

GROUP ACTION ON INTUITIONISTIC FUZZY IDEALS OF RINGS

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ABSTRACT. Let G be a group acting on a ring R . We will define the group action of G on an intuitionistic fuzzy set of R . We will introduce intuitionistic fuzzy G -prime ideals of a ring and we will prove that every intuitionistic fuzzy G -prime ideal is the largest G -invariant intuitionistic fuzzy ideal of R contained in the intuitionistic fuzzy prime ideal which is uniquely determined up to G -orbits.

1. Introduction

K.T. Atanassov introduced the notion of intuitionistic fuzzy set as a generalization of the notion of fuzzy sets in 1986[2]. In[5] Banerjee and Basnet applied the concept of intuitionistic fuzzy sets to the theory of rings, and introduced the notions of intuitionistic fuzzy subrings and intuitionistic fuzzy ideals of a ring. Y.B. Jun and C.H.Park studied some properties of intrinsic product of intuitionistic fuzzy subrings and ideal in rings[9].

In this paper we will define an intuitionistic fuzzy prime ideal using the intrinsic product of intuitionistic fuzzy sets[9]. The study of groups acting on rings is an attempt to develop Galois theory for rings. R.P.Sharma and S.Sharma extended the group action to the fuzzy ideals of a ring with group action on it and studied some relations between the fuzzy G -prime ideals of a ring and the fuzzy prime ideals of that ring.

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In this paper we will investigate the relation between an intuitionistic fuzzy G -prime ideal of a ring and the intuitionistic fuzzy prime ideal of the given ring under the new definition of intuitionistic fuzzy prime ideals defined by this paper.

2. Preliminaries

As an generalization of the notion of fuzzy sets in a set S , Atanassov introduced the concept of an intuitionistic fuzzy set (IF set for short) A defined on a non-empty set S as objects having the form

$$A = \{ \langle x, \mu_A(x), \gamma_A(x) \rangle \mid x \in S \}.$$

where μ_A, γ_A are functions from S into the closed interval $[0, 1]$, which denote the degree of membership (namely μ_A) and the degree of non-membership (namely γ_A) of each element $x \in S$ to A respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for all $x \in S$. For the sake of simplicity, we shall use the symbol $A = (\mu_A, \gamma_A)$ for the IF set $A = \{ \langle x, \mu_A(x), \gamma_A(x) \rangle \mid x \in S \}$. Let $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ be IFs in a set M . We define

- $A \subseteq B \Leftrightarrow (\forall x \in M) (\mu_A(x) \leq \mu_B(x), \gamma_A(x) \geq \gamma_B(x))$.
- $A = B \Leftrightarrow A \subseteq B$ and $B \subseteq A$.
- $A \cap B = (\mu_A \wedge \mu_B, \gamma_A \vee \gamma_B)$.
- $A \cup B = (\mu_A \vee \mu_B, \gamma_A \wedge \gamma_B)$.
- $0_{\sim} = (0, 1)$ and $1_{\sim} = (1, 0)$.

Let $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ be IF set in a set S . We define the product $A \circ B = (\mu_{A \circ B}, \gamma_{A \circ B})$ in R given by

$$\mu_{A \circ B}(x) := \begin{cases} \bigvee_{x=ab} \min\{\mu_A(a), \mu_B(b)\} & \text{if } x \text{ is factorizable in } R \\ 0 & \text{otherwise,} \end{cases}$$

$$\gamma_{A \circ B}(x) := \begin{cases} \bigwedge_{x=ab} \min\{\gamma_A(a), \gamma_B(b)\} & \text{if } x \text{ is factorizable in } R \\ 1 & \text{otherwise} \end{cases}$$

The notion of an intrinsic product of two IF sets in a ring R was introduced by Y.B.Jun and C.H.Park as follows.

DEFINITION 2.1[9]. Let $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ be IF sets in a ring R . The intrinsic product of $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ is defined to be the IF set $A * B = (\mu_{A*B}, \gamma_{A*B})$ in R given by

$$\mu_{A*B}(x) := \bigvee_{x = \sum_{\text{finite}} a_i b_i} \min \left\{ \begin{array}{l} \mu_A(a_1), \mu_A(a_2), \dots, \mu_A(a_m), \\ \mu_B(b_1), \mu_B(b_2), \dots, \mu_B(b_m) \end{array} \right\}$$

$$\gamma_{A*B}(x) := \bigwedge_{x = \sum_{\text{finite}} a_i b_i} \max \left\{ \begin{array}{l} \gamma_A(a_1), \gamma_A(a_2), \dots, \gamma_A(a_m), \\ \gamma_B(b_1), \gamma_B(b_2), \dots, \gamma_B(b_m) \end{array} \right\}$$

if we can express $x = a_1 b_1 + a_2 b_2 + \dots + a_m b_m$ for some $a_i, b_i \in R$ and for some positive integer m where each $a_i b_i \neq 0$. Otherwise, we define $A * B = 0_{\sim}$, i.e., $\mu_{A*B}(x) = 0$ and $\gamma_{A*B}(x) = 1$.

DEFINITION 2.2. An IF sets $A = (\mu_A, \gamma_A)$ in a ring R is called an IF ideal of R if it satisfies the following conditions:

- (i) $(\forall x, y \in R) (\mu_A(x - y) \geq \min\{\mu_A(x), \mu_A(y)\})$,
- (ii) $(\forall x, y \in R) (\gamma_A(x - y) \leq \max\{\gamma_A(x), \gamma_A(y)\})$,
- (iii) $(\forall x, y \in R) (\mu_A(xy) \geq \max\{\mu_A(x), \mu_A(y)\})$,
- (iv) $(\forall x, y \in R) (\gamma_A(xy) \leq \min\{\gamma_A(x), \gamma_A(y)\})$,

Y.B.Jun and C.H.Park showed the following results in [9]

LEMMA 2.3. Let A, B and C be IF ideals of a ring R . Then

- (i) $A + B$ is an IF ideal.
- (ii) $A * (B * C) = (A * B) * C$.
- (iii) $A * (B + C) \subseteq (A * B) + (A * C)$.

3. IF prime ideals

We assume that R is a ring with identity denoted by 1 and G is a finite group such that G acts on R . The identity of the group G will usually also be denoted by 1. We define the group action of G on an IF set of a ring R .

DEFINITION 3.1. The group action of G on an IF set A of R is given by $A^g = \{ \langle x, \mu_A(x^g), \gamma_A(x^g) \rangle \mid x \in R \}$ where $g \in G$.

Under this definition, A^g is clearly an IF set if A is an IF set. We can denote A^g as $A^g = (\mu_{A^g}, \gamma_{A^g})$ if A is denoted by $A = (\mu_A, \gamma_A)$ where $\mu_{A^g}(x) = \mu_A(x^g)$ and $\gamma_{A^g}(x) = \gamma_A(x^g)$. Also we can easily know that if A is an IF ideal then A^g is also an IF ideal.

LEMMA 3.2. *Let A be an IF ideal of R . Then A^g is also an IF ideal of R .*

Proof. It is easily proved by the fact that $(x - y)^g = x^g - y^g$ and $(xy)^g = x^g y^g$ for every every $x, y \in R$.

We will define IF prime ideals of a ring R as following

DEFINITION 3.3. *An IF ideal P of a ring is called an IF prime ideal if for two IF ideals A and B , $A * B \subseteq P$ imply that either $A \subseteq P$ or $B \subseteq P$.*

Instead of $A * B \subseteq P$, if $A \circ B \subseteq P$ imply that either $A \subseteq P$ or $B \subseteq P$ then since $A \circ B \subseteq A * B \subseteq P$, imply that either $A \subseteq P$ or $B \subseteq P$. But the converse is not generally true.

LEMMA 3.4. *Let $P = (\mu_P, \gamma_P)$ be an IF prime ideal of R . Then P^g is also an IF prime ideal of R .*

Proof. Let $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ be IF ideals of R and $A * B \subseteq P^g$. At first we claim that $A^{g^{-1}} * B^{g^{-1}} \subseteq P$. Since $A^{g^{-1}} = (\mu_{A^{g^{-1}}}, \gamma_{A^{g^{-1}}})$ and $B^{g^{-1}} = (\mu_{B^{g^{-1}}}, \gamma_{B^{g^{-1}}})$ it is sufficient to show that $\mu_{A^{g^{-1}} * B^{g^{-1}}} \subseteq \mu_P$ and $\gamma_{A^{g^{-1}} * B^{g^{-1}}} \supseteq \gamma_P$.

For every $r \in R$,

$$\begin{aligned} & \mu_{A^{g^{-1}} * B^{g^{-1}}}(r) \\ &= \bigvee_{r = \sum_{finite} a_i b_i} (\min(\mu_A(a_1^{g^{-1}}), \dots, \mu_A(a_n^{g^{-1}}), \mu_B(b_1^{g^{-1}}), \dots, \mu_B(b_n^{g^{-1}}))) \\ &= \bigvee_{r^{g^{-1}} = \sum_{finite} a_i^{g^{-1}} b_i^{g^{-1}}} (\min(\mu_A(a_1^{g^{-1}}), \dots, \mu_A(a_n^{g^{-1}}), \mu_B(b_1^{g^{-1}}), \dots, \mu_B(b_n^{g^{-1}}))) \\ &\leq \mu_{A * B}(r^{g^{-1}}) \leq \mu_{P^g}(r^{g^{-1}}) = \mu_P(r) \end{aligned}$$

and

$$\begin{aligned} & \gamma_{A^{g^{-1}} * B^{g^{-1}}}(r) \\ &= \bigwedge_{r = \sum_{finite} a_i b_i} (\max(\gamma_A(a_1^{g^{-1}}), \dots, \gamma_A(a_n^{g^{-1}}), \gamma_B(b_1^{g^{-1}}), \dots, \gamma_B(b_n^{g^{-1}}))) \\ &= \bigwedge_{r^{g^{-1}} = \sum_{finite} a_i^{g^{-1}} b_i^{g^{-1}}} (\max(\gamma_A(a_1^{g^{-1}}), \dots, \gamma_A(a_n^{g^{-1}}), \gamma_B(b_1^{g^{-1}}), \dots, \gamma_B(b_n^{g^{-1}}))) \\ &\geq \gamma_{A * B}(r^{g^{-1}}) \geq \gamma_{P^g}(r^{g^{-1}}) = \gamma_P(r) \end{aligned}$$

Thus we know that $A^{g^{-1}} * B^{g^{-1}} \subseteq P$ and $A^{g^{-1}} \subseteq P$ or $B^{g^{-1}} \subseteq P$. Thus $A \subseteq P^g$ or $B \subseteq P^g$.

We will define G -invariant IF ideals of a ring R .

DEFINITION 3.5. *Let A be an IF ideal of a ring R . Then A is said to be a G -invariant IF ideal of R if and only if $\mu_{A^g}(r) = \mu_A(r^g) \geq \mu_A(r)$ and $\gamma_{A^g}(r) = \gamma_A(r^g) \leq \gamma_A(r)$ for all $r \in R, g \in G$.*

From this definition, we can easily know that if A is G -invariant IF ideal of R if and only if $A^g = A$ for all $g \in G$ since $\gamma_A(r) = \gamma_A((r^g)^{g^{-1}}) \leq \gamma_A(r^g) \leq \gamma_A(r)$ (Similarly we can get $\mu_A = \mu_A^g$).

LEMMA 3.6. *Let A be an IF ideal of a ring R and let $A^G = \bigcap_{g \in G} A^g$. Then A^G is the largest G -invariant IF ideal of R contained in A .*

Proof. Clearly A^G is an IF ideal since every intersection of IF ideals of a ring R is also IF ideal. Let B be a G -invariant IF ideal contained in A . Then since $A^G = \bigcap_{g \in G} A^g$ for all $g \in G$, we obtain $B \subseteq A^G$.

Let $A^G = (\mu_{A^G}, \gamma_{A^G})$ then it is easily obtained that $\mu_{A^G}(r) = \min_{g \in G}(\mu_A(r^g))$ and $\gamma_{A^G}(r) = \max_{g \in G}(\gamma_A(r^g))$. From Lemma 3.6 we know that an IF ideal A of R is G -invariant of R if and only if $A = A^G$.

LEMMA 3.7. *If $A = (\mu_A, \gamma_A)$, $B = (\mu_B, \gamma_B)$ are G -invariant IF ideals of R , then $A * B = (\mu_{A * B}, \gamma_{A * B})$ is a G -invariant IF ideal.*

Proof. Let $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$. We will show that $\mu_{A * B}(r) = \mu_{A * B}(r^g)$ and $\gamma_{A * B}(r^g) = \gamma_{A * B}(r)$ for every $g \in G$ and $r \in R$ where $A * B = (\mu_{A * B}, \gamma_{A * B})$. Since $r^g = \sum_{finite} a_i b_i$ if and only

if $r = \sum_{finite} a_i^{g^{-1}} b_i^{g^{-1}}$ and $\mu_{A^G} = \mu_A$, $\gamma_{A^G} = \gamma_A$, $\mu_{B^G} = \mu_B$ and $\gamma_{B^G} = \gamma_B$, we know that for every $r \in R$ and every $g \in G$,

$$\begin{aligned} \mu_{A*B}(r^g) &= \bigvee_{r^g = \sum_{finite} a_i b_i} (\min(\mu_A(a_1), \mu_A(a_2), \dots, \mu_A(a_n), \mu_B(b_1), \\ &\quad \dots, \mu_B(b_n))) \\ &= \bigvee_{r = \sum a_i^{g^{-1}} b_i^{g^{-1}}} (\min(\mu_A(a_1^{g^{-1}}), \dots, \mu_A(a_n^{g^{-1}}), \mu_B(b_1^{g^{-1}}), \\ &\quad \dots, \mu_B(b_n^{g^{-1}}))) \\ &= \mu_{A*B}(r) \end{aligned}$$

Similarly we can know that $\gamma_{A*B}(r^g) = \gamma_{A*B}(r)$. Thus $A * B$ is G -invariant.

4. IF G -prime ideals

DEFINITION 4.1. Let $\{A_n = (\mu_{A_n}, \gamma_{A_n}) | n \in I\}$ be a nonempty collection of IF sets of a ring R . Then defined the IF set

$$\bigcup_{n \in I} A_n = (\mu_{\bigcup_{n \in I} A_n}, \gamma_{\bigcup_{n \in I} A_n})$$

by $\mu_{\bigcup_{n \in I} A_n} = \bigwedge_{n \in I} \mu_{A_n}$ and $\gamma_{\bigcup_{n \in I} A_n} = \bigvee_{n \in I} \gamma_{A_n}$.

LEMMA 4.2. Let $\{A_n\}$ be a chain of IF ideals of R . Then $\bigcup_n A_n$ is an IF ideal of R .

Proof. To prove the Lemma 4.1, we will prove that for every $x, y \in R$

$$\min(\vee_n(\mu_{A_n}(x), \vee_n(\mu_{A_n}(y))) = \vee_n(\min(\mu_{A_n}(x), \mu_{A_n}(y)))$$

and

$$\max(\wedge_n(\gamma_{A_n}(x), \wedge_n(\gamma_{A_n}(y))) = \wedge_n(\max(\gamma_{A_n}(x), \gamma_{A_n}(y)))$$

where $\mu_{A_i} \leq \mu_{A_{i+1}}$ and $\gamma_{A_i} \geq \gamma_{A_{i+1}}$. Assume that

$$k = \max(\wedge_n(\gamma_{A_n}(x), \wedge_n(\gamma_{A_n}(y))) < \wedge_n(\max(\gamma_{A_n}(x), \gamma_{A_n}(y))) = \ell.$$

Then $\wedge_n(\gamma_{A_n}(x)) < l$ and $\wedge_n(\gamma_{A_n}(y)) < \ell$. Thus there exist s, t such that $\gamma_{A_s}(x) < \ell$ and $\gamma_{A_t}(y) < \ell$ by the property of infimum. Then

for some m such that m is larger than s and t , $\gamma_{A_s}(x) \geq \gamma_{A_m}(x)$ and $\gamma_{A_s}(y) \geq \gamma_{A_m}(y)$. Thus $\max(\gamma_{A_m}(x), \gamma_{A_m}(y)) < l$, which is contradiction to the fact that $\wedge_n(\max(\gamma_{A_n}(x), \gamma_{A_n}(y))) = l$. On the other hand assume that

$$k = \max(\wedge_n(\gamma_{A_n}(x), \wedge_n(\gamma_{A_n}(y))) > \wedge_n(\max(\gamma_{A_n}(x), \gamma_{A_n}(y))) = l.$$

In this case we can assume that $k = \wedge_n(\gamma_{A_n}(x))$ without generality. Then there exists some m such that $\max(\gamma_{A_m}(x), \gamma_{A_m}(y)) < k$. This is contradiction to the fact $k \leq \gamma_{A_m}(x)$. Thus we know that

$$\max(\wedge_n(\gamma_{A_n}(x), \wedge_n(\gamma_{A_n}(y))) = \wedge_n(\max(\mu_{A_n}(x), \mu_{A_n}(y)))$$

and similarly we can know that

$$\min(\vee_n(\mu_{A_n}(x), \vee_n(\mu_{A_n}(y))) = \vee_n(\min(\mu_{A_n}(x), \mu_{A_n}(y))).$$

Let $x, y \in R$,

$$\begin{aligned} \mu_{\cup_n A_n}(x - y) &= \bigvee_n \mu_{A_n}(x - y) \\ &\geq \vee_n(\min(\mu_{A_n}(x), \mu_{A_n}(y))) \\ &= \min(\bigvee_n \mu_{A_n}(x), \bigvee_n \mu_{A_n}(y)) \\ &= \min(\mu_{\cup_n A_n}(x), \mu_{\cup_n A_n}(y)) \end{aligned}$$

and

$$\begin{aligned} \gamma_{\cup_n A_n}(x - y) &= \bigwedge_n \gamma_{A_n}(x - y) \\ &\leq \wedge_n(\max(\gamma_{A_n}(x), \gamma_{A_n}(y))) \\ &= \max(\bigwedge_n \gamma_{A_n}(x), \bigwedge_n \gamma_{A_n}(y)) \\ &= \max(\gamma_{\cup_n A_n}(x), \gamma_{\cup_n A_n}(y)) \end{aligned}$$

by the above results. Moreover, we can know that

$$\begin{aligned} \mu_{\cup_n A_n}(xy) &= \bigvee_n \mu_{A_n}(xy) \\ &\geq \vee_n(\max(\mu_{A_n}(x), \mu_{A_n}(y))) \\ &\geq \bigvee_n \mu_{A_n}(x) = \mu_{\cup_n A_n}(x) \end{aligned}$$

and

$$\begin{aligned} \mu_{\cup_n A_n}(xy) &= \bigvee_n \mu_{A_n}(xy) \\ &\geq \vee_n(\max(\mu_{A_n}(x), \mu_{A_n}(y))) \\ &\geq \bigvee_n \mu_{A_n}(y) = \mu_{\cup_n A_n}(y) \end{aligned}$$

It follows that

$$\mu_{\cup_n A_n}(xy) \geq \max(\mu_{\cup_n A_n}(x), \mu_{\cup_n A_n}(y))$$

And also we can get the fact that

$$\gamma_{\cup_n A_n}(xy) \leq \min(\gamma_{\cup_n A_n}(x), \gamma_{\cup_n A_n}(y))$$

Thus we know that $\bigcup_n A_n$ is an IF ideal of R .

An IF G -prime ideal of a ring R is defined as followings.

DEFINITION 4.3. A non constant G -invariant IF ideal P of a ring R is called IF G -prime ideal if for any two G -invariant IF ideals A and B of R , $A * B \subseteq P$ implies that either $A \subseteq P$ or $B \subseteq P$.

From this definition, we will prove the following theorem.

THEOREM 4.4. If P is an IF prime ideal of R , then P^G is an IF G -prime ideal of R . Conversely, if Q is an IF G -prime ideal of R , then there exists an IF prime ideal P of R such that $P^G = Q$ where P is unique up to its G -orbit.

Proof. Let P be an IF prime ideal of R and let A and B be two G -invariant IF ideals of R such that $A * B \subseteq P^G$. Then $A * B \subseteq P$ since $P^G \subseteq P$. Since P is an IF prime ideal of R , either $A \subseteq P$ or $B \subseteq P$. Thus $A \subseteq P^G$ or $B \subseteq P^G$ since P^G is the largest G -invariant IF ideal contained in P . Conversely, let Q be an IF G -prime ideal of R . Let

$$S = \{A, \text{ an IF ideal of } R \mid A^G \subseteq Q, Q \subseteq A\}$$

Since $Q^G \subseteq Q$, S is not empty. Let $C = \{A_i\}$ be a chain of S . Then $\cup A_i$ is an IF ideal of R by lemma 4.1. Moreover we can get the fact that $(\cup A_i)^G \subseteq Q$. Let $\cup A_i = (\mu, \gamma)$ and $Q = (\mu_Q, \gamma_Q)$, then clearly $\mu = \bigvee_i \mu_{A_i}$ and $\gamma = \bigwedge_i \gamma_{A_i}$ where $A_i = (\mu_{A_i}, \gamma_{A_i})$. For every $x \in R$,

$$\mu^G(x) = \min_{g \in G} \left(\bigvee_i (\mu_{A_i}(x^g)) \right) = \bigvee_i \left(\min_{g \in G} (\mu_{A_i}(x^g)) \right) \leq \mu_Q(x)$$

since each A_i^G is contained in Q . And

$$\gamma^G(x) = \max_{g \in G} (\bigwedge_i (\gamma_{A_i}(x))) = \bigwedge_i (\max_{g \in G} (\gamma_{A_i}(x))) \geq \gamma_Q(x).$$

Third equality of each above term is followed by a generalization of fact that

$$\min(\vee_n(\mu_{A_n}(x), \vee_n(\mu_{A_n}(y))) = \vee_n(\min(\mu_{A_n}(x), \mu_{A_n}(y)))$$

and

$$\max(\wedge_n(\gamma_{A_n}(x), \wedge_n(\gamma_{A_n}(y))) = \wedge_n(\max(\gamma_{A_n}(x), \gamma_{A_n}(y)))$$

which are shown in Lemma 4.1. Thus we know that $(\cup A_i)^G \subseteq Q$ and $(\cup A_i)^G \in S$. From this fact we can apply Zorn's lemma to S and let P be a maximal such IF ideal. Suppose that A and B are IF ideals of R such that $A * B \subseteq P$ and $P \subseteq A, P \subseteq B$. Then $(A * B)^G \subseteq P^G \subseteq Q$. Since $A^G * B^G \subseteq A * B$ and $A^G * B^G$ is G -invariant, $A^G * B^G \subseteq (A * B)^G$. Thus $A^G * B^G \subseteq (A * B)^G \subseteq Q$ implies $A^G \subseteq Q$ or $B^G \subseteq Q$. By the maximality of P , we know that $A \subseteq P$ or $B \subseteq P$ and P is IF prime ideal. On one hand since $Q \subset P, Q^G \subseteq P^G$ and thus we know that $P^G = Q$. Suppose that there exists another IF prime ideal T of R such that $T^G = Q$. Since $*_{g \in G} P^g \subseteq \cap_{g \in G} P^g = P^G \subseteq T, P^g \subseteq T$ for some $g \in G$ by the primeness of T . Thus $P \subseteq T^{g^{-1}}$ and $P = T^{g^{-1}}$ since the fact that $(T^{g^{-1}})^G = T^G \subseteq Q$ implies that T is contained in the set S . Thus we know that P is unique up to G -orbits.

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