

# Dynamics Modeling of a Gas Engine-Driven Heat Pump in Cooling Mode

Younggy Shin\*

Department of Mechanical Engineering, Sejong University,  
Seoul 143-747, Korea

Hooncheul Yang, Choon-Seob Tae, Cheol-Yong Jang, Soo Cho

Building Energy Research Center, Korea Institute of Energy Research,  
Daejeon 305-343, Korea

The present study has been conducted to simulate dynamics of a gas engine-driven heat pump (GHP) for design of control algorithm. The dynamics modeling of a GHP was based on conservation laws of mass and energy. For automatic control of refrigerant pressures, actuators such as engine speed, outdoor fans, coolant three-way valves and liquid injection valves were PI or P controlled. The simulation results were found to be realistic enough to apply for control algorithm design. The model can be applied to build a virtual real-time GHP system so that it interfaces with a real controller in purpose of prototyping control algorithm.

**Key Words :** GHP (Gas Engine-Driven Heat Pump), Automatic Control, Heat Pump, Realtime Simulator, PID Control, Compressor, Unloading

## Nomenclature

### Notation

$c_{pg}$  : Specific heat at constant pressure of refrigerant gas [kJ/kg °C]  
 $h$  : Enthalpy [kJ/kg]  
 $\dot{m}$  : Mass flow rate of refrigerant gas [kg/s]  
 $N$  : Engine speed [rpm]  
 $PD$  : Rate of change of displacement volume of a compressor [m<sup>3</sup>/s]  
 $p$  : Pressure [Pa]  
 $\dot{Q}$  : Volumetric flow rate [m<sup>3</sup>/s]  
 $t$  : Time [sec]  
 $T$  : Temperature [°C or K], torque [N-m]  
 $u$  : Internal energy [kJ/kg]  
 $UA$  : Overall thermal conductance [W/°C]  
 $V$  : Volume [m<sup>3</sup>]  
 $v$  : Specific volume [m<sup>3</sup>/kg]  
 $w$  : Specific work [J/kg]

### Greek symbols

$\eta_v$  : Volumetric efficiency of a compressor  
 $\eta_m$  : Mechanical efficiency of a compressor  
 $\theta$  : Rotation angle of an engine crankshaft [radian]  
 $\rho$  : Density of refrigerant [kg/m<sup>3</sup>]  
 $\tau$  : Time constant [sec]

### Subscripts

*amb* : Hot gas bypass valve  
*byb* : Hot gas bypass valve  
*cd* : Condenser  
*comp* : Compressor  
*cool* : Engine coolant  
*disp* : Engine displacement volume  
*eng* : Engine  
*ev* : Evaporator  
*f* : Saturated liquid  
*g* : Saturated vapor  
*gas* : Refrigerant in gas phase  
*i* : Inlet  
*indoor* : Indoor  
*j* : Any of indoor units  
*liq* : Liquid control valve

\* Corresponding Author,

E-mail : ygshin@sejong.ac.kr

TEL : +82-2-3408-3284; FAX : +82-2-3408-3333

Department of Mechanical Engineering, Sejong University, Seoul 143-747, Korea. (Manuscript Received August 25, 2005; Revised January 3, 2006)

*load* : Cooling load

*o* : Outlet

## 1. Introduction

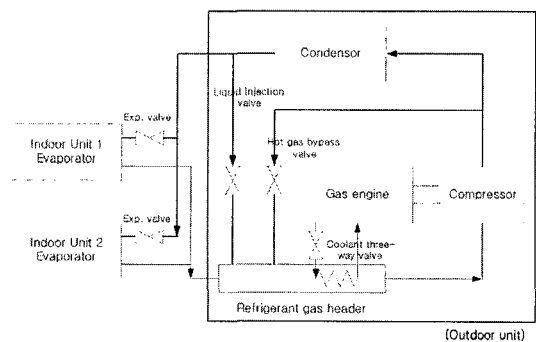
Large commercial buildings have adopted conventionally central HVAC systems which use turbo-chillers and air ducts. However, the central system has the following demerits: need of full-time mechanic staff, high maintenance cost and inability of zonal separate HVAC control. In order to overcome such shortcomings, nowadays HVAC systems are in the trend of transition from the central one to several multi-split air conditioners. They have indoor cooling units which exchange heat with an outdoor unit by controlling refrigerant flow rates through them. As the technology associated with the multi-split air conditioner system reaches maturity, its market portion will grow further. The key part of the technology is the control algorithm that makes the system run stably against rapid transitions of cooling load.

A big problem in developing such technology is that domestic companies' research capability is too limited to catch up the competitiveness of Japanese products in short period because the control technology is highly complicated and domestic companies' investment on the technology was relatively negligible. Secure establishment of GHP (gas engine-driven heat pump) control algorithm is a prerequisite to product competitiveness. Heat carrier of conventional HVAC systems is either air or water in form of its single phase and hence simple PID loops or two-position mode control sufficed for control. However, the heat carrier in GHP is the refrigerant that flows around the system undergoing phase changes for heat exchange. The control algorithm has to distribute the refrigerant in appropriate proportion among indoor units depending on cooling load and recover lubricants from stagnant locations always at the level of optimal COP. To test such control algorithm, a large and expensive thermal environment chamber has to be facilitated. However such facility can be afforded by only a couple of big companies. Thus the purpose

of the study is to suggest a dynamics model that can serve to build a real time virtual GHP system which makes it possible to test control algorithm on a desk-top environment. The study also shows an example of evaluating control performance on the developed dynamics model.

## 2. Background for GHP Control

Figure 1 shows the schematic of GHP to deal with cooling loads coming from indoor cooling units. Normal cooling load according to weather condition or indoor activity can be processed by engine speed variation. However, extraordinary operation conditions, such as sudden transition of cooling load by powering on or off indoor units, GHP start-up and system operation at very low loads, for example, 'sub-idle cooling load', which refers to the cooling load smaller than that corresponding to idle engine speed, should be assisted by auxiliary actuators, such as a liquid injection valve, a hot gas bypass valve and a coolant three-way valve, in addition to engine speed. The liquid injection valve acts as an emergency device to lower degree of overheat at the compressor inlet by injecting pressurized liquid refrigerant near the compressor inlet, which evaporates and hence cools down the refrigerant entering the compressor. The hot gas bypass valve serves as an emergency tool to regulate refrigerant pressures beyond the limits of normal pressure control. It works by bypassing pressurized hot gas back into the compressor inlet, whereby it is possible to lower excessive high pressure at the compressor



**Fig. 1** Schematic of GHP for control

exit and to increase excessive low pressure at the compressor inlet whenever needed.

The operable speed range of a gas engine is between 800 rpm and 2,400 rpm so its turn-down ratio is about 3:1. It is quite narrow compared to the least range of cooling load that is 8:1 in terms of number of indoor units attached to a GHP system. To increase the turndown ratio, loading/unloading of compressor is applied. However, when it comes to the sub-idle cooling load, the cooling capacity at idle speed is yet too big to deal with the load. It will lead to frequent starts and stops of the system. To avoid the frequent switching, artificial cooling load is generated. It is made by injecting liquid refrigerant into the gas header and supplying necessary vaporization heat by increased coolant flow rate through a coolant three-way valve to the gas header where the refrigerant exchanges heat with the coolant in a compact plate-type heat exchanger.

On the other hand, when GHP pressures deviate significantly from its set values, which is mainly caused by switching indoor units frequently, emergency control mode is activated where liquid refrigerant is injected and hot refrigerant gas is bypassed back to the inlet of compressors.

Figure 2 shows the concept of prototyping a controller interfaced with a virtual real-time GHP plant that is programmed using a dynamics model to be developed in the study. GHP is a system heavily coupled among many inputs and outputs and requires a huge and expensive calorimeter to test its control performance because of various cooling loads associated with its many indoor

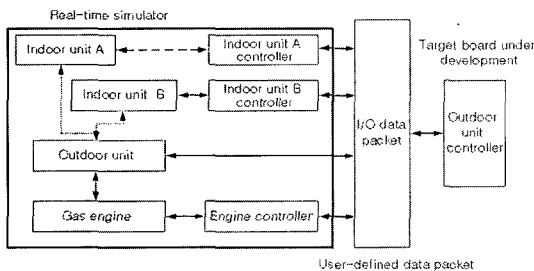


Fig. 2 Controller prototyping environment using a real-time simulator

units. To develop GHP control algorithm and prototype its controller in short time and less cost, it is necessary to develop a virtual real-time GHP plant to replace a GHP and a calorimeter. If a virtual real-time GHP plant can be realized by modeling GHP dynamics and making it run in real-time exchanging its inputs and outputs with a target controller via data communication, for example, through serial port, it will save a lot of time and cost for development of a GHP system.

### 3. Dynamics Modeling

#### 3.1 Compressor

A P-V diagram of a reciprocating compressor is shown in Fig. 3. Its volumetric efficiency and refrigerant flow rate are defined respectively in Equations (1) and (2) (McQuiston et al, 2000).

$$\eta_v = \frac{\dot{m}_{comp} v_3}{PD} \tag{1}$$

$$\dot{m}_{comp} = \left[ 1 + C - C \left( \frac{p_c}{p_b} \right)^{1/n} \right] \frac{PD}{v_b} \tag{2}$$

Here, polytropic exponent  $n$  should be determined by experiment, but it is a common practice to approximate it with the isentropic exponent  $k$ . For the refrigerant R-22,  $k$  is 1.16. The clearance coefficient  $C$  is defined as follows and its value is about 0.05.

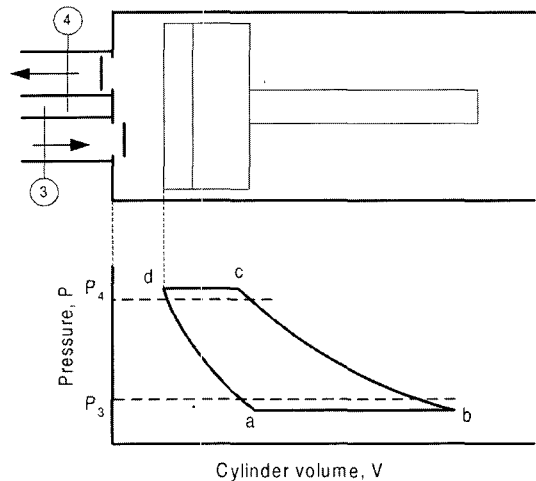


Fig. 3 P-V diagram of the compressor

$$C = \frac{V_d}{V_b - V_d} \quad (3)$$

Work per cycle and power consumed are calculated from the following relations.

$$w_{comp} = \frac{n}{(n-1)} p_b v_b \left[ \left( \frac{p_c}{p_b} \right)^{(n-1)/n} - 1 \right] \quad (4)$$

$$\dot{W} = \frac{\dot{m}_{comp} w_{comp}}{\eta_m} \quad (5)$$

### 3.2 Evaporator

Cooling load is processed by vaporizing liquid refrigerant in the evaporator and superheat of the refrigerant is regulated by an electronic expansion valve located at the exit of the evaporator. Heat balance equation in the evaporator is described as follows.

$$m_{ev} \frac{dh_{ev}}{dt} = \dot{Q}_{load} - \dot{m}_{ev,j} (h_{ev,o} - h_{ev,i}) \quad (6)$$

Cooling load in the evaporator can be expressed by multiplication of enthalpy difference between the inlet and exit of the evaporator and the vapor throughput. Since superheat of R22 is controlled within 5°C, its sensible heat is less than 1% of latent heat. So contribution of the sensible heat to the cooling load is neglected for simplicity as follows.

$$\dot{Q}_{load} = UA_{indoor} (T_{indoor} - T_{ev,sat}) \quad (7)$$

In addition, Equation (6) is rearranged to obtain a first-order differential equation with respect to the enthalpy at the evaporator exit by assuming that the internal energy in the evaporator is approximated to the enthalpy at the evaporator exit.

$$m_{ev} \frac{dh_{ev,o}}{dt} + \dot{m}_{ev,j} h_{ev,o} \approx \dot{Q}_{load} + \dot{m}_{ev,j} h_{ev,i} \quad (8)$$

Pressure build-up in the evaporator can be explained from the following consideration. Evaporation of refrigerant by cooling load increases pressure in the evaporator whereas suction of refrigerant gas into the compressor decreases the pressure. At steady state, evaporation amount by cooling load is balanced by the amount of refrigerant gas sucked into a compressor. As cooling load increases, evaporation amount increases and the pressure in the evaporator increases when

a compressor runs at fixed speed at the moment. So the compressor speed should be increased to keep the pressure constant.

### 3.3 Gas header

All of refrigerant gas leaving each indoor unit is collected in a refrigerant gas header and sucked into a compressor. As shown in Fig. 4, the gas in the course of heading toward the refrigerant gas header receives additional heat and, in some cases, additional refrigerant in gas or liquid phase from auxiliary devices, such as a liquid injection valve, a hot gas bypass valve and an engine coolant heat exchanger, to control superheat, refrigerant pressures and temperatures.

The behavior of the pressure of the superheat gas in the gas header can be approximated by the following ideal gas relation.

$$P_{ev} V_{gas} = m_{gas} R T_{gas} \quad (9)$$

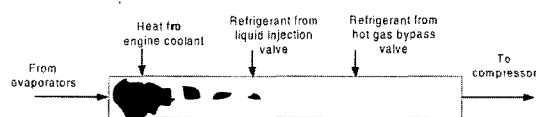
Varying cooling load changes the amount of refrigerant evaporated from each indoor unit and the intake amount by the compressor and hence changing the amount remained in the gas header, which leads to the change of the pressure in the evaporator described by the following relation.

$$\frac{dP_{ev}}{dt} = \frac{R}{V_{gas}} \left( T_{gas} \frac{dm_{gas}}{dt} + m_{gas} \frac{dT_{gas}}{dt} \right) \quad (10)$$

At that moment, the rate of change of the mass of the superheat gas accumulated in the gas header is determined by the following mass flow rates from each indoor unit, a liquid injection valve, a hot gas bypass valve and by the intake amount into the compressor.

$$\frac{dm_{gas}}{dt} = \sum_{j=1}^n \dot{m}_{ev,j} + \dot{m}_{liq} + \dot{m}_{byp} - \dot{m}_{comp} \quad (11)$$

The heat balance equation to simulate temperature of the refrigerant vapor in the gas header is as follows :



**Fig. 4** Heat and mass balance in the gas header for auxiliary control

$$m_{gas} \frac{du_{gas}}{dt} = \sum_{j=1}^n (\dot{m}h)_{ev,o,j} + \dot{Q}_{cool} + (\dot{m}h)_{liq} + (\dot{m}h)_{byp} - (\dot{m}h)_{comp,i} \quad (12)$$

The following approximation also holds :

$$h_{byp} \approx h_{comp,i} + w_{comp} \quad (12a)$$

$$h_{liq} \approx h_{comp,i} + h_{fg} \quad (12b)$$

Since the internal energy  $u_{gas}$  can be approximated to the enthalpy of the refrigerant at the compressor inlet  $h_{comp,i}$ , combining Eqs. (11)–(12b), the following differential equation with respect to  $h_{comp,i}$  can be derived.

$$m_{gas} \frac{dh_{comp,i}}{dt} \approx \sum_{j=1}^n (\dot{m}h)_{ev,o,j} + \dot{Q}_{cool} + \dot{m}_{byp} w_{comp} - \dot{m}_{liq} h_{fg} - \left( \sum_{j=1}^n \dot{m}_{ev,o,j} - \frac{dm_{gas}}{dt} \right) h_{comp,i} \quad (13)$$

The superheat at the compressor inlet can be estimated by solving Eq. (13).

### 3.4 Condenser

Application of the above argument to the compressor leads to the following relation :

$$\frac{dm_{cd,g}}{dt} = \dot{m}_{comp} - \dot{m}_{byp} - \dot{m}_{cd,f} \approx 0 \quad (14)$$

$$M_{cd Cp, steel} \frac{dT_{cd, sat}}{dt} = -\dot{Q}_{cd} + (h_{comp, out} - h_{cd, f}) \dot{m}_{cd, f} \approx 0 \quad (15)$$

$$\dot{Q}_{cd} \cong UA_{cd} (T_{cd, sat} - T_{amb}) \quad (16)$$

$$P_{cd} \approx P_{sat} = fn(T_{cd, sat}) \quad (17)$$

In Equations (14) and (15), transient terms are neglected for ease of calculation. The condensation temperature at the condenser,  $T_{cd, sat}$ , is solved from Equations (14) ~ (16). Finally, the condenser pressure is obtained from the Equation (17).

### 3.5 Expansion valve

The opening of an electronic expansion valve is determined by number of pulse steps applied to the valve stepper. An experimental result shows that the flow rate through the valve has a following linear relation with respect to number of pulse steps to a stepper (Kim et al., 1995).

$$\dot{m}_{ev,j} \propto steps \times \sqrt{\rho \Delta p} \quad (18)$$

### 3.6 Engine dynamics

Engine power is regulated by the opening angle of a throttle valve through which a mixture of air and CNG (compressed natural gas) flows. Since mixture flow rate is non-linear with respect to the opening angle of the valve due to geometric characteristics of its butterfly valve type, the following relation is assumed for simplicity.

$$\dot{m}_{max} / \dot{m}_{mix, max} = bmep / bmep_{max} = \sin \phi \quad (19)$$

$bmep$  stands for brake mean effective pressure and is defined as mean pressure applied to the piston top surface by the gas in the combustion chamber during one engine cycle (Heywood, 1988). Its magnitude is proportional to the amount of the mixture trapped in the combustion chamber and its maximum is about 850 kPa for naturally aspirated spark-ignition engines of modern passenger cars (Heywood, 1988).

$$2\pi T_{eng} = bmep V_{disp} / 2 \quad (20)$$

$$\dot{W}_{eng} = T_{eng} \theta = 2\pi T_{eng} N / 60 \quad (21)$$

The dynamic relation with respect to engine speed  $N$  with a compressor connected to the engine is as follows :

$$J_{eng} \ddot{\theta} = T_{eng} - T_{comp} - B\dot{\theta} \quad (22)$$

Here,  $J$  is the moment of inertia of the engine connected to a compressor. Laplace transformation leads to the following transfer function :

$$\frac{N(s)}{T(s)} = \frac{30/\pi}{Js+B} = \left( \frac{30}{\pi B} \right) \frac{1}{\tau_{eng}s+1} \quad (23)$$

Here,  $T$  is net torque obtained by subtracting compressor-absorbed torque from engine-generated torque. The compressor-absorbed torque is determined by circulating flow rate of the refrigerant and pressure difference across the compressor and so forth.

## 4. GHP Control Algorithm

The GHP control algorithm in cooling mode is as follows. Each indoor unit deals with its indoor cooling load by controlling superheat of

the refrigerant at its exit. The resulting flow rate through indoor units and the volumetric displacement rate (=rotational speed times displacement volume) of the compressor determine the refrigerant pressure at the inlet of the compressor. The outdoor unit controls the pressure within some specified range by regulating the volumetric displacement rate. Thus the resulting rate reflects magnitude of cooling load. A technical challenge of the heat pump with multi-split indoor units is limited variation of cooling capacity compared to the wide range of cooling load variation imposed by many indoor units. The turndown ratio of engine speed is about 3:1. To increase the turndown ratio, loading/unloading of individual compressor cylinders is also employed. In spite of such implementation, continuous GHP operation is not possible for very low cooling loads (for example, when a single indoor unit is in service at light load). In that case, the GHP might repeat starts and stops frequently to cope with such low loads and, as a result, adverse effect on system durability and poor control performance is inevitable. Thus, an idea is suggested that the system is supplied with artificial cooling load whose amount makes it run continuously. The load is generated by circulating part of hot engine coolant through the plate-type heat exchanger installed near the inlet of the compressor. The heat exchanger exchanges heat with the refrigerant and the coolant throughput is controlled by the opening angle of the coolant three-way valve. This type of artificial cooling load generation is a widely-accepted technique. Fig. 5 shows an example of control algorithm to implement such idea (Yanmar Ltd., 2003). In this study the same idea was adopted for simulation.

Since thermal dynamics of a GHP system corresponds to linear combination of first-order transfer functions of many mechanical components although they are coupled complicatedly to form the system, a system response to any control effort approaches stably to a certain steady-state value without severe excursions. Therefore, it is possible to construct a stable control system by designing a cascade-type control algorithm where fast control loops are placed as inner ones

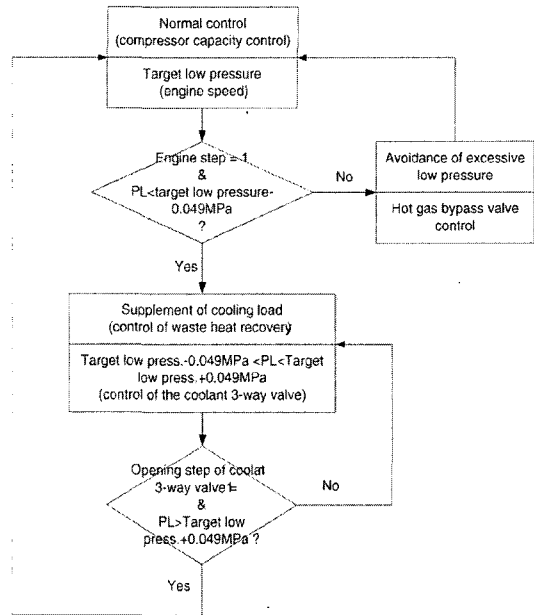


Fig. 5 Control algorithm of Yanmar GHP

(Corripio, 1990). In this study, cascaded PI loops were applied to all components that need feedback control loops. For example, engine speed and opening angle of the coolant three-way valve were PI-controlled for the refrigerant low pressure. And the inverter speed of the outdoor fan motor was P-controlled for the refrigerant high pressure.

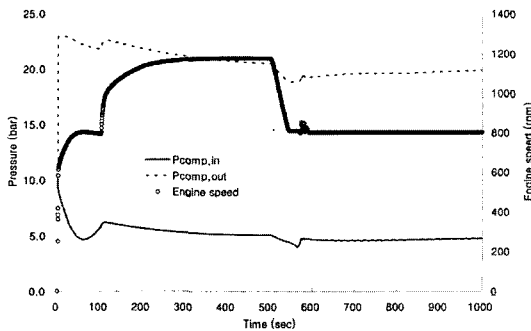
## 5. Simulation Result

Figure 6 shows an example of simulated GHP operation obtained by programming the modeling result and control algorithm on SIMULINK® (Mathworks Co., 2002). The target compressor was a four-cylinder reciprocating type with the total displacement volume of 554.2 cm<sup>3</sup>. Two indoor units are exposed to 35°C of indoor temperature as shown in Fig. 1. For sake of continuous operation, the gas engine is forced to run at its lowest speed of 800 rpm even for a cooling load which is lower than the minimal cooling capacity that the GHP system can produce. One of main concerns in view of control performance is the behavior of refrigerant low pressure and COP at such low cooling loads under which the coolant

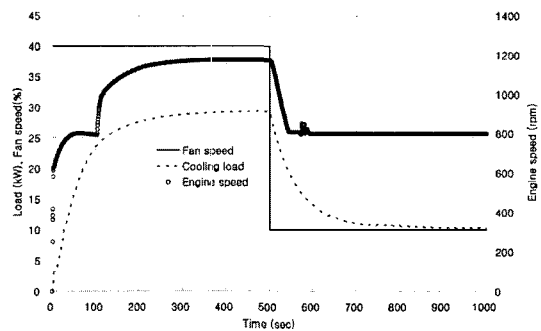
three-way valve and the liquid-injection valve become main actuators to control the pressure and temperature of the refrigerant at the compressor inlet. To monitor the concern, the cooling load was set to be 30 kW for the first 500 sec and 10 kW later on as shown in Fig. 6(b). For the variation of cooling load, Fig. 6(a) shows that simulated refrigerant high and low pressures follow well the target pressures of 20 bar and 5 bar, respectively. Since engine speed can not be reduced below 800 rpm to prevent refrigerant pressure drop, liquid refrigerant has to be injected

into the refrigerant gas header and it should be vaporized by the hot engine coolant via the plate-type coolant heat exchanger, as shown in Fig. 6(d), so that the engine keeps running at 800 rpm. Fig. 6(c) represents refrigerant temperatures at the inlet and exit of the compressor.

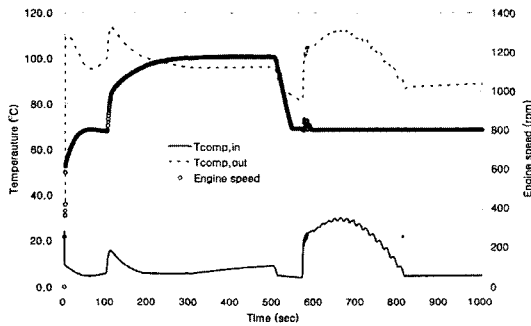
Fig. 6(e) shows COP for the same period. It is shown that COP is about 3.7 for normal load, but it decreases below 2 for very low cooling load. Therefore, the limit of low load for continuous operation must be designed with energy saving target taken into account.



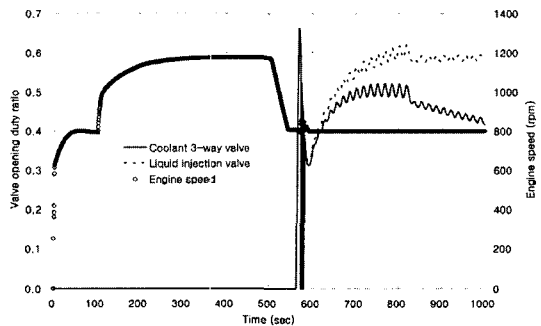
(a) Refrigerant pressures



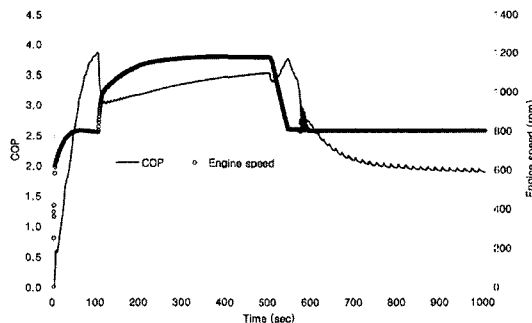
(b) Fan speed and cooling load



(c) Refrigerant temperatures at compressor



(d) Coolant 3-way and liquid injection valves



(e) COP

Fig. 6 Simulation results

## 6. Conclusions

The following conclusions were obtained from the study of simulating GHP system dynamics to design control algorithm and evaluate control performance :

(1) Modeling the refrigerant cycle is quite complicated due to two-phase flow heat transfer phenomena. However, through reasoning of simplified physics to derive differential equations, a simple model was obtained which simulates thermal dynamics to serve as a plant for control loop design.

(2) The behavior of simulated temperatures, pressures and COP was found to be close to those of real systems. It means that the model can be served as a virtual GHP system to test many algorithm candidates. Further development of the model would make it possible to evaluate energy savings and control performance quantitatively with respect to various control strategies.

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