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ON A CLASS OF γ - PREOPEN SETS IN A TOPOLOGICAL SPACE

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ABSTRACT. In this paper we introduce the concept of γ - preopen sets in a topological space together with its corresponding γ-preclosure and γ-preinterior operators and a new class of topology $\tau_{\gamma p}$ which is generated by the class of γ - preopen sets. Also we introduce γ - pre T_i spaces ($i = 0, \frac{1}{2}, 1, 2$) and study some of its properties and we proved that if γ is a regular operation, then$(X, \tau_{\gamma p})$ is a γ - pre $T_{\frac{1}{2}}$ space. Finally we introduce (γ, β)-precontinuous mappings and study some of its properties.

1. Introduction

The concept of preopen sets and semi preopen sets was introduced respectively by Mashhour etal [6] and Andrijevic [1]. Andrijevic[2] introduced a new class of topology generated by preopen sets and corresponding closure and interior operators. Kasahara [4] defined the concept of an operation on topological spaces and introduced the concept of α- closed graphs of an operation. Ogata [7] called the operation α (respectively α - closed set) as γ- operation (respectively γ- closed set) and introduced the notion of τ_γ which is the collection of γ- open sets in a topological space. Also he introduced the concept of $\gamma - T_i(i = 0, \frac{1}{2}, 1, 2)$ and characterized $\gamma - T_i$ spaces using the notion of γ-closed and γ- open sets.

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In this paper in section 2 we introduce the concept of γ- preopen sets, which is analogous to preopen sets in a topological space and study some of the basic properties. In section 3 we introduce the concept of γ - semi preopen sets in a topological space and define the corresponding γ- semi precloure and γ- semi preinterior operators. In section 4 we introduce a new topology $\tau_{\gamma p}$ generated by γ- preopen sets. In section 5 we introduce the concept of γ- pre T_i spaces $(i = 0, \frac{1}{2}, 1, 2)$ and characterize γ- pre T_i spaces using the notion of γ- preclosed and γ- preopen sets and we proved that $(X, \tau_{\gamma p})$ space is a γ - pre $T_{\frac{1}{2}}$ space. Finally in section 6 we introduce (γ, β)- precontinuous mappings and study some of its properties

2. Preopen sets

In this section we introduce the concept of γ - preopen sets and study some of their basic properties

DEFINITION 2.1. [6] Let (X, τ) be a topological space and a subset $A \subseteq X$ is a called preopen set if $A \subseteq int(cl(A))$.

DEFINITION 2.2. [6] Let (X, τ) be a topological spaces and $A \subseteq X$, then

(i) preinterior of A is defined by union of all preopen sets contained in A and it is denoted by pint(A).
(ii) preclosure of A is defined by intersection of all preclosed set containing A and it is denoted by pcl(A).

DEFINITION 2.3. [1] Let (X, τ) be a topological space a subset A of X is said to be a semi preopen set if and only if there exits a preopen set U such that $U \subseteq A \subseteq cl(U)$.

DEFINITION 2.4. [2] Let (X, τ) be a topological space. A subset A of X is called a γ- set if $A \cap B \in PO(X)$ for all $B \in PO(X)$.

DEFINITION 2.5. [4] Let (X, τ) be a topological space. An operation γ on a topology τ is a mapping from τ into the power set $P(X)$ of X such that $V \subseteq V^\gamma$ for each $V \in \tau$ where V^γ denotes the value of γ at V.

DEFINITION 2.6. [7] A subset A of topological space is called γ-open set if for each $x \in A$ there exists a open set U such that $x \in U$ and $U^\gamma \subseteq A$. τ_γ -denoted the set of all γ- open sets in a topological space.

DEFINITION 2.7. [7] Let (X, τ) be a topological space and γ be an operation on τ . Then for any subset A of X, $\tau_\gamma - cl(A) = \bigcap\{F : A \subseteq F \text{ and } X - F \in \tau_\gamma\}$.

DEFINITION 2.8. [8] Let (X, τ) be a topological space and γ be an operation on τ . Then for any subset A of X, $\tau_\gamma - int(A) = \bigcup\{G : G \subseteq A \text{ and } G \in \tau_\gamma\}$.

DEFINITION 2.9. Let (X, τ) be a topological space and be an operation on . A subset A of X is said to be γ- preopen set if $A \subseteq \tau_\gamma - int(\tau_\gamma - cl(A))$.

REMARK 2.10. The set of all γ - preopen set in a topological space (X, τ) is denoted as $\tau_\gamma - PO(X)$.

EXAMPLE 2.11. Let $X = \{a, b, c, d\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}\}$. We define the operation $\gamma : \tau \to P(X)$as follows : for every $A \in \tau$, $A^\gamma = int\ cl(A)$ if $A = \{a\}$ and $A^\gamma = cl(A)$ if $A \neq \{a\}$, then $\tau_\gamma - PO(X) = \{\phi, X, \{a\}, \{c\}, \{a, c\}, \{a, b\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$.

THEOREM 2.12. *If A is a γ- open set in (X, τ), then A is a γ-preopen set.*

Proof. Proof follows from the Definition 2.9 and Remark 3.8[8]. ☐

REMARK 2.13. The converse of the above theorem need not be true. In the Example 2.11 $\{a, b\}$ is a γ- preopen set but not a γ- open set.

REMARK 2.14. The concept of γ- preopen and preopen are independent.

In Example 2.11 $\{b, c\}$ is preopen but not a γ- preopen set. Similarly the set $\{a, c, d\}$ is γ- preopen but not a preopen set.

THEOREM 2.15. *If (X, τ) be a γ - regular space then the concept of γ - preopen and preopen coincide*

Proof. Proof follows from the Proposition 2.4[7] and Remark 3.8[8]. □

THEOREM 2.16. *Let $\{A_\alpha\}_{\alpha\in J}$ be the collection of γ - preopen set in a topological space (X, τ), then $\cup_{\alpha\in J}A_\alpha$ is also a γ - preopen set.*

Proof. Since each A_α is γ- preopen and $A_\alpha \subseteq \bigcup_{\alpha\in J} A_\alpha$, implies that

$$\bigcup_{\alpha\in J} A_\alpha \subseteq \tau_\gamma - int\left(\tau_\gamma - cl\Big(\bigcup A_\alpha\Big)\right).$$

Hence is a γ- preopen set in (X, τ). □

REMARK 2.17. If A and B are two γ - preopen set in (X, τ), then the following example shows that $A \cap B$ need not be a γ- preopen set in (X, τ).

Let $X = \{a, b, c\} = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ and define an operation $\gamma : \tau \to P(X)$ by $A^\gamma = A$ if $b \in A$ and $A^\gamma = cl(A)$ if $b \notin A$. Then $A = \{a, c\}, B = \{b, c\}$ are γ - preopen set but $A \cap B = \{c\}$ is not a γ - preopen set.

LEMMA 2.18. *Let (X, τ) be a topological space and γ be an operation on τ and A be the subset of X. Then the following are holds good:*

(i) $\tau_\gamma - cl(\tau_\gamma, -cl(A)) = \tau_\gamma - cl(A)$
(ii) $\tau_\gamma - int(\tau_\gamma - int(A)) = \tau_\gamma - int(A)$
(iii) $\tau_\gamma - cl(A) = (X - \tau_\gamma - int(X - A))$.
(iV) $\tau_\gamma - int(A) = (X - \tau_\gamma - cl(X - A))$.

Proof. Proof of (i),(ii),(iii) and (iv) follows from Definition 2.7 and 2.8. □

LEMMA 2.19. *Let (X, τ) be a topological space and γ be a regular operation on τ. Then*

(i) *for every γ - open set G and every subset $A \subseteq X$ we have $\tau_\gamma - cl(A) \cap G \subseteq \tau_\gamma - cl(A \cap G)$*

(ii) *for every γ - closed set F and every subset $A \subseteq X$ we have* $\tau_\gamma - int(A \cup F) \subseteq \tau_\gamma - int(A) \cup F$.

Proof. (i) Let $x \in \tau_\gamma - cl(A) \cap G$, then $x \in \tau_\gamma - cl(A)$ and $x \in G$. Let V be the γ- open set containing x. Then by Proposition2.9[7] $V \cap G$ is also γ - open set containing x. Since $x \in \tau_\gamma - cl(A)$, implies $(V \cap G) \cap A \neq \phi$. This implies $V \cap (A \cap G) \neq \phi$. This is true for every V containing x, hence by Proposition3.3[7] $x \in \tau_\gamma - cl(A \cap G)$. Therefore $\tau_\gamma - cl(A) \cap G \subseteq \tau_\gamma - cl(A \cap G)$.

(ii) Proof follows from (i) and Lemma 2.18(iv). □

THEOREM 2.20. *Let (X, τ) be a topological space and γ be a regular operation on τ. Let A be a γ - preopen and U be the γ- open subset of X then $A \cap U$ is also γ - preopen set.*

Proof. Proof follows from the Proposition 2.9[7] and Lemma 2.18. □

DEFINITION 2.21. Let (X, τ) be a topological space, a subset A of X is said to be

(i) γ - dense set if $\tau_\gamma - cl(A) = X$
(ii) γ - nowhere dense set if $\tau_\gamma - int(\tau_\gamma - cl(A)) = \phi$.

THEOREM 2.22. *Let (X, τ) be a topological space and γ be a regular operation on τ, then a subset N of X is γ - nowhere dense set if and only if any one of the following condition holds:*

(i) $\tau_\gamma - cl(X - \tau_\gamma - cl(N)) = X$
(ii) $N \subseteq \tau_\gamma - cl(X - \tau_\gamma - cl(N))$
(iii) *Every nonempty γ- open set U contains a non empty γ- open set A disjoint with N.*

Proof. (i) $\tau_\gamma - int(\tau_\gamma - cl(N)) = \phi$ if and only if $X - (\tau_\gamma - cl(X - (\tau_\gamma - cl(N))) = \phi$ (by Lemma 2.18 (iii)) if and only if $X \subseteq \tau_\gamma - cl(X - (\tau_\gamma - cl(N)))$ if and only if $X = \tau_\gamma - cl(X - (\tau_\gamma - cl(N)))$.

(ii) $N \subseteq X = \tau_\gamma - cl(X - (\tau_\gamma - cl(N)))$ by (i). Conversely, $N \subseteq \tau_\gamma - cl(X - (\tau_\gamma - cl(N)))$, implies $\tau_\gamma - cl(N) \subseteq \tau_\gamma - cl(X - (\tau_\gamma - cl(N)))$. Since $X = \tau_\gamma - cl(N) \cup (X - (\tau_\gamma - cl(N)))$, implies $X \subseteq \tau_\gamma - cl(X -$

$(\tau_\gamma - cl(N))) \cup (X - (\tau_\gamma - cl(N))) = \tau_\gamma - cl(X - (\tau_\gamma - cl(N)))$. Hence $X = \tau_\gamma - cl(X - (\tau_\gamma - cl(N)))$.

(iii) Let N be a γ-nowhere dense subset of X, then $\tau_\gamma - int(X - \tau_\gamma - cl(N)) = \phi$. This implies $\tau_\gamma - cl(N)$ does not contain any γ - open set. Hence for any nonempty γ-open set U, $U - (\tau_\gamma - cl(N)) \neq \phi$. Thus by Proposition2.9(ii)[7] $A = U - (\tau_\gamma - cl(N))$ is a non empty γ- open set contained in U and disjoint with N. Conversely, if for any given non empty γ - open set U, there exists a non empty γ- open set A such that $A \subseteq U$ and $A \cap N = \phi$, then $N \subseteq X - A$, $\tau_\gamma - cl(N) \subseteq \tau$, $cl(X - A) = (X - A)$. Therefore $U - (\tau_\gamma - cl(N)) \supseteq U - (X - A) = U \cap A = A \neq \phi$. Thus $\tau_\gamma - cl(N)$ does not contain any nonempty γ - open set. This implies $\tau_\gamma - int(X - \tau_\gamma - cl(N)) = \phi$. Hence N is γ - nowhere dense set in X. ☐

THEOREM 2.23. *Let (X, τ) be a topological space and γ be an operation on τ , then every singleton set $\{x\}$ is either γ - preopen or γ - nowhere dense set .*

Proof. Suppose $\{x\}$ is not γ-preopen then $\tau_\gamma - int(X - \tau_\gamma - cl(\{x\})) = \phi$. This implies $\{x\}$ is τ_γ - nowhere dense set in X. ☐

DEFINITION 2.24. A topological space is said to be γ - submaximal if every γ - dense subset of X is γ - open.

THEOREM 2.25. *Let (X, τ) be a topological space in which every γ - preopen set is γ -open then (X, τ) is γ - submaximal.*

Proof. Let A be a γ - dense subset of (X, τ). Then $A \subseteq \tau_\gamma - int(\tau_\gamma - cl(A))$. This implies A is a γ - preopen. Therefore from the assumption it is γ - open. Hence (X, τ) is γ - submaximal. ☐

DEFINITION 2.26. Let (X, τ) be a topological space, A subset A of X is called γ - preclosed if and only if $X - A$ is γ - preopen, equivalently a subset A of X is γ - preclosed if and only if $\tau_\gamma - cl(\tau_\gamma - int(A)) \subseteq A$.

REMARK 2.27. The family of all γ- preclosed set in (X, τ) is denoted by $\tau_\gamma - PF(X)$.

DEFINITION 2.28. Let (X,τ) be a topological space and A be a subset of X. Then τ_γ - preclosure of A is defined as intersection of all γ- preclosed set containing A. That is, $\tau_\gamma - pcl(A) = \cap\{F : X - F \in \tau_\gamma - PO(X) \text{ and } A \subseteq F\}$.

DEFINITION 2.29. Let (X,τ) be a topological space and A be a subset of X. Then τ_γ- preinterior of A is defined as union of all γ-preopen set contained in A.

That is, $\tau_\gamma - pint(A) = \cup\{U : U \in \tau_\gamma - PO(X) \text{ and } U \subseteq A\}$.

THEOREM 2.30. *Let (X,τ) be a topological space and A be a subset of X, then*

(i) $\tau_\gamma - pint(A)$ *is γ - preopen set contained in A.*
(ii) $\tau_\gamma - pcl(A)$ *is γ - preclosed set containing A.*
(iii) *A is γ - preclosed if and only if $\tau_\gamma - pcl(A) = A$.*
(iv) *A is γ - preopen if and only if $\tau_\gamma - pint(A) = A$.*

Proof. Proof (i) follows from the Definition 2.29 and Theorem 2.16. Proof (ii) follows from the Definitions 2.28 and Theorem 2.16.
Proof (iii) and (iv) follows from the Definition 2.28, (ii) and Definition 2.29, (i) respectively. ☐

THEOREM 2.31. *Let (X,τ) be a topological space and γ be a regular operation on τ and A be a subset of X. Then the following holds good:*
(i) $\tau_\gamma - pcl(A) = A \cup \tau_\gamma - cl(\tau_\gamma - int(A))$.
(ii)$\tau_\gamma - pint(A) = A \cap \tau_\gamma - int(\tau_\gamma - cl(A))$.

Proof. (i) $\tau_\gamma - cl(\tau_\gamma - int(A \cup \tau_\gamma - cl(\tau_\gamma - int(A)))) \subseteq \tau_\gamma - cl(\tau_\gamma - int(A) \cup \tau_\gamma - cl(\tau_\gamma - int(A)))$(by Lemma 2.19 (ii)) $\subseteq \tau_\gamma - cl(\tau_\gamma - int(A)) \cup \tau_\gamma - cl(\tau_\gamma - int(A)) = \tau_\gamma - cl(\tau_\gamma - int(A)) \subseteq A \cup (\tau_\gamma - cl(\tau_\gamma - int(A))$. Hence $A \cup (\tau_\gamma - cl(\tau_\gamma - int(A))$is a γ - preclosed set containing A. Therefore $\tau_\gamma - pcl(A) \subseteq A \cup (\tau_\gamma - cl(\tau_\gamma - int(A))$. Conversely $\tau_\gamma - cl(\tau_\gamma - int(A)) \subseteq \tau_\gamma - cl(\tau_\gamma - int(\tau_\gamma - pcl(A)) \subseteq \tau_\gamma - pcl(A)$since $\tau_\gamma - pcl(A)$is γ - preclosed set. Hence $\tau_\gamma - pcl(A) = A \cup \tau_\gamma - cl(\tau_\gamma - int(A))$

(ii) Proof follows from (i) and Lemma 2.19 (i). ☐

COROLLARY 2.32. *Let (X,τ) be a topological space γ be a regular operation on τ and A be the subset of X. Then*

(i) $\tau_\gamma - pint(\tau_\gamma - cl(A)) = \tau_\gamma - int(\tau_\gamma - cl(A))$
(ii) $\tau_\gamma - pcl(\tau_\gamma - int(A)) = \tau_\gamma - cl(\tau_\gamma - int(A))$
(iii) $\tau_\gamma - int(\tau_\gamma - pcl(A)) = \tau_\gamma - int(\tau_\gamma - cl(\tau_\gamma - int(A)))$
(iv) $\tau_\gamma - cl(\tau_\gamma - pint(A)) = \tau_\gamma - cl(\tau_\gamma - int(\tau_\gamma - cl(A)))$

Proof. i) By Theorem 2.31 (ii)$\tau_\gamma - pint(\tau_\gamma - cl(A)) = \tau_\gamma - cl(A) \cap \tau_\gamma - int(\tau_\gamma - cl(\tau_\gamma - cl(A))) = \tau_\gamma - cl(A) \cap \tau_\gamma - int(\tau_\gamma - cl(A)) = \tau_\gamma - int(\tau_\gamma - cl(A))$.

(ii) By Theorem 2.31 (i)$\tau_\gamma - pcl(\tau_\gamma - int(A)) = \tau_\gamma - int(A) \cup \tau_\gamma - cl(\tau_\gamma - int(\tau_\gamma - int(A))) = \tau_\gamma - cl(\tau_\gamma - int(A))$.

(iii) and (iv) follows from (i) and (ii) respectively. □

THEOREM 2.33. *Let (X, τ) be a topological space and γ be a regular operation on τ and A be the subset of X. Then $\tau_\gamma - pcl(\tau_\gamma - pint(A)) = \tau_\gamma - pint(A) \cup (\tau_\gamma - cl(\tau_\gamma - int(A)))$.*

Proof. Since $\tau_\gamma \subseteq \tau_\gamma - PO(X)$, implies $\tau_\gamma - int(A) \subseteq \tau_\gamma - pint(A) \subseteq A$. Hence $\tau_\gamma - int(\tau_\gamma - pint(A)) = \tau_\gamma - int(A)$. By Theorem 2.31(i) $\tau_\gamma - pcl(\tau_\gamma - pint(A)) = \tau_\gamma - pint(A) \cap \tau_\gamma - cl(\tau_\gamma - int(\tau_\gamma - pint(A)) = \tau_\gamma - pint(A) \cup \tau_\gamma - cl(\tau_\gamma - int(A))$. □

3. γ - semi preopen set

In this section we introduce the concept of γ - semi preopen sets and study some of their basic properties.

DEFINITION 3.1. A subset A of a topological space (X, τ) is γ - semi preopen if and only if there exists a γ- preopen set U in X such that $U \subseteq A \subseteq \tau_\gamma - cl(U)$. The family of all γ - semi preopen set in (X, τ) is denoted by $\tau_\gamma - SPO(X)$

REMARK 3.2. If A is γ- preopen set in (X, τ), then it is γ - semi preopen. But converse need not be true.

Proof. Proof follows from the Definition 3.1 □

Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}, \{c\}, \{a, c\}, \{a, b\}\}$ and $\gamma : \tau \to P(X)$ be an operation defined as follows $A^\gamma = A$ if $A = \{a\}$, and

$A^{\gamma} = A \cup \{c\}$ if $A \neq \{a\}$. Then $\{a, b\}$ is γ-semi preopen but not γ - preopen.

THEOREM 3.3. *Let τ be a regular operation on a topological space (X, τ) and A be a subset of X, then A is γ - semi preopen set if and only if $A \subseteq \tau_\gamma - cl(\tau_\gamma - int(\tau_\gamma - cl(A))$.*

Proof. Let A be a γ - semi preopen set, then there exists a γ - preopen set U such that $U \subsetneq A \subsetneq \tau_\gamma - cl(U)$. This implies that $\tau_\gamma - cl(A) = \tau_\gamma - cl(U)$. By Theorem 2.30(ii) $\tau_\gamma - cl(U)$ is γ - pre-closed set, implies $A \subsetneq \tau_\gamma - cl(U) \subsetneq \tau_\gamma - cl(\tau_\gamma - int(\tau_\gamma - cl(U)) = \tau_\gamma - cl(\tau_\gamma - int(\tau_\gamma - cl(A))$.

Conversely, $A \subseteq \tau_\gamma - cl(\tau_\gamma - int(\tau_\gamma - cl(A)))$. Let $U = A \cap \tau_\gamma - int(\tau_\gamma - cl(A))$, then by Theorem 2.31(ii) and Theorem 2.30 (i) U is γ- preopen set. Now we have $\tau_\gamma - int(\tau_\gamma - cl(A)) = \tau_\gamma - cl(A \cap \tau_\gamma - int(\tau_\gamma - cl(A)) \subseteq \tau_\gamma - cl(A \cap \tau_\gamma - int(\tau_\gamma - cl(A)) = \tau_\gamma - cl(U)$. Since $A \subseteq \tau_\gamma - cl(A) \subseteq \tau_\gamma - cl((\tau_\gamma - cl(\tau_\gamma - int(\tau_\gamma - cl(A))) = \tau_\gamma - cl(\tau_\gamma - int(\tau_\gamma - cl(A)) \subseteq \tau_\gamma - cl(U)$. Hence A is γ - semi preopen set. □

THEOREM 3.4. *Let (X, τ) be a topological space and be a operation on and let $\{A_\alpha : \alpha \in J\}$ be the set of all γ - semi preopen set in (X, τ).Then $\bigcup_{\alpha \in J} A_\alpha$ is also a γ - semi preopen set.*

Proof. Since each A_α is γ - semi preopen set, implies there exists a γ - preopen set U_α such that $U \subseteq A \subseteq \tau_\gamma - cl(U)$. Hence $\bigcup_{\alpha \in J} U_\alpha \subseteq \bigcup_{\alpha \in J} A_\alpha \subseteq \tau_\gamma - cl(U_\alpha) \subseteq \tau_\gamma - cl(\bigcup_{\alpha \in J} U_\alpha)$. By Theorem 2.16 is a γ - preopen set hence is a γ - semi preopen set. □

REMARK 3.5. If A and B are two γ - semi preopen sets in a topological space (X, τ), then $A \cap B$ need not be a γ- semi preopen set.

In the Example given in Remark 3.2 both $\{a, b\}$ and $\{b, c\}$ are γ - semi preopen but $A \cap B = \{b\}$ is not a γ - semi preopen set.

THEOREM 3.6. *Let (X, τ) be a topological space and γ be a regular operation on τ and V be a γ - open set and A be a γ - semi preopen set, then $V \cap A$ is also γ - semi preopen set.*

Proof. By Theorem3.3 and Lemma 219(i) we have $V \cap A \subseteq V \cap \tau_\gamma - cl(\tau_\gamma - int(\tau_\gamma - cl(A)) \subseteq \tau_\gamma - cl(V \cap \tau_\gamma - int(\tau_\gamma - cl(A)) = \tau_\gamma - cl(\tau_\gamma - int(V \cap \tau_\gamma - cl(A)) \subseteq \tau_\gamma - cl(\tau_\gamma - int(\tau_\gamma - cl(V \cap A))$.This implies $V \cap A$ is a γ - semi preopen set. □

DEFINITION 3.7. Let (X, τ) be a topological space. A subset A of $X - A$ is said to be a γ- semi preclosed set if and only if $X - A$ is γ-semi preopen set in (X, τ).

REMARK 3.8. The family of all γ-semi preclosed sets in (X, τ) is denoted by $\tau_\gamma - SPF(X)$.

THEOREM 3.9. *Let (X, τ) be a topological space and γ be a regular operation τ . Then*

(i) for any subset B of X is γ - semi preclosed if and only if τ_γ-int(τ_γ-cl(τ_γ -int(A)) $\subseteq$ A (ii) if $\{B_\alpha : \alpha \in J\}$ be the family of γ - semi preclosed sets in (X, τ), then $\bigcap B_\alpha$ is also a γ -semi preclosed set. (iii)if F is γ- closed and A is γ- semi preclosed then $F \cup B$ is also γ - semi preclosed.

Proof. Proof (i), (ii) and (iii) are follows from Theorem 3.3, 3.4 and 3.6 respectively. □

DEFINITION 3.10. Let (X, τ) be a topological space and A be a subset of X, then γ-semi preclosure of A and γ-semi preinterior of A is defined as
$\tau_\gamma - spcl(A) = \cup\{F_\alpha : A \subseteq F_\alpha$ and $F \subseteq \tau_\gamma - SPF(X)\}$
and $\tau_\gamma - spint(A) = \cap\{G_\alpha : G_\alpha \subseteq A$ and $G \subseteq \tau_\gamma - SPO(X)\}$

REMARK 3.11. Let (X, τ) be a topological space and A be the subset of X then
$(i)\tau_\gamma - spcl(A)$ is γ - semi preclosed set containing A
$(ii)\tau_\gamma - spint(A)$ is γ - semi preopen set contained in A.

Proof. Proof of (i) and (ii) follows from the Definition 3.10 and Theorem 3.9(ii) and Theorem 3.4 respectively. □

THEOREM 3.12. *Let (X, τ) be a topological space and γ be an regular operation on τ and A be a subset of X. Then $\tau_\gamma - spcl(A) = A \cup \tau_\gamma - int(\tau_\gamma - cl(\tau_\gamma - int(A)))$.*

Proof. $\tau_\gamma - int(\tau_\gamma - cl(\tau_\gamma - int(A \cup \tau_\gamma - int(\tau_\gamma - cl(\tau_\gamma - int(A))))) \subseteq \tau_\gamma - int(\tau_\gamma - cl(\tau_\gamma - int(A) \cup \tau_\gamma - int(\tau_\gamma - int(\tau_\gamma - cl(\tau_\gamma - int(A))))) \subseteq \tau_\gamma - int(\tau_\gamma - cl(\tau_\gamma - int(A)) \cup \tau_\gamma - cl(\tau_\gamma - int(\tau_\gamma - cl(\tau_\gamma - int(A)))) \subseteq \tau_\gamma - int(\tau_\gamma - cl(\tau_\gamma - int(A)))$(Since $\tau_\gamma - cl(\tau_\gamma - int(A)))$ is a γ - preclosed set) $\subseteq A \cup \tau_\gamma - int(\tau_\gamma - cl(\tau_\gamma - int(A)))$. Hence $A \cup \tau_\gamma - int(\tau_\gamma - cl(\tau_\gamma - int(A)))$ is a γ - semi preclosed set and thus $\tau_\gamma - spcl(A) \subseteq A \cup \tau_\gamma - int(\tau_\gamma - cl(\tau_\gamma - int(A)))$.

Conversely, Since $\tau_\gamma - spcl(A)$ is γ- semi preclosed set, then by Theorem3.9(ii) we have $\tau_\gamma - int(\tau_\gamma - cl(\tau_\gamma - int(A))) \subseteq \tau_\gamma - int(\tau_\gamma - cl(\tau_\gamma - int(\tau_\gamma - spcl(A))) \subseteq \tau_\gamma - spcl(A)$. Hence $A \cup \tau_\gamma - int(\tau_\gamma - cl(\tau_\gamma - int(A))) \subseteq \tau_\gamma - spcl(A)$.

□

THEOREM 3.13. *Let (X, τ) be a topological space and γ be a regular operation on τ and A be any subset of X, then $\tau_\gamma - spint(A) = A \cap \tau_\gamma - cl(\tau_\gamma - int(\tau_\gamma - cl(A)))$.*

Proof. $A \cap \tau_\gamma - cl(\tau_\gamma - int(\tau_\gamma - cl(A))) \subseteq \tau_\gamma - cl(\tau_\gamma - int(\tau_\gamma - cl(A))) = \tau_\gamma - cl(\tau_\gamma - int(\tau_\gamma - cl(A) \cap \tau_\gamma - int(\tau_\gamma - cl(A)))) \subseteq \tau_\gamma - cl(\tau_\gamma - int(\tau_\gamma - cl(A \cap \tau_\gamma - int(\tau_\gamma - cl(A)))))$(by Lemma 2.19(i)) $\subseteq \tau_\gamma - cl(\tau_\gamma - int(\tau_\gamma - cl(A \cap \tau_\gamma - cl(\tau_\gamma - int(\tau_\gamma - cl(A))))))$.Hence$A \cap \tau_\gamma - cl(\tau_\gamma - int(\tau_\gamma - cl(A))) \in \tau_\gamma - SPO(X)$ by the Theorem 3.3. Therefore $A \cap \tau_\gamma - cl(\tau_\gamma - int(\tau_\gamma - cl(A))) \subseteq \tau_\gamma - spint(A)$.

Conversely, Since $\tau_\gamma - spint(A)$ is γ - semi preopen set, implies $\tau_\gamma - spint(A) \subseteq \tau_\gamma - cl(\tau_\gamma - int(\tau_\gamma - cl(\tau_\gamma - spint(A)) \subseteq \tau_\gamma - cl(\tau_\gamma - int(\tau_\gamma - cl(A)))$. Hence $\tau_\gamma - spint(A) \subseteq A \cap \tau_\gamma - cl(\tau_\gamma - int(\tau_\gamma - cl(A))$. □

4. Topology generated by γ - preopen sets

Now we study the properties of the topology generated by γ -pre open sets.

DEFINITION 4.1. Let (X, τ) be a topological space. A subset A of X is called a $\tau_\gamma - \gamma$ set if $A \cap B \in \tau_\gamma - PO(X)$ for every $B \in \tau_\gamma - PO(X)$.

The set of all $\tau_\gamma - \gamma$ set in a topological space (X, τ) is denoted by $\tau_{\gamma p}$.

REMARK 4.2. $\tau_{\gamma p} \subsetneq \tau_\gamma - PO(X)$ for any τ on X.

DEFINITION 4.3. Let (X, τ) be a topological space and A be a subset of X, then A is said to be $\tau_{\gamma p}$ - closed if and only if $X - A \in \tau_{\gamma p}$.

DEFINITION 4.4. Let (X, τ) be a topological space and A be the subset of X. τ_p-interior of A and $\tau_{\gamma p}$-closure of A is defined as $\tau_{\gamma p} - int(A) = \bigcup\{U : U \in \tau_{\gamma p}\text{and}U \subseteq A\}$ and $\tau_{\gamma p} - cl(A) = \bigcap\{F : F \in X - \tau_{\gamma p}\text{and}A \subseteq F\}$ respectively.

THEOREM 4.5. *Let (X, τ) be a topological space and A be a subset of X, then A is closed in $(X, \tau_{\gamma p})$ if and only if $A \cup B$ is γ- preclosed for every γ- preclosed set B in (X, τ)*

Proof. Let B be a γ - preclosed set in (X, τ) then $X - B$ is γ - preopen. Since A is closed in $(X, \tau_{\gamma p})$ implies $X - A \in \tau_{\gamma p}$. Hence $(X - A) \cap (X - B) \in \tau_\gamma - PO(X)$. Therefore $A \cup B$ is γ- preclosed. Conversely, if $A \cup B$ is γ-preclosed for every γ - preclosed set B, then $A \cup B = (X - A) \cap (X - B)$ is γ- preopen. This implies $X - A \in \tau_{\gamma p}$. Therefore A is closed in $(X, \tau_{\gamma p})$. □

THEOREM 4.6. *Let γ be a regular operation on (X, τ) and A be a subset of X. Then*

(i) $\tau_{\gamma p} - int(\tau_\gamma - cl(A)) = \tau_\gamma - int(\tau_\gamma - cl(A))$
(ii) $\tau_{\gamma p} - cl(\tau_\gamma - int(A)) = \tau_\gamma - cl(\tau_\gamma - int(A))$

Proof. (i) By Definition of $\tau_{\gamma p}$ and Theorem 2.20 $\tau_\gamma \subsetneq \tau_{\gamma p}$ we have $\tau_\gamma - int(\tau_\gamma - cl(A)) \subseteq (\tau_{\gamma p} - int(\tau_\gamma - cl(A))$. Hence by Remark 4.2 and Corollary 2.32 (ii) we have $(\tau_{\gamma p} - int(\tau_\gamma - cl(A)) \subseteq \tau_\gamma - pint(\tau_\gamma - cl(A)) = \tau_\gamma - int(\tau_\gamma - cl(A))$. Hence $\tau_{\gamma p} - int(\tau_\gamma - cl(A)) = \tau_\gamma - int(\tau_\gamma - cl(A))$.
(ii) Proof follows from Remark 4.2 and Corollary 2.32(i) □

THEOREM 4.7. *$\tau_{\gamma p}$ is a topology on X*

Proof. It is obvious that $\phi \in \tau_{\gamma p}$ and $X \in \tau_{\gamma p}$. Let $\{A_\alpha : \alpha \in J\}$ be a collection of $\tau_{\gamma p}$ set in (X, τ), then $A_\alpha \cap B \in \tau_\gamma - PO(X)$ for all $B \in \tau_\gamma - PO(X)$ and every $\alpha \in J$. Hence $\cup(A_\alpha \cap B) \in \tau_\gamma - PO(X)$, this implies $(\cup(A_\alpha)) \cap B \in \tau_\gamma - PO(X)$. Therefore $\cup A_\alpha \in \tau_{\gamma p}$. If $C, D \in \tau_{\gamma p}$, then $(C \cap D) \cap B = C \cap (D \cap B) \in \tau_\gamma - PO(X)$ for all $B \in \tau_\gamma - PO(X)$. This implies $(C \cap D) \in \tau_\gamma - PO(X)$. Hence $\tau_{\gamma p}$ is a topology on X. □

5. Separation axioms

In this section we investigate general operator approaches on T_i spaces where $i = 0, \frac{1}{2}, 1, 2$. Let $\gamma : \tau \to P(X)$ be an operation on a topology τ.

DEFINITION 5.1. A topological space (X, τ) is called a γ-pre T_0 space if for each distinct points $x, y \in X$ there exists a γ-preopen set U such that either $x \in U$ and $y \notin U$ or $y \in U$ and $x \notin U$.

DEFINITION 5.2. A topological space (X, τ) is called a γ- pre T_1 space if for each distinct points $x, y \in X$ there exist γ-preopen sets U and V contain x and y respectively such that $y \notin U$ and $x \notin V$.

DEFINITION 5.3. A space (X, τ) is called γ-pre T_2 space if for each $x, y \in X$, there exist γ-preopen sets U and V such that $x \in U$ and $y \in V$ and $U \cap V = \phi$.

DEFINITION 5.4. Let (X, τ) be a topological space. A subset A of X is called γ- pre g.closed if $\tau_\gamma - pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is a γ-preopen set.

REMARK 5.5. From the definition every γ-preclosed set is γ- pre g.closed set.

DEFINITION 5.6. A topological space (X, τ) is called γ-pre $T_{\frac{1}{2}}$ space if for each γ-pre g.closed set of (X, τ) is γ-preclosed.

THEOREM 5.7. *For a point $x \in X$, $x \in \tau_\gamma$-pcl(A) if and only if $V \cap A \neq \phi$ for any $V \in \tau_\gamma$- PO(X) such that $x \in V$.*

Proof. Let F_0 be the set of all $y \in X$ such that $V \cap A \neq \phi$ for any $V \in \tau_\gamma - PO(X)$ and $y \in V$. Now to prove that $\tau_\gamma - pcl(A) = F_0$. Let us assume $x \in \tau_\gamma - pcl(A)$ and $x \notin F_0$. Then there exists a γ-preopen set U of x such that $U \cap A = \phi$. This implies $A \subseteq X - U$. Therefore $\tau_\gamma - pcl(A) \subseteq X - U$. Hence $x \notin \tau_\gamma - pcl(A)$. This is a contradiction. Hence $\tau_\gamma - pcl(A) \subseteq F_0$. Conversely, let F be a set such that $A \subseteq F$ and $X - F \in \tau_\gamma - PO(X)$. Let $x \notin F$ then we have $x \in X - F$ and $(X - F) \cap A = \phi$. This implies $x \notin F_0$. Therefore $F_0 \subseteq F$. Hence $F_0 \subseteq \tau_\gamma - pcl(A)$. □

THEOREM 5.8. *Let (X, τ) be a topological space and A be a subset of X, then A is γ-pre g.closed if and only if $\tau_\gamma - pcl(\{x\}) \cap A \neq \phi$ holds for every $x \in \tau_\gamma - pcl(A)$.*

Proof. Let U be any γ- preopen set such that $A \subseteq U$. Let $x \in \tau_\gamma - pcl(A)$. By assumption there exists a point $z \in \tau_\gamma - pcl(\{x\})$ and $z \in A \subseteq U$. Therefore from Theorem 5.7 $U \cap \{x\} \neq \phi$. This implies $x \in U$. Hence A is γ- pre g.closed set.
Conversely, suppose there exists a point $x \in \tau_\gamma - pcl(A)$ such that $\tau_\gamma - pcl(\{x\}) \cap A = \phi$. Since $\tau_\gamma - pcl(\{x\})$ is a γ-preclosed set implies $X - (\tau_\gamma - pcl(\{x\}))$ is a γ-preopen set. Since $A \in X - (\tau_\gamma - pcl(\{x\}))$ and A is γ - pre g.closed set, implies $\tau_\gamma - pcl(A) \subseteq (X - \tau_\gamma - pcl(\{x\}))$. Hence $x \notin \tau_\gamma - pcl(A)$. This is a contradiction. □

THEOREM 5.9. *Let (X, τ) be a topological space and A be the γ - pre g.closed set in (X, τ). Then $\tau_\gamma - pcl(A) - A$ does not contain non empty γ - closed set.*

Proof. Suppose there exists a non empty γ- preclosed set F such that $F \subseteq \tau_\gamma - pcl(A) - A$. Let $x \in F$, then $x \in \tau_\gamma - pcl(A)$, implies $F \cap A = \tau_\gamma - pcl(F) \cap A \supset \tau_\gamma - pcl(\{x\}) \cap A \neq \phi$ and hence $F \cap A \neq \phi$. This is a contradiction. □

THEOREM 5.10. *For each $x \in X$, $\{x\}$ is γ-preclosed or $X - \{x\}$ is γ - pre g.closed.*

Proof. Suppose that $\{x\}$ is not γ-preclosed. Then $X - \{x\}$ is not γ-preopen. Let U be a γ-preopen set such that $X - \{x\} \subseteq U$. Then $U = X$. Therefore $\tau_\gamma - pcl(X - \{x\}) \subseteq U$. Hence $X - \{x\}$ is γ -pre g.closed. □

THEOREM 5.11. *A topological space (X, τ) is a γ-pre $T_{\frac{1}{2}}$ space if and only if for each $x \in X$, $\{x\}$ is γ - preopen or γ- preclosed.*

Proof. Suppose $\{x\}$ is not γ-preclosed. Then it follows from the assumption and Theorem 5.10 $\{x\}$ is γ-preopen.

Conversely, Let F be a γ-pre g.closed set in (X, τ). Let $x \in \tau_\gamma - pcl(F)$, then by the assumption $\{x\}$ is either γ-preopen of γ-preclosed.

Case(i): Suppose $\{x\}$ is γ- preopen, then by Theorem 5.7 $\{x\} \cap F \neq \phi$. This implies $\tau_\gamma - pcl(F) = F$. Therefore (X, τ) is a γ-pre $T_{\frac{1}{2}}$ space.

Case(ii): Suppose $\{x\}$ is γ- preclosed. Let us assume $x \notin F$ then, $\{x\} \in \tau_\gamma - pcl(F) - F$. This is a Contradiction. Hence $x \in F$. Therefore (X, τ) is a $\gamma - preT_{\frac{1}{2}}$ space. □

THEOREM 5.12. *A space (X, τ) is γ - pre T_1 if and only if for any $x \in X$, $\{x\}$ is γ - pre closed.*

Proof. Proof follows from the Definitions 5.2 and 2.7. □

REMARK 5.13. From the Theorems 5.10, 5.11, 5.12 we have

$$\gamma - preT_2 \longrightarrow \gamma - preT_1 \longrightarrow \gamma - preT_{\frac{1}{2}} \longrightarrow \gamma - preT_0.$$

THEOREM 5.14. *Let (X, τ) be a topological space and γ be a regular operation on τ. The topological space $(X, \tau_{\gamma p})$ is a γ- pre $T_{\frac{1}{2}}$ space.*

Proof. By Theorem 5.11 to prove $(X, \tau_{\gamma\ p})$ is a γ - pre $T_{\frac{1}{2}}$ space it is enough to prove that every $x \in X$, $\{x\}$ is either γ - preopen or γ-preclosed in $(X, \tau_{\gamma\ p})$. Suppose $\{x\} \in \tau_{\gamma\ p}$, then by Remark 4.3 $\{x\}$ is γ- preopen. Suppose $\{x\} \notin \tau_{\gamma\ p}$, then there exist a γ- preopen set A such that $\{x\} \cap A$ is not γ-preopen. This implies that $\{x\}$ is not γ-preopen and so $\tau_\gamma - int(\tau_\gamma - cl\{x\}) = \phi$. This implies $\{x\}$ is γ-nowhere dense subset of X. This implies $\tau_\gamma - cl(X - (\tau_\gamma - cl(\{x\}))) = X$. Hence By Lemma 2.18 (iv) $\tau_\gamma - cl(\tau_\gamma - int(X - \{x\}) = X$. Since $\tau_\gamma - int(X - \{x\}) \subseteq X - \{x\}$, implies $\tau_\gamma - cl(X - \{x\}) = X$. Hence $X - \{x\}$ is a γ- preopen set in (X, τ). This implies $\{x\}$ is γ-preclosed. Hence $(X, \tau_{\gamma\ p})$ is a $\gamma - preT_{\frac{1}{2}}$-space.

□

6. (γ,β)-precontinuous mappings

In this section we introduce the concept of (γ,β)-precontinuous mappings and study some of its basic properties. Through out this section let (X,τ) and (Y,σ) be two topological spaces and let $\gamma:\tau\rightarrow P(X)$ and $\beta:\tau\rightarrow P(Y)$ be the operations on τ and σ respectively.

Definition 6.1. A mapping $f:(X,\tau)\rightarrow(Y,\sigma)$ is said to be (γ,β)-precontinuous if for each x of X and each β-preopen set V containing $f(x)$, there exists a γ-preopen set U such that $x\in U$ and $f(U)\in V$.

Remark 6.2. By Theorem 2.12 we have every (γ,β) continuos mapping is (γ,β)-pre continuous. But converse need not be true.

Remark 6.3. If both (X,τ) and (Y,σ) are regular space, then the concept (γ,β)-pre continuity and precontinuity coincide.

Theorem 6.4. *Let* $f:(X,\tau)\rightarrow(Y,\sigma)$ *be a* (γ,β)*-precontinuos mapping. Then*

(i) $f(\tau_\gamma - pcl(A))\subseteq\tau_\beta - pcl(f(A))$ *holds for every subset of* A *of* X

(ii) *for any* β*-preclosed set* B *of* (Y,σ), $f^{-1}(B)$ *is* γ*- preclosed in* (X,τ).

Proof. (i) Let $y\in f(\tau_\gamma - pcl(A))$ and V be the γ-preopen set containing y, then there exists a point $x\in X$ and a γ-preopen set U such that $f(x)=y, x\in U$ and $f(U)\subseteq V$. Since $x\in\tau_\gamma - pcl(A)$, we have $U\cap A\neq\phi$, and hence $\phi\neq f(U\cap A)\subseteq f(U)\cap f(A)\subset V\cap f(A)$. This implies $x\in\tau_\gamma - cl(f(A))$.

(ii) It is sufficient to prove that (i) implies (ii). Let B be the β-preclosed set in (Y,σ). That is $\tau_\gamma - pcl(B)=B$. By using (i) $f(\tau_\gamma - pcl(f^{-1}(B))\subseteq\tau_\beta - pcl(f(f^{-1}(B))=\tau_\beta - pcl(B)=B$ holds. Therefore $\tau_\gamma - pcl(f^{-1}(B))\subseteq f^{-1}(B)$ and hence $f^{-1}(B)=\tau_\gamma - pcl(f^{-1}(B))$. Hence $f^{-1}(B)$ is γ- preclosed set in(X,τ). ☐

Definition 6.5. A mapping $f:(X,\tau)\rightarrow(Y,\sigma)$ is said to be (γ,β)-preclosed if for any γ-preclosed set A of (X,τ), $f(A)$ is β-preclosed.

DEFINITION 6.6. Let $id : (X, \tau) \to P(X)$ be the identity operation. If f is (id, β)-preclosed then $f(F)$ is β-preclosed for any preclosed set F of (X, τ).

THEOREM 6.7. *If f is bijective mapping and $f^{-1}(Y, \sigma) \to (X, \tau)$ is (id, β)-precontinuous then f is (id, β)-preclosed.*

Proof. Proof follows from the Definitions 6.5 and 6.6. □

THEOREM 6.8. *Suppose $f : (X, \tau) \to (Y, \sigma)$ is (γ, β)-precontinuos and f is (γ, β)- preclosed then*

(i) *for every γ-pre g.closed set A of (X, τ), the image $f(A)$ is β-pre g.closed.*
(ii) *for every β-pre g.closed set B of (Y, σ), then $f^{-1}(B)$ is also β-pre g.closed.*

Proof. (i) Let V be any β-preopen set in (Y, σ) such that $f(A) \subseteq V$, then by Theorem 6.4 (ii) $f^{-1}(V)$ is γ-preopen. Since A is γ-pre g.closed and $A \subseteq f^{-1}(V)$, we have $\tau_\gamma - pcl(A) \subseteq f^{-1}(V)$ and hence $f(\tau_\gamma - pcl(A)) \subseteq V$. By assumption $f(\tau_\gamma - pcl(A))$ is a β-preclosed set, therefore $\tau_\beta - pcl(f(A)) \subseteq \tau_\beta - pcl(f(\tau_\gamma - pcl(A)) = f(\tau_\gamma - pcl(A)) \subseteq V$. This implies $f(A)$ is β-pre g.closed.

(ii) Let U be any γ-preopen set such that $f^{-1}(B) \subseteq U$. Let $F = \tau_\gamma - pcl(f^{-1}(B)) \cap (X - U)$, then F is γ-preclosed set in (X, τ). This implies $f(F)$ is β-preclosed set in (Y, σ). Since $f(F) = f(\tau_\gamma - pcl(f^{-1}(B)) \cap (X - U)) \subseteq \tau_\beta - pcl(B) \cap f(X - U) \subseteq \delta\tau_\beta - pcl(B) \cap (Y - B)$. This implies $f(F) = \phi$ and hence $F = \phi$. Therefore $\tau_\gamma - pcl(f^{-1}(B)) \subseteq U$, implies $f^{-1}(B)$ is γ-pre g.closed. □

THEOREM 6.9. *Suppose $f : (X, \tau) \to (Y, \sigma)$ is (γ, β)-precontinuous and (γ, β)-preclosed, then*

(i) *if f is injective and (Y, σ) is β-pre $T_{\frac{1}{2}}$, then (X, τ) is γ-pre $T_{\frac{1}{2}}$.*
(ii) *if f is surjective and (X, τ) is γ-pre $T_{\frac{1}{2}}$, then (Y, σ) is β- pre $T_{\frac{1}{2}}$.*

Proof. (i) Let A be a γ-pre g.closed set of (X, τ). Now to prove that A is γ-preclosed. By assumption it is obtained that $f(A)$ is β-pre g.closed and hence $f(A)$ is β-preclosed. Since f is (γ, β)-precontinuous, implies $f^{-1}(f(A))$ is γ-preclosed. Therefore A is γ- preclosed. Hence (X,τ) is γ-pre $T_{\frac{1}{2}}$.

(ii) Let B be a β- pre g.closed set in (Y,σ). Then $f^{-1}(B)$ is γ-preclosed since (X,τ) is γ-pre $T_{\frac{1}{2}}$ space. It follows from the assumption that B is γ-preclosed. ☐

DEFINITION 6.10. A mapping $f:(X,\tau)\to(Y,\sigma)$ is a said to be (γ,β)-prehomeomorphic, if f is bijective, (γ,β)-precontinuous and f^{-1} is (γ,β) - precontinuos.

THEOREM 6.11. *Let $f:(X,\tau)\to(Y,\sigma)$ is (γ,β)-prehomeomorphism. If (X,τ) is γ-pre $T_{\frac{1}{2}}$ then (Y,σ) is β-pre $T_{\frac{1}{2}}$.*

Proof. Let $\{y\}$ be a singleton set of (Y,σ), then there exists a point x of X such that $y=f(x)$. It follows from the assumption and Theorem 5.11 that $\{x\}$ is γ-preopen or γ-preclosed. By using Theorem 6.4 (ii) $\{y\}$ is β- preopen or β-preclosed. This implies (Y,σ) is β-pre $T_{\frac{1}{2}}$ space. ☐

THEOREM 6.12. *Suppose $\gamma:\tau\to P(X)$ is a regular operation. Then (X,τ) is γ-pre $T_{\frac{1}{2}}$.*

Proof. By Proposition 2.9[7], we have (X,τ) is a topological space. Now to prove (X,τ) is γ-pre $T_{\frac{1}{2}}$, it is enough to show that $\{x\}$ is γ-preopen or γ- preclosed.

Case (i): Suppose $\{x\}\in\tau_\gamma$, then by Theorem 3.4 $\{x\}$ is γ- preopen.

Case (ii): Suppose $\{x\}\notin\tau_\gamma$, then $\gamma-cl(\tau_\gamma-int\{x\})=\tau_\gamma-cl(\phi)=\phi\subset\{x\}$. Therefore $\{x\}$ is γ-preclosed. ☐

THEOREM 6.13. *Suppose $\gamma:\tau\to P(X)$ is a regular operation. Then (X,τ) is γ-pre $T_{\frac{1}{2}}$ if and only if (X,τ_γ) is pre $T_{\frac{1}{2}}$.*

Proof. By Proposition 2.9[7], we have (X,τ_γ) is a topological space. By Theorem 2.27 [3] it is pre $T_{\frac{1}{2}}$ space.

Conversely, If (X,τ_γ) is pre $T_{\frac{1}{2}}$, then $\{x\}$ is preopen or preclosed in (X,τ). Hence it is γ-preopen or γ- preclosed in (X,τ). ☐

THEOREM 6.14. *Let $f:(X,\tau)\to(Y,\sigma)$ be a (γ,β) - precontinuous and injective. If (Y,σ) is β-pre T_2(resp.γ-pre T_1), then (X,τ) is γ-pre T_2(resp. γ- pre T_1).*

Proof. Suppose (Y, σ) is β- pre T_2. Let x and y be distinct points of X. Then there exists two β-preopen set U and V such that $f(x) \in U$, $f(y) \in V$ and $U \cap V = \phi$. Since f is (γ, β)-pre continuous there exists two γ- preopen set W and S such that $f(W) \subseteq U$ and $f(S) \subseteq V$, implies $W \cap S = \phi$. This implies (X, τ) is γ- pre T_2. In similar way we prove (X, τ) is γ-pre T_1 whenever (X, τ) is β- pre T_1. □

THEOREM 6.15. *Suppose $\gamma : \tau \to P(X)$ is regular, then (X, τ) is γ- pre T_2 if and only if (X, τ) is pre T_2.*

Proof. Proof is straight forward from the Definition 5.3 and Proposition 2.9 [7]. □

REFERENCES

[1] D. Andrijevic, Semi preopen sets, Math. Vesnik, 38 (1986), 24-32.
[2] D. Andrijevic, On the topology generated by preopen sets, Math. Vesnik, 39 (1987), 367-376.
[3] H.Maki, J.Umehara and T. Noiri, Every topological space is pre T?, Mem. Fac. Sci. Kochi Univ. (Math), 17 (1996), 33-42.
[4] S.Kasahara, Operation - compact spaces, Math. Japonica, 24 (1979), 97-105.
[5] N.Levine, Semi -open sets and semi continuity in topological spaces, Amer. Math. Monthly 70(1963), 36-41.
[6] A.S. Mashhour, M.E.Abd El-Monsef and S.N.El-Deep, On pre-continuous and weak continuous mappings, Proc.,Math., Phys., Soc., Egypt, 53(1982), 47-53.
[7] H.Ogata, Operation on topological spaces and associated topology, Math. Japonica, 36 (1991), 175-184.
[8] G.Sai Sundara Krishnan, A new class of semi open sets in a topological space, Proc. NCMCM-2003, Allied Publishers, New Delhi, pp-305-311.

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