larger than the baseline
- Drag coefficient (C_D) must be smaller than the baseline
- Fuel weight (W_{fuel}) must be smaller than the baseline.

Other constraints about lift (L) and displacement at the tip of the wing (d_{tip}) are selected, too.

- Lift (L) is larger than take-off gross weight.
- Displacement at the tip of the wing (d_{tip}) is within 1% of the baseline.

This design problem is comprised of seven design variables (semi-span, sweep angle out, sweep angle in, c/c_{root} at 30% span, taper ratio, t/c at root and t/c at tip) as depicted in Fig. 2 and design space consists of parameters related to the planform of the wing as summarized in Table 1. c/c_{root} at 30% span, taper ratio, t/c at root and t/c at tip are expressed as follows.

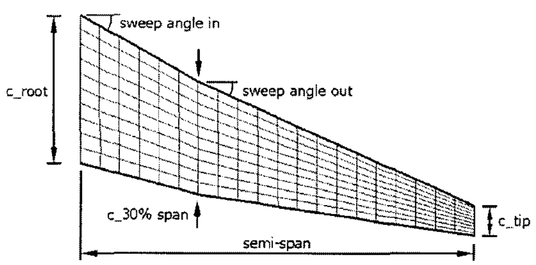

Fig. 2 Design variables of transport wing

Table 1 Design space of transport wing problem

Design Variables	Minimum	Baseline	Maximum
Semi-span (m)	12.696	14.220	15.744
Sweep angle out (deg)	19.5	24.5	29.5
Sweep angle in (deg)	23.0	30.0	37.0
c/c_{root} at 30% span	0.711	0.761	0.811
Taper ratio	0.184	0.204	0.224
t/c root	0.111	0.131	0.151
t/c tip	0.063	0.083	0.103

$$
\begin{aligned}
c/c_{root}\ \text{at 30\% span} &= \frac{\text{chord at 30\% position of semi span}}{\text{chord at wing root}} = \frac{c\_30\%\ span}{c\_root} \\
\text{taper ratio} &= \frac{\text{chord at wing tip}}{\text{chord at wing root}} = \frac{c\_tip}{c\_roor} \\
t/c\ \text{at root} &= \frac{\text{thickness at wing root}}{\text{chord at wing root}} = \frac{t\_root}{c\_root} \\
t/c\ \text{at tip} &= \frac{\text{thickness at wing tip}}{\text{chord at wing tip}} = \frac{t\_tip}{c\_tip}
\end{aligned} \tag{4}
$$

The initial value of each design variable is determined based on DC-9 specification. For this problem, the flight condition should be like following. The aircraft cruises at 7,620 m above the ground with Mach number 0.75. Angle of attack is considered to be zero and take-off gross weight is 49,000 kg.

4.2 Transform to CO formulation

As shown in Fig. 3, the transport wing design problem is formulated into two level optimization consisting of a system level and two subspaces.

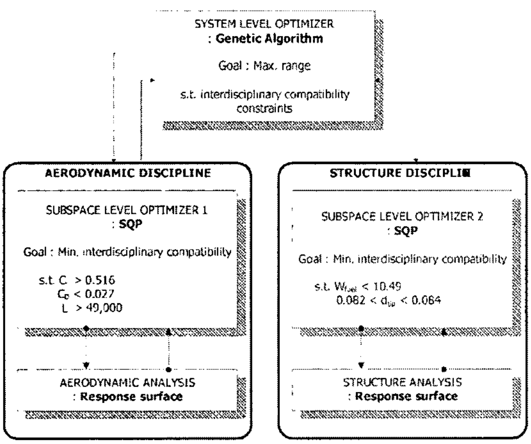

Fig. 3 Transport wing design in collaborative optimization

- *System level*

$$
\begin{aligned}
&\textit{Max.} \quad F(z) = Range \\
&\textit{subject to} \quad \sum_i (x_{i1}-z_i)^2 + \sum_j (y_{j1}-z_j)^2 = 0 \\
&\qquad\qquad (\textit{aerodynamic compatibility}) \\
&\qquad\qquad \sum_i (x_{i2}-z_i)^2 + \sum_j (y_{j2}-z_j)^2 = 0 \\
&\qquad\qquad (\textit{strucural compatibility}) \\
&\textit{where, } z = [\textit{seven design variables, } C_L, C_D, L, W_{fuel}, d_{tip}] \\
&\qquad x_1 = [\textit{seven design variables}],\ y_1 = [C_L, C_D, L] \\
&\qquad x_2 = [\textit{seven design variables}],\ y_2 = [W_{fuel}, d_{tip}]
\end{aligned} \tag{5}
$$

- *Subspace level : Aerodynamics*

$$
\begin{aligned}
&\textit{Max.} \quad g_1(x_1, y_1) = \sum_i (x_{i1}-z_i)^2 + \sum_j (y_{j1}-z_j)^2 \\
&\textit{subject to} \quad C_L \geq C_{L,baseline} \\
&\qquad\qquad C_D \leq C_{D,baseline} \\
&\qquad\qquad L \geq \textit{take-off gross weight} \\
&\textit{where, } z = [\textit{seven design variables, } C_L, C_D, L] \\
&\qquad x_1 = [\textit{seven design variables}],\ y_1 = [C_L, C_D, L]
\end{aligned} \tag{6}
$$

- *Subspace level : Structure*

$$
\begin{aligned}
&\textit{Max.} \quad g_2(x_2, y_2) = \sum_i (x_{i2}-z_i)^2 + \sum_j (y_{j2}-z_j)^2 \\
&\textit{subject to} \quad W_{fuel} \leq W_{fuel,baseline} \\
&\qquad\qquad |d_{tip}| \leq 0.01 \times d_{tip,baseline} \\
&\textit{where, } z = [\textit{seven design variables, } W_{fuel}, d_{tip}] \\
&\qquad x_2 = [\textit{seven design variables}],\ y_2 = [W_{fuel}, d_{tip}]
\end{aligned} \tag{7}
$$

The system level maximizes range subject to the compatibility conditions, and sends each subspace the target values (z) including seven design variables and the interdisciplinary outputs (C_L, C_D and L to aerodynamic subspace, W_{fuel} and d_{tip} to structural subspace). In this process, x_1, x_2, y_1 and y_2 are treated as fixed parameters. After each subspace receives target values from the system level, aerodynamic subspace finds x_1 and y_1 to minimize the interdisciplinary compatibility (g_1) with satisfying C_L, C_D and L constraints. Concurrently, structural subspace also searches for x_2 and y_2 to minimize itself objective function (g_2) subject to W_{fuel} and d_{tip} conditions. And then, subspaces return x_1, x_2, y_1 and y_2 to the system level. Repeated this process and finished the optimization, the target values (z) in the system level agree to design variables (x) and res-

Table 2 Results of regression analysis for response surface models. (Transport wing)

	Range	C_L	C_D	Lift	W_{fuel}	d_{tip}
R^2	0.9994	0.9999	0.9991	0.9999	0.9999	0.9999
R^2_{adj}	0.9992	0.9999	0.9987	0.9999	0.9999	0.9999
RMS	0.0064	0.0001	0.0098	0.0003	0.0002	0.0001

ponses (y) in the subspace level because of the compatibility constraints in the system level.

4.3 Disciplinary analyses

For disciplinary analyses, vortex lattice method (VLM) is used for aerodynamic analysis and Wing-box modeling for structural analysis. The original code is decomposed along aerodynamic and structural disciplines. Here, Weissinger method is applied as a VLM, in which aerodynamic force is computed from the planar geometry of the lift surface created by the superposition of vortex filaments, and trapezoidal vortex ring is distributed on the lift surface to consider the effect of mean camber line of the wing section. In addition, Prandtl-Glauert rule is used, under the assumption of small disturbance, to enable consideration of compressibility. Induced drag, skin-friction drag, profile drag and wave drag are considered as to compute total drag. Induced drag is computed by Treffz Plane analysis, profile drag by empirical equation and wave drag by Crest-Critical Mach number method. Besides, the wing structure is modeled by 20 segments in a direction of span. Based on the fact that the leading edge and the trailing edge take a little role in transferring the load from the wing to the fuselage, the wing-box endures main load applied to the wing. Upper and lower skin, spar and rib consist of the wing-box. More details are given in Yoon's research (Yoon et al., 1999).

4.4 Construction of response surface models and regression analysis

In this design problem, 144 experimental points are selected by central-composite design and second order full polynomial regression model is used to build the response surface models for Range, C_L, C_D, L/D, W_{fuel} and d_{tip}. For the validation of response surface models constructed, R^2 and R^2_{adj} and Root Mean Square (RMS) are estimated and summarized in Table 2. R^2_{adj} is more than 0.99 for all response surface models, which shows that response surface models catch the characteristics of the design space.

4.5 Optimization results of the transonic wing

As CO using GA and response surface method is performed, the range increases from 3081.6 km of the baseline to 4053.6 km. Also the same design problem is solved by MDF and the original CO, to compare with their results. MDF is used and gradient-based method is applied as an optimization algorithm (SQP is used). The biggest difference between these two MDO methods is whether the problem is decomposed in line with disciplines. In implementing of the wing design by MDF, aerodynamic and structural analyses are conducted together. Results are represented in Table 3 and Fig. 4.

The optimized semi-span is longer and sweep angles are smaller than that of the baseline. Moreover, L/D is increased by about 30% in the aerodynamic discipline and, W_{wing} is grown but W_{fuel} is almost constant in Table 3 since the optimization of the structural discipline is subjected to the change of W_{fuel}. As mentioned

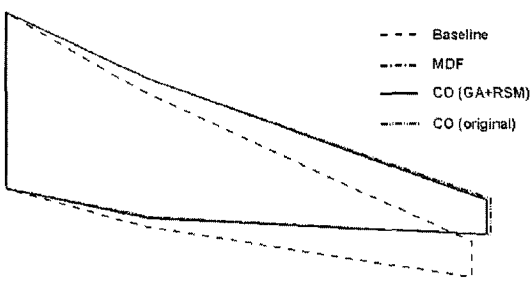

Fig. 4 Optimized wing planform. (Transport wing)

Table 3 Comparison of the Optimization Results. (Transport wing)

Design Variables	Baseline	*MDF*	CO (GA+RSM)	CO (original)
Semi-span (m)	14.220	14.660	14.678	14.781
Sweep angle out (deg)	24.5	19.5	19.6	19.5
Sweep angle in (deg)	30.0	25.2	25.0	25.1
c/c_{root} at 30% span	0.761	0.780	0.783	0.775
Taper ratio	0.204	0.205	0.204	0.205
t/c root	0.131	0.111	0.111	0.111
t/c tip	0.083	0.082	0.082	0.082
Output Values	**Baseline**	**MDF**	**CO (GA+RSM)**	**CO (original)**
L/D	18.79	24.56	24.56	24.54
W_{fuel} (ton)	10.49	10.49	10.55	10.50
W_{wing} (ton)	4.58	5.54	5.54	5.69
Range (km)	3081.6	4028.4	4053.6	4026.6
CPU time (min)		1.1	3	32

above, these results in the improvement of range.

In Table 3, results of *CO* using *GA* and response surface method have difference of about 0.7% for those of *MDF* and also same for original *CO*. Through this fact, we can confirm that *CO* using *GA* and response surface method shows good agreement of optimized results with other *MDO* methods. Besides, computing time is spent by 1/10 of conventional *CO* and comes close to *MDF*.

5. Fighter Wing Design

5.1 Definition

A fighter wing design is applied to *CO* using *GA* and response surface method as a practical design problem. The wing for a fighter aircraft of T-50 is modeled simply. As seen in the previous problem, we conduct multidisciplinary optimization consisting of aerodynamic and structural disciplines.

In general, a supersonic fighter is maneuvered at various flight conditions. As the single-point design of the wing, which considers only one flight condition like the cruise, has no significant meaning, the multi-point design should be carried out by taking into account various flight conditions. But, because this study is accomplished to validate the possibility of *CO* using *GA* and response surface method, the representative flight condition for the fighter wing and the required design objectives are carefully selected and determined as follows.

$$\begin{aligned} &Max. \quad L/D \\ &subject\ to \quad C_L \geq C_{L,baseline} \\ &\qquad\qquad\quad C_D \leq C_{D,baseline} \\ &\qquad\qquad\quad d_{tip} \leq d_{tip,baseline} \end{aligned} \tag{8}$$

The objective function in Eq. (8) means the higher L/D at the cruse speed flight condition is favorable to extend the flying range. C_L and C_D constraints are selected to meet the requirement that the aerodynamic performance of a designed wing should be at least as good as that of the baseline wing. d_{tip} constraint means that the wing tip displacement of the optimized wing must be less than that of the baseline wing, and plays a key role in the structural stability of the wing. The designed wing is structurally more stable and stiffer than the baseline wing by imposing this constraint. d_{tip} used for the structural constraint is measured at the trailing edge.

The design space in this study consists of parameters related to the planform and the structural

Table 4 Design space of fighter wing problem

Design Variables	Minimum	Baseline	Maximum
Sweep angle (deg)	30.0	35.0	40.0
Aspect ratio	2.5	3.5	4.5
Twist (deg)	0.0	−2.5	−5.0
Ref. area	1.211	1.346	1.480
Taper ratio	0.216	0.240	0.264
Thickness of root lower skin (cm)	0.0	0.24	0.52
Thickness of tip lower skin (cm)	0.0	0.24	0.52
Thickness of root upper skin (cm)	0.0	0.24	0.52
Thickness of tip upper skin (cm)	0.0	0.24	0.52

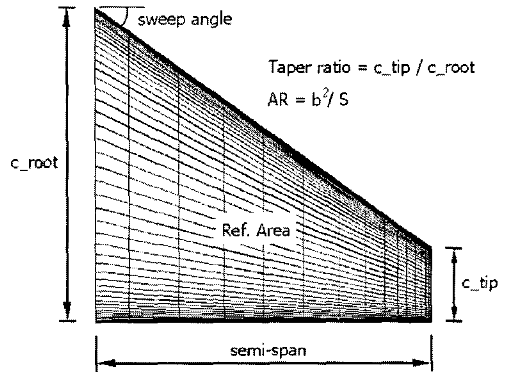

Fig. 5 Design variables of fighter wing

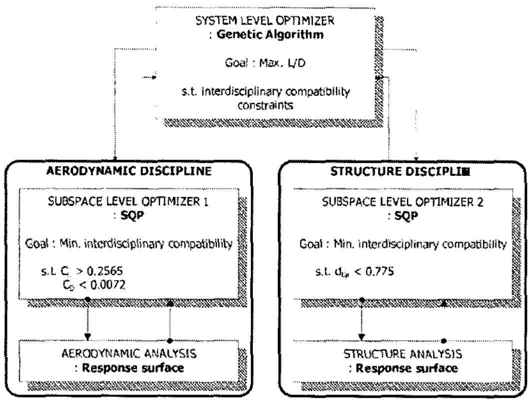

Fig. 6 Fighter wing design in collaborative optimization

skin thickness of the wing as summarized in Table 4. The sweep angle, the aspect ratio (AR), the linear twist angle, the area and the taper ratio of wing are chosen for the design variables, which determine the wing planform uniquely. Its design variables are well depicted in Fig. 5.

Four structural design variables are added to determine the upper and lower wing skin thickness. From the minimum thickness determined by the structural ultimate loading conditions, the skin thickness of the wing root and tip are increased by the amount of the structural design variables and the skin thicknesses of the intermediate region are determined by the linear interpolation between the skin thicknesses of the wing root and tip.

The number of design variables is 9 in total, and the ranges of design variables are summarized in Table 4. Here, the main wing of T-50 is selected as the baseline wing of the optimization. The aircraft cruises at 12,000 m above the ground with Mach number 0.87 and angle of attack is two degree.

5.2 Transform to CO formulation

As shown in Fig. 6, the fighter wing design problem is formulated into *CO* consisting of a system level and two subspaces.

• *System level*

$$
\begin{aligned}
&\textit{Max.} \quad F(z) = L/D \\
&\textit{subject to} \quad \sum_i (x_{i1}-z_i)^2 + \sum_j (y_{j1}-z_j)^2 = 0 \\
&\qquad\qquad (\textit{aerodynamic compatibility}) \\
&\qquad\qquad \sum_i (x_{i2}-z_i)^2 + \sum_j (y_{j2}-z_j)^2 = 0 \qquad (9) \\
&\qquad\qquad (\textit{strucural compatibility}) \\
&\textit{where, } z = [\textit{nine design variables}, C_L, C_D, d_{tip}] \\
&\qquad x_1 = [\textit{five design variables}],\ y_1 = [C_L, C_D] \\
&\qquad x_2 = [\textit{nine design variables}],\ y_2 = [d_{tip}]
\end{aligned}
$$

- *Subspace level : Aerodynamics*

$$\begin{aligned} &Max. \quad g_1(x_1, y_1) = \sum_i (x_{i1} - z_i)^2 + \sum_j (y_{j1} - z_j)^2 \\ &subject\ to \quad C_L \geq C_{L,baseline} \\ &\qquad\qquad\quad C_D \leq C_{D,baseline} \\ &where,\ z = [five\ design\ variables,\ C_L,\ C_D] \\ &\qquad x_1 = [five\ design\ variables],\ y_1 = [C_L,\ C_D] \end{aligned} \tag{10}$$

- *Subspace level : Structure*

$$\begin{aligned} &Max. \quad g_2(x_2, y_2) = \sum_i (x_{i2} - z_i)^2 + \sum_j (y_{j2} - z_j)^2 \\ &subject\ to \quad d_{tip} \leq d_{tip,baseline} \\ &where,\ z = [nine\ design\ variables,\ d_{tip}] \\ &\qquad x_2 = [nine\ design\ variables],\ y_2 = [d_{tip}] \end{aligned} \tag{11}$$

From Eqs. (9) ~ (11), one thing has to be pointed out. The aerodynamic subspace only has five design variables to prune aerodynamic analysis of irrelevant structural variables such as skin thickness, viz sweep angle, aspect ratio, twist angle, wing area and taper ratio. But the structural subspace treats nine design variables in total.

The system level maximizes L/D subject to the compatibility conditions, treating x_1, x_2, y_1 and y_2 as fixed parameters. And then, it sends the target values (z) to the aerodynamic and the structural subspace. Here, the aerodynamics subspace receives five design variables and interdisciplinary outputs (C_L and C_D) and the structural subspace takes nine design variables and d_{tip}.

After given the target values from the system level, the aerodynamic subspace is conducted to get optimum values that minimize discrepancy (g_1) between target values and design variables with satisfying C_L, and C_D constraints. Concurrently, the structural subspace also searches for x_2 and y_2 to minimize the interdisciplinary compatibility (g_2) subject to dtip conditions. Each discipline determines its own new values for the subspace design variables (x_1 and x_2) and the interdisciplinary responses (y_1 and y_2). The new values of the subspace design variables have different values according to disciplines and they are again sent to the system level to further improve the design.

This process is repeated until the design gets maximum L/D and satisfies all constraints at the same time.

5.3 Disciplinary Analyses

For disciplinary analyses, three-dimensional Euler equation for aerodynamic analysis and nine-node shell mixed finite element method for structural analysis are used. The original code is decomposed along aerodynamic and structural disciplines.

For aerodynamics, the three-dimensional Euler equation is used to calculate the transonic aerodynamic properties of the fighter wing. In this study, Van Leer's flux vector splitting is employed to calculate the Jacobian matrix and Roe's flux difference splitting to solve the flux vector. To increase the order of spatial accuracy, flux vectors on the cell interface are computed by MUSCL (Monotone Upstream-centered Schemes for Conservation Law) extrapolation scheme. To avoid the unexpected oscillation of the solution around the discontinuous flow field, MUSCL scheme is tapped with Van Albada limiter. In this regard, Beam-Warming's AF-ADI (Approximate Factorization-Alternating Direction Implicit) scheme is employed as time integration method. To accelerate the convergence of the numerical analysis and reduce the computational time, local time step, saw tooth cycle multi-grid method and the implicit residual smoothing are adopted as well. O-H type grid is used as the wing mesh for computational fluid dynamics (CFD) calculation as shown in Fig. 7. More detailed aerodynamic

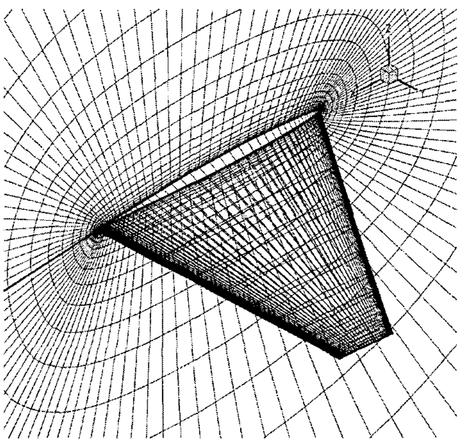

Fig. 7 Three-dimensional wing mesh for CFD calculation. (O-H type)

analysis of this study has been described on references (Kim et al., 2002a ; 2005 ; Jeon et al., 2004).

For the structural analysis of the wing, nine-node shell mixed finite element is utilized. The element has three translational degrees of freedom (*DOF*) and two rotational *DOF* per node as shown in Fig. 8 ; therefore, each element has 45 *DOF*. However, for the modeling of complicated structures such as wing boxes, the normal direction of the surface may not be continuous. In such cases, since the rotational deformation of the discontinuous surface cannot be expressed with only two rotational *DOF*s per node, "drilling degrees of freedom" is adopted for the elements (Cook et al., 1989). To combine *CFD* with computational structural mechanics (*CSM*), non-uniform bi-cubic spline composite surface method is applied to transform *CFD* mesh to *CSM* mesh as depicted in Fig. 9.

To determine the minimum structural size of the wing components, DaDT (durability and damage tolerance) allowable method is used for spar, rib, and lower skin subjected to tension forces. This method based on that the maximum principal stress of each element must not exceed the DaDT allowable stress. Also, the minimum size of the upper skin thickness is determined to withstand the buckling whose load is acquired by the analysis of an idealized equivalent rectangular panel. In the process of the multidisciplinary design, the minimum size of the structural component calculated by the above-mentioned methods is used as structural constraints (Kim et al., 2002b ; 2005).

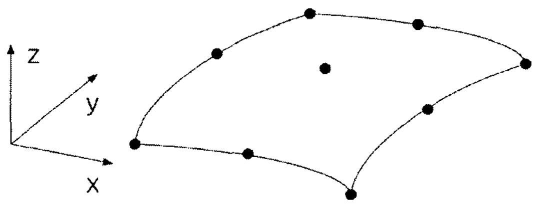

Fig. 8 Nine-node shell mixed element

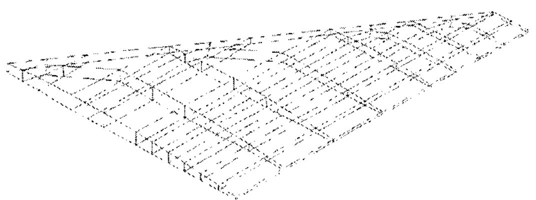

Fig. 9 CSM model of the wing

Four structural design variables selected in this work are most important design parameters because of the largest compressive and tension loading.

5.4 Construction of response surface models and regression analysis

The number of design variables is 9 in total, and approximately 64 calculations are sufficient to produce accurate response surface models. 64 experimental points are chosen through the D-optimal experimental design, and second order full polynomial regression model is used to build the response surface models for C_L, C_D, L/D and d_{tip}. R^2 and R^2_{adj} and Root Mean Square (*RMS*) are estimated and summarized in Table 5. R^2_{adj} is more than 0.97 for all response surface models, which guarantees the reliable prediction capability of the response surface models.

Table 5 Results of regression analysis for response surface models. (Fighter wing)

	C_L	$C_{D,I}$	L/D	d_{tip}
R^2	0.9994	0.9970	0.9991	0.9986
R^2_{adj}	0.9956	0.9789	0.9937	0.9904
RMS	0.0254	0.0932	0.0192	0.0867

5.5 Optimization results of the fighter wing

Similarly to the previous problem, *CO* using *GA* and response surface method is compared with *MDF*. On the other hand, if *CO* or *MDF* is performed with analyses of Euler equations and *FEM*, it is obvious that the cost is very expensive. Hence, response surface models are used as all analyses in this problem and the fighter wing is optimized. In addition, we only exhibited results of *MDF* to validate the accuracy of *CO* using *GA* and response surface method.

As shown in Table 6 and Fig. 10, L/D rises to about 26% from 35.63 of the baseline to 45.11 of the optimized shape. Besides, we can verify that results of *CO* are almost agreed to those of

Table 6 Comparison of the Optimization Results. (Fighter wing)

Design Variables	Baseline	MDF	CO (GA+RSM)
Sweep angle (deg)	35.0	40.0	40.0
Aspect ratio	3.5	4.50	4.49
Twist (deg)	−2.5	−2.861	−2.921
Ref. area	1.346	1.211	1.211
Taper ratio	0.240	0.231	0.240
Thickness of root lower skin (cm)	0.24	0.46	0.48
Thickness of tip lower skin (cm)	0.24	0.48	0.48
Thickness of root upper skin (cm)	0.24	0.44	0.44
Thickness of tip upper skin (cm)	0.24	0.51	0.51
Output Values		**MDF**	**CO (GA+RSM)**
C_L	0.2565	0.2568	0.2570
C_D	0.0072	0.0057	0.0057
d_{tip} (cm)	0.775	0.774	0.771
L/D	35.63	44.78	45.11
CPU time (min)		1.5	15

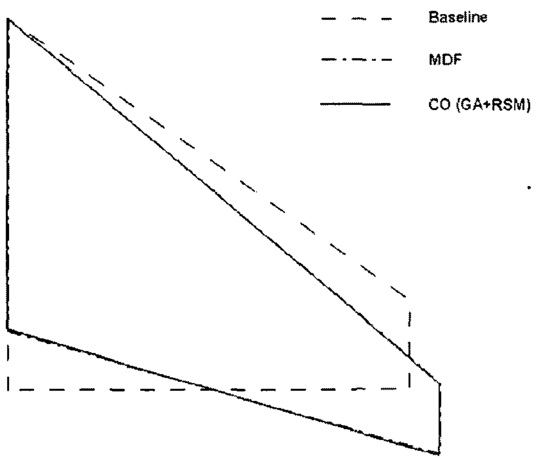

Fig. 10 Optimized wing planform. (Fighter wing)

MDF. The optimized reference area is decreased from the baseline, then resulting in the diminution of lift and drag. By the way, because the optimizer of aerodynamic discipline is subjected to the decrease of C_L, there is not loss of C_L. In comparison with *MDF* and *CO* using *GA* and response surface method, there is about 4% discrepancy for the taper ratio and the thickness of root lower skin, and within 2% difference for other variables and outputs. In other words, it is shown that *MDF* and *CO* using *GA* and response surface method have similar optimum.

6. Conclusions

In this research, aircraft wing design problems are solved using *CO* that analyses are replaced with response surface models and the use of *GA* is limited to the system level optimization. From this approach, we can verify that disciplinary autonomy and accuracy are maintained and it is also possible to converge efficiently. Based on results, the following conclusions can be drawn.

First, we could achieve aero-structural multidisciplinary optimization of the aircraft wing design with guaranteeing autonomy of subspaces. Because the system level controls the discrepancy of the interdisciplinary variables that each discipline minimizes, all disciplines don't directly communicate with others. Consequently, we could have maintained the autonomy of subspaces during carrying out the optimization design. Second, *CO* with response surface models guarantees the accuracy of optimum solution and efficient convergence. Results of *CO* using *GA* and response surface method show within 4% difference, compared with conventional *MDF* and *CO*. In CPU time, it spends about 10% of the original *CO* on the optimization process. Hence, replacing analyses with response surface models improves more efficiency of convergence than the original *CO* does. Third, since *GA* has no concern with the initial condition, the use of *GA* at the system level optimizer reduces the difficulty of the convergence caused by the interdisciplinary compatibility constraints of the system level. Finally, through the achievement of two *MDO* problems, it has been confirmed that *CO* using *GA* and response surface method might have good possibility of application in real problems. Even if *CO* using *GA* and response surface method requires more CPU time than *MDF*, it does not need the interface to connect with other analyses because it guarantees the autonomy of each analysis. Therefore, *CO* using *GA* and response surface method is available for real problems, such as a large-scale *MDO* problem.

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