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A Comparative Study of Mathematics Tests in China and UK[1]

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This paper uses a composite difficulties model developed by the author (Bao 2002a, 2002b) to compare the characteristics of two sample mathematics tests in China and UK. The initial findings were described through five difficulty factors on several difficulty levels. According the initial findings, the author then tries to analyze the styles of mathematics problem-solving and the curriculum backgrounds in these two countries.

Keywords: difficulty model, mathematics test, international comparison, problem-solving
ZDM Classification: C33, C80, D53, U43
MSC2000 Classification: 97C30, 97C40, 97D10, 97U40

1. INTRODUCTION

Regarding the differences in the mathematical achievement between East Asian and Western countries, many researchers try to explain why East Asian students consistently out-perform their counterparts in the West. Some initial findings include: The influences of Confucian cultural heritage, which makes the students hard-working and achievement-oriented (Wong 1996, 1998), the world-class mathematics curriculum (Ginsberg et al. 2005), teachers' profound understanding of fundamental mathematics (Ma 1999), more time spent on after-school instruction and studying (Fuligni & Stevenson 1995), having more positive attitudes toward achievement, and stronger belief that studying is the major means for academic success (Chen & Stevenson 1995), and so on. Park (2004) also reviewed and discussed some factors contributing to East Asian students' success in international mathematics and science comparative studies, which include the number system, selection-oriented education environment, attitudes of students towards the test,

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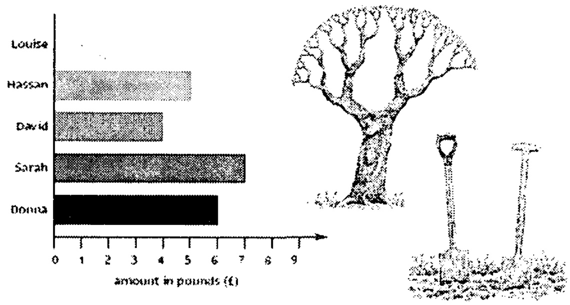
and teacher education.

The purpose of this paper is to compare the difficulty characteristics of mathematics problems in some Chinese and British tests, and then to discuss the different problem-solving styles between Chinese and British junior high school students.

Let us first see two sample items from Key Stage 3 Assess and Review Test in UK and Suzhou City High school entrance examination in China:

Sample A (GCSE 2001, UK): Five children collect money to plant trees. Here is a bar chart of the amount they have raised so far. Their target is £40 altogether. How much more money do they need to reach the target?



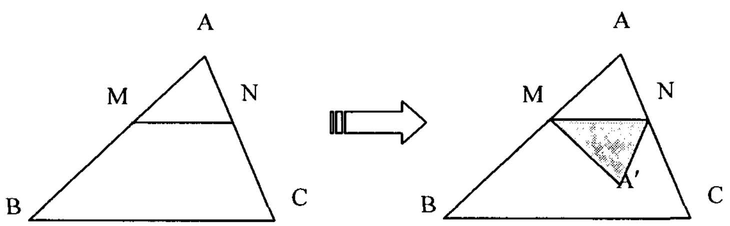
Sample B (HSEE 2001, China): Given a triangle ABC (Figure (1)) with area of 25 cm^2. The size of line segment BC is 10 cm. $\angle B$ and $\angle C$ are acute angles. M is a movable point on side AB (M is different to A and B). Through point M make a line segment MN so that it parallel to BC and intersect with AC at point N. The size of line segment MN is labelled with x cm.

(1) Use x to represent the area of triangle AMN.

(2) Turn the small triangle ΔAMN around line segment MN. The new place of point A is re-labelled with A′ and the overlapped area of triangle $A'MN$ and quadrangle $BCNM$ is y cm^2 (Figure (2)).

a. Find an equation for y in terms of x, and the domain of x.

b. Find the maximum of the overlapped area y and the value of x when y gets its maximum point.



(1) (2)

Each of the two samples is the last item (so it is the most difficult item in the test) in its tests, but they seem quite different. The solution of sample A only needs two steps:

S_1: From the bar chart to find the sum of the amount they have already raised, which is 3+5+4+7+6 = 25(£).

S_2: Find the money they needed, which is 40 – 25 = 15(£).

We can use a flow chart to show the problem-solving process of the sample item:

$$\boxed{S_1} \longrightarrow \boxed{S_2}$$

Figure 1: Problem-solving process of sample A

Comparing to the first sample, sample B needs much more steps as following:

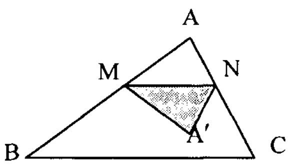
S_1: $MN // BC \Rightarrow \angle AMN = \angle ABC,\ \angle ANM = \angle ACB$ (According the given T_0 and theorem T_1: A straight line falling on parallel straight lines makes the corresponding angles equal to one another.)

S_2: Prove $\Delta AMN \backsim \Delta ABC$. (According theorem T_2: If one triangle has two angles equal to two angles of another triangle, then the two triangles are similar to each other.)

S_3: Find $\frac{S_{\Delta AMN}}{S_{\Delta ABC}} = \left(\frac{x}{BC}\right)^2$. (According theorem T_3: If two similar triangles have a scale factor of a:b, then the ratio of their areas is $a^2:b^2$).

S_4: Find $S_{\Delta AMN} = \frac{1}{4}x^2$. This is the end of the solution of (1). (According the property of proportional T_4)

S_5: There are three situations according the location of point A′, as the following figures:



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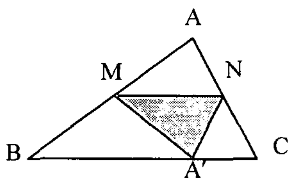

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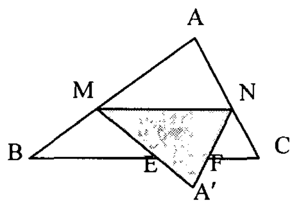

(5)

S_6: If point A′ is on the side of BC, as figure (4), then the line segment MN bisects the height of ΔABC on base BC (According the property of congruent triangles T_5).

S_7: Find $x = \frac{BC}{2} = 5$ (According the given T_0 and theorem T_6: If a straight line is drawn parallel to one of the sides of a triangle, then it cuts the sides of the triangle proportionally).

S_8: So y = $\frac{1}{4}x^2 = \frac{25}{4}$ (According step S_4).

S_9: If point A′ is inside quadrangle *BCNM*, as figure (3), then $0 < x < 5$, and also $y = \frac{1}{4}x^2$. (According step S_4 and S_7).

S_{10}: When $0 < x \le 5$, function $y = \frac{1}{4}x^2$ is monotonically increasing, so y gets its maximum at $x = 5$. The maximum is $\frac{1}{4}\times(5)^2 = \frac{25}{4}$. (According the property of quadratic function T_6)

S_{11}: If point A′ is outside quadrangle *BCNM*, as figure (5), then find $5<x<10$ (According step S_7 and given T_0)

S_{12}: Find y = $S_{\Delta A'MN} - S_{\Delta A'EF}$ (Given T_0).

S_{13}: Find $EF // MN$ (Given T_0).

S_{14}: Prove $\Delta A'EF \backsim \Delta A'MN$ (According T_0, T_1 and T_2).

S_{15}: Prove $\Delta A'EF \backsim \Delta ABC$ (According step S_2, and theorem T_7: If two triangles are similar to another triangle, they are similar each other).

S_{16}: Find that the height h of ΔABC (base on side *BC*) is 5 (According T_0 and the area formula of triangle T_8).

S_{17}: From $\Delta A'EF \backsim \Delta ABC$, get $\frac{S_{\Delta A'EF}}{S_{\Delta ABC}} = \left(\frac{h_2}{5}\right)^2$ (Suppose the height of $\Delta A'EF$ on base *EF* is h_2, according step S_{14}, and theorem T_3).

S_{18}: Find $y = -\frac{3}{4}x^2 + 10x - 25$, $5 < x < 10$ (According the property of proportional T_4, then simplify the equation).

S_{19}: Re-write the equation $y = -\frac{3}{4}x^2 + 10x - 25$ to $y = -\frac{3}{4}(x - \frac{20}{3})^2 + \frac{25}{3}$ (Using "completing the square" method M1).

S_{20}: Find $y_{\max} = \frac{25}{3}$ at $x = \frac{20}{3}$ (According the maximum theorem of quadratic function T_9)

S_{21}: Comparing the results in S_8, S_{10} and S_{20}, get the absolutely maximum of *y*, which is $\frac{25}{3}$ at $x = \frac{20}{3}$. (The end)

We also use a flow chart to show the problem-solving process of sample B:

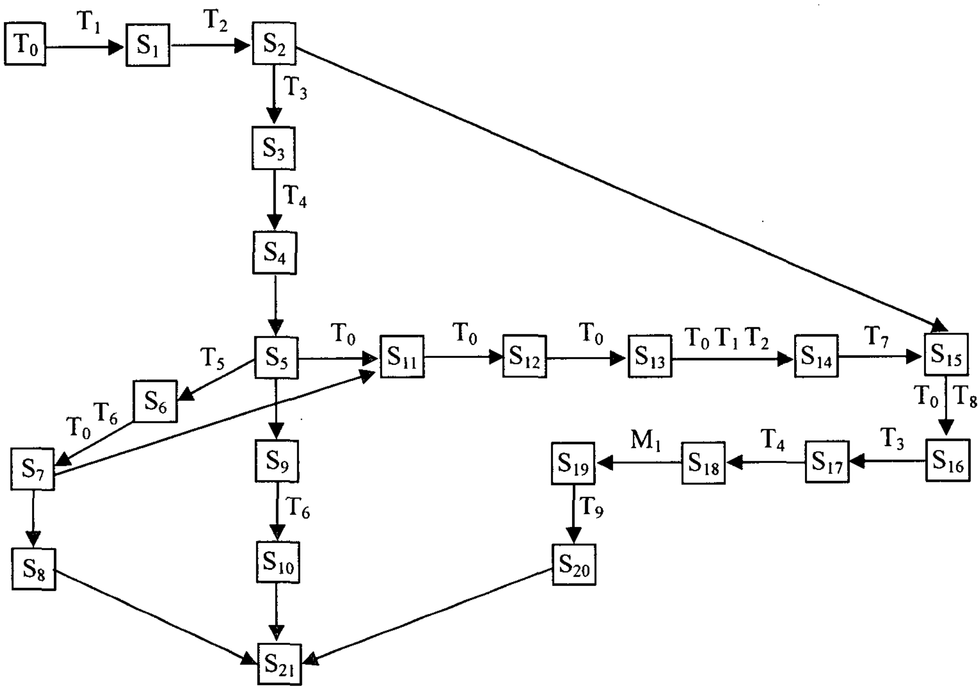

Figure 2: Problem-solving process of sample B

Comparing the processes of sample A and sample B (Figures 1 and 2), the big difference is easy to find. For example, the Chinese sample has much more steps and needs more mathematics knowledge, which includes many theorems and different methods. Although the two sample items belong to the same grade level, their difficulty characteristics and levels are certainly not same. This leads to the following research questions:

- How to assess the difficulty characteristics and levels of mathematics tests?
- What are the differences of difficulty characteristics and levels of mathematics tests between China and UK? And then
- How do these differences affect styles of mathematics problem-solving?

The research methodology for this study will use the one that undertaken in another related study by the author. Bao (Bao 2002a, 2002b) developed a composite difficulty model which compared Chinese with British mathematics curricula. The initial findings of this study show that there are significant differences between the two mathematics curricula. For example, the Chinese mathematics curriculum attaches more importance to the understanding of mathematical knowledge and methods, than to activities which investigate the use of mathematics. That is, in Chinese textbooks there are few mathematical problems related to the daily life of students. In addition, the level of "two basics" in the Chinese mathematics curriculum is much more advanced than that found in the British mathematics curriculum; and so on.

In this paper, however, we want to know whether or not the case is similar to the mathematics tests in China and UK and how it affects students' problem-solving.

2. THE COMPOSITE DIFFICULTY MODEL

The difficulty levels of mathematics tests are used to be measured by the difficulty index, that is, the average percent of students responding correctly. However, the difficulty index is not a suitable tool for comparative study on different tests in different countries. Actually, that measured by the difficulty index are the levels of students' performance, not the difficult levels of mathematics tests. Furthermore, the difficulty index of a test item can only show how many students answer the item correctly. It cannot reveal the difficulty characteristics of the item.

Some researchers have tried to find new models for comparison of mathematics test. Ginsberg; Leinwand; Anstrom; Pollock & Witt (2005) characterized each mathematics problem in three ways: the approximate number of steps to arrive at a solution; whether the solution strategy involved solving for an unknown intermediate variable; and whether the solution strategy involved merely a routine application of a formula or definition (e.g., a straightforward, obvious, or commonplace application of mathematical knowledge) or, alternatively, a nonroutine strategy or approach to solving the problem.

Heinze; Cheng & Yang (2004) divided test items into three levels of competency:

(I) basic competency (applying facts and rules, *e.g.* for calculations),
(II) argumentative competency (one-step-argumentation), and
(III) argumentative competency (combining several arguments).

Heller & Heller (1999) listed twenty-one difficulty characteristics of a problem, which include: no explicit target variable, unfamiliar context, atypical situation, unusual target variable, simplifying assumptions, and more than two subparts.

Nohara (2001) has compared the difficulty of test items in NAEP, TIMSS-R and PISA. He identified several factors that could contribute to the relative difficulty of the assessments. Among them, four are likely to make items more difficult for most students in most cases. These include the response type, the context of the item, requirements for multi-step reasoning, and the amount of computation. Figure 3 presents each of these factors together for each assessment on four-line graphs.

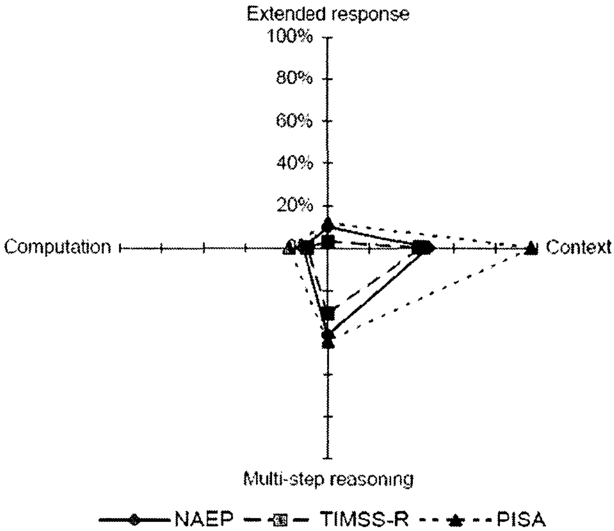

Figure 3: Mathematics difficulty factors

Based on Nohara's model of overall difficulty, I developed a new model for composite difficulty of mathematics tests, which has five factors (see Figure 4).

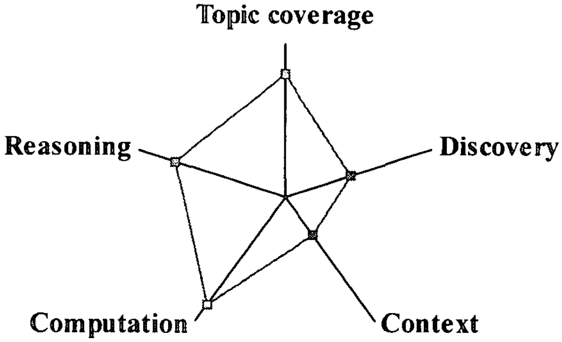

Figure 4: Composite difficulty model (Bao 2002a, 2002b)

Each factor in the model is further divided into several levels (Table 1).

Table 1. Difficult levels of composite difficulty factors (Bao 2002a, 2002b)

Factors	Levels			
Discovery	Knowing	Understanding	Investigating	
Context	None	Personal life	Public affair	Scientific situation
Computation	None	Number computation	Simple symbolic computatio n	Complex symbolic computatio n
Reasoning	None	Simple Reasoning	Complex Reasoning (more than two steps)	
Topic coverage	Single topic	Two topics	Above three topics	

The above model is used to assess the composite difficulty of a mathematics problem. In order to evaluate the composite difficulty of a mathematics test, the difficulty index of each factor will be estimated by the following formula:

$$d_i = \frac{\sum_j n_{ij} d_{ij}}{n} \qquad (\sum_j n_{ij} = n; \qquad i = 1,2,3,4,5; \qquad j = 1,2,\Lambda) \qquad (*)$$

Where d_i $(i = 1, 2, 3, 4, 5)$ corresponds to five factors; d_{ij} represents the i-th power index of the j-th level; n_{ij} is the total number of items which belong to the j-th level of the i-th factor, the sum of n_{ij} is n (total items in the test).

3. THE SAMPLES

The British samples are from official GCSE 1996, which include three levels: foundation, intermediate and higher. There are totally 140 items. The Chinese samples are of HSEE (High School Entrance Examination) in Suzhou City. The total number of items is 126.

Table 2 shows the numbers and percentages of items on each level for each factor in the two samples.

4. THE INITIAL FINDINGS

Based on Table 2, this part will first compare difficulty levels on each difficulty factor of GCSE with HSEE, and try to find any difficulty characteristics of these tests. Then, the composite difficulties of the two samples will be analysed.

Table 2. Numbers, percentages and weighted means of items on each difficulty level

Factors	Levels		Number of Items		Percentages		Weighted Means	
			GCSE	HSEE	GCSE	HSEE	GCSE	HSEE
Investigation	Knowing		87	53	62.14	42.06	1.41	1.58
	Understanding		49	73	35	57.94		
	Investigating		4	0	2.86	0		
Context	None		64	106	45.71	84.13	1.66	1.27
	Personal		56	6	40	4.76		
	Public		19	14	13.57	11.11		
	Scientific		0	0	0	0		
Computation	None		50	23	35.71	18.25	1.83	3.33
	Number		73	40	52.14	31.75		
	Simple Symbolic	1 step	9	12	11.43	22.22		
		2 steps	7	16				
	Complex Symbolic	3 steps	1	13	0.71	27.78		
		4 steps	0	9				
		5 steps	0	7				
		6 steps	0	6				
Reasoning	None		53	28	37.86	22.22	1.73	3.33
	Simple	1 step	73	29	61.43	40.48		
		2 steps	13	22				
	Complex	3 steps	1	15	0.71	37.30		
		4 steps	0	18				
		5 steps	0	5				
		6 steps	0	2				
		7 steps	0	1				
		8 steps	0	3				
		11 Steps	0	2				
		14 steps	0	1				
Topic Coverage	Single		71	28	50.71	22.22	1.55	2.83
	Two		61	33	43.57	26.19		
	Above Three	Thee	8	30	5.71	51.59		
		Four	0	14				
		Five	0	11				
		Six	0	8				
		Seven	0	2				

Note 1. The weighted mean is calculated by formula (*).

2. Percentage and weighted mean are rounded to 0.01.

4.1 Differences on Discovery Levels

As shown in Table 2, 64.14% of items in GCSE belong to the "Knowing" level, whereas with the HSEE sample is 40.06 %. The percentages for items belonging to the "Understanding" level in each sample are respectively 35% and 57.94%; the percentages for items belonging to the "Investigating" level are 2.86% and 0% (see Fig. 5).

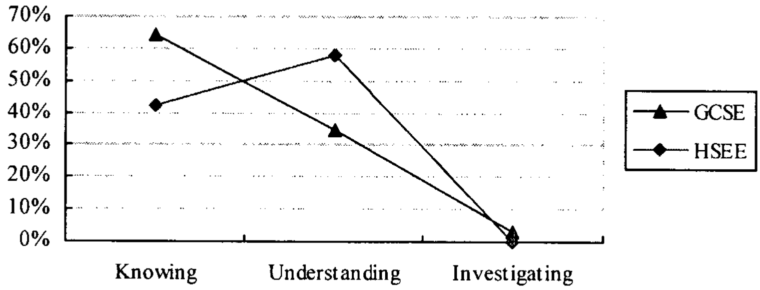

Figure 5: Comparison on discovery levels between GCSE and HSEE

Figure 5 shows that the percentages in HSEE, compared to GCSE, are lower on the "Knowing" level, but higher on the "Understanding" level. On the "Investigating" level, the two samples are nearly the same, which means that the most items in the two samples are traditional mathematics problems.

4.2 Differences on Context levels

The contexts of mathematics problems can be divided into four levels by the "distance" of the material to the students (Bao 2002a & OECD 2000). The closest is (personal) daily life, next is public/occupational life, and the most distant contexts for students are scientific ones. As shown in Table 2, 84.13% of items in HSEE do not provide any real-life contexts, whereas in GCSE sample the percentage is only 45.71%. The percentages of items belonging to "Personal" context level in each sample are respectively 4.76% and 40%; the percentages of items on "Public" context level are 11.11% and 13.57%. There are no any items related to "Scientific" contexts in both samples (see Fig. 6).

The Figure 6 shows that there are more items in GCSE connected to real-life situations. In HSEE, almost all the items are "pure mathematics" problems. The few items which are related to real-life contexts in HSEE are usually associated with a traditional workplace.

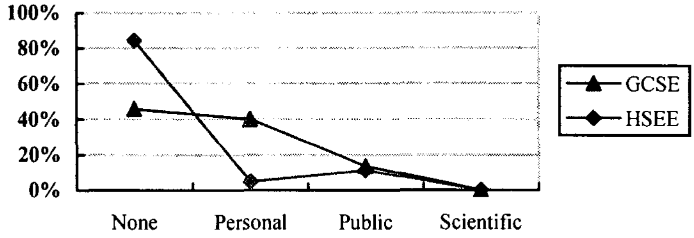

Figure 6: Comparison on context levels between GCSE and HSEE

It is necessary to note, however, that the percentage of items that have “scientific” contexts is still very low both in China and in UK. The new teaching and learning styles, such as “Project Learning,” “Cross Subjects activity,” “Scientific Investigation,” are increasingly popular in many countries, although this leads us to question whether scientific situations can be better integrated into the mathematics tests. Obviously, this issue requires further research.

4.3 Differences on Computation Level

Figure 7 shows the percentages of items for different difficulty levels of computation in GCSE and HSEE.

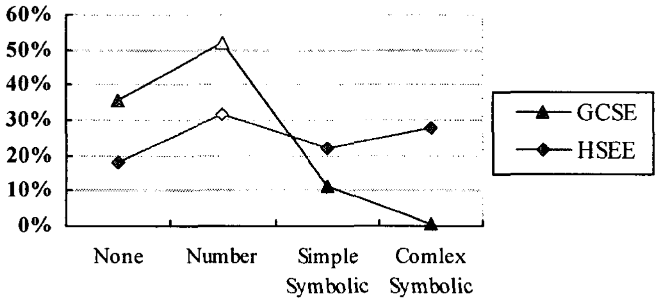

Figure 7: Comparison on computation level between GCSE and HSEE

According the above figure, there are much more items that require “Complex symbolic computation” in HSEE than in GCSE.

4.4 Differences on Reasoning Levels

Mathematical reasoning is a strong point in traditional Chinese mathematics curriculu m, especially in the geometry curriculum. Compared to the samples of GCSE, there is a v ery strong sense of mathematics reasoning in HSEE, as seen in the following figure.

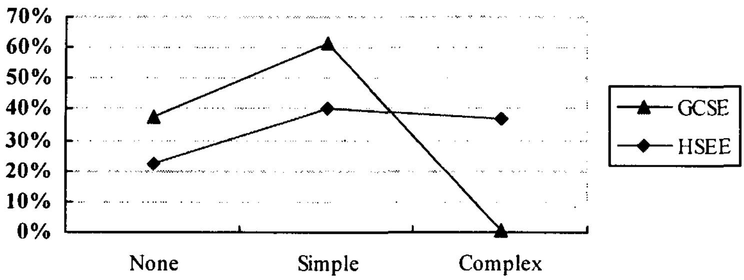

Figure 8: Comparison on reasoning level between GCSE and HSEE

From Table 2 we can see that there is only one item belonging to the "Compl ex reasoning level" in GCSE. But in HSEE, 32 items need more than 3 steps of formal mathematic reasoning. The largest number of reasoning steps needed by sin gle item in HSEE is 14. It's certainly a long way for students to get the solution of the item.

4.5 Differences on Topic coverage levels

In this paper, the concept "Topic coverage" indicates the number of mathematics topics in a single item. The following figure is based on Table 2.

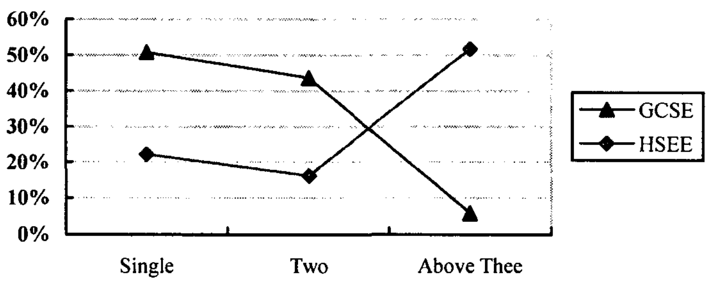

Figure 9: Comparison on topic coverage level between GCSE and HSEE

Figure 9 shows that more than half items in GCSE only have a "single topic" and less than 5.71% of items cover "Three topics." But in HSEE, more than half items cover at least three different mathematics topics.

4.6 Differences on Composite difficulties

In the above five sections, we have compared the difficulty levels on different factors with the samples of GCSE and HSEE.

In this section, we will use the composite difficulty model to compare these two samples as a whole. The following figure is based Table 2.

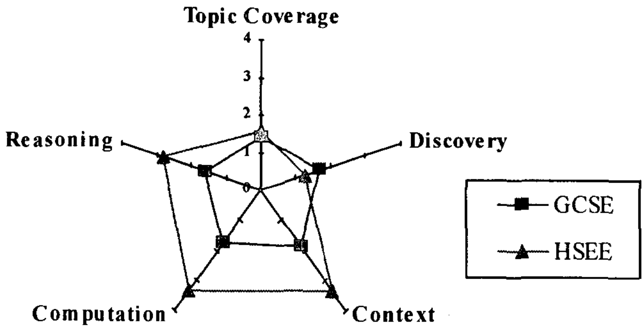

Figure 10: Comparison on composite difficulty between GCSE and HSEE

From Figure 10, we can summarize some initial findings on the comparison between GCSE and HSEE, as following:

- On the "Discovery" levels, although the Chinese sample is a bit higher than that of the British sample, the most items in GCSE and HSEE are actually the traditional mathematics problems. There are very few items both in the two samples that we can call "mathematical investigation."
- On the "Context" levels, the Chinese tests are lower than British ones. In the samples of HSEE, there are almost no problems with the contexts related students' daily life, and the items in GCSE are set more often in real-world contexts, especially the contexts related to students' personal life.
- On the "computation," "reasoning" and "topic coverage" levels, the Chinese tests are much higher than the British tests. Because the three factors are the main components of the traditional Chinese mathematics "Two basics," it seems that the Chinese mathematics teaching has very strong "Two basics"
- Based on the five difficulty factors, the Chinese sample as a whole seems much more difficult than the British sample.

5. THE STYLES OF PROBLEM-SOLVING

The above differences between Chinese and British tests, which also appear between

Chinese and British mathematics curricula and classrooms, will certainly affect students' styles of mathematics problem-solving in these two countries.

In China, The Multi-steps, logic-based and knowledge-rich mathematics problems lead to the following characteristics of mathematics problem-solving (Figure 11):

- Students usually need to complete three different activities during problem-solving: extending the problem to different variations (一题多变), using different methods to solve the problem (一题多解) and applying the solutions to different situations (一法多用). This is called "The trilogy of problem-solving (解题三部曲)" by some Chinese mathematics teachers (Bao; Wong; Yi & Gu 2003).
- The basic strategy to solve a difficult new problem is by reducing it to a solved or easier problem. This is called "The reducing strategy (化归策略)". In order to use a reducing strategy, students must remember some typical samples (典型例题) in each area of mathematics contents. These typical samples will become step-stones during problem-solving.
- Students' style of problem-solving is somewhat similar to the work of a "pure mathematician": formal deduction, theorem-based reasoning, multi-steps complex symbolic operation, rich of mathematics knowledge, but no realistic contexts.

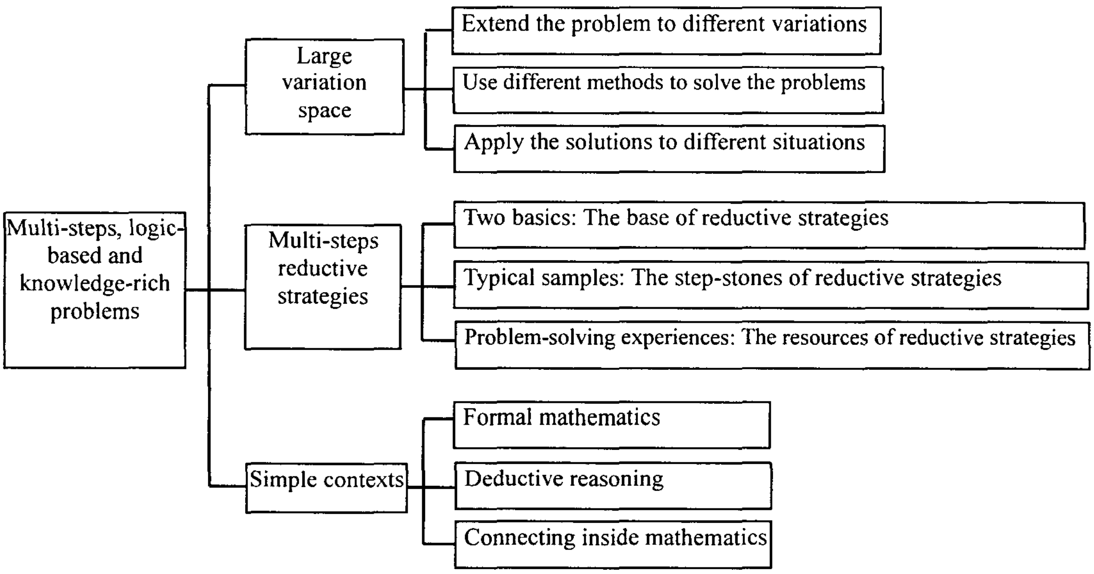

Figure 11: The characteristics of Chinese mathematics problem-solving

Meanwhile, the style of British problem solvers seems quite different with their Chinese counterparts. The solutions of most mathematics problems in British textbooks and tests only need one or two steps with none or simple reasoning. Students' exercises are often similar to sample problems in textbooks or demonstrated by teachers in classrooms. In most situations, what students need to do is executing a routine procedure or using "trial and error" method. Although there are also some variations for some problems, what changed are usually not the mathematics contents or methods, but the realistic contexts.

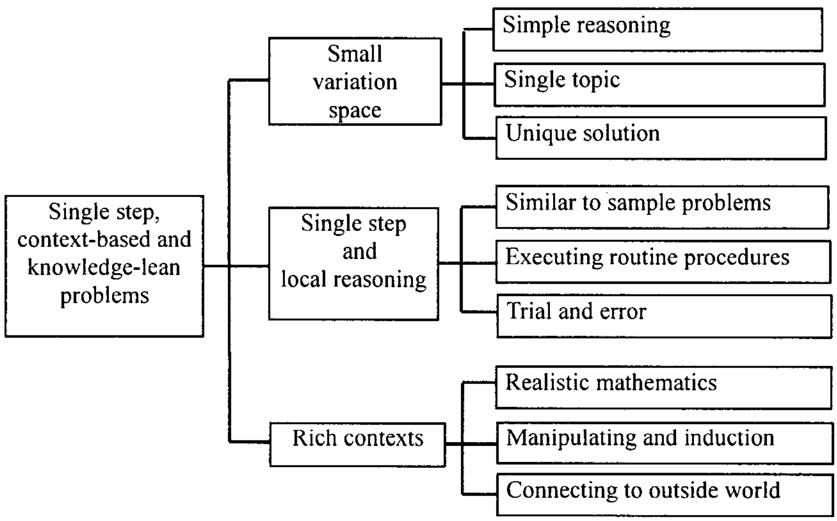

Figure 12: The characteristics of British mathematics problem-solving

The different mathematics problems may also affect students' attitudes towards mathematics problem-solving. Some findings show that the typical beliefs of the West students include "There is only one correct way to solve any mathematics problem – usually the rule the teacher has most recently demonstrated to the class", "Mathematics problems have one and only one right answer" and "Students who have understood the mathematics they have studied will be able to solve any assigned problem in five minutes or less" (*e.g.*, Schoenfeld 1992; Stevenson, Stigler & Lee 1986).

6. THE CURRICULUM BACKGROUND

Some differences of mathematics problems and problem-solving between Chinese and British mathematics tests can be traced to the differences between Chinese and British mathematics curricula. For example, we list all chapters of year 8 textbooks of MEP (Mathematics Enhancement Programme 2001) and PEP (People's Education Press 2001) in Table 3.

Table 3. Contents of MEP and PEP textbooks (year 8)

MEP textbooks (UK)		PEP textbooks (China)
	Y8A	Algebra II
1	Mathematical Diagrams	Ch 8. Factorization
2	Factors	Ch 9. Rational Expressions
3	Pythagoras' Theorem	Ch 10. Evolution of Numbers
4	Rounding and Estimating	Ch 11. Square Root Expressions
5	Data Analysis	Geometry II
6	Nets and Surface Area	Ch 3. Triangles
7	Ratio and Proportion	Ch 4. Quadrilaterals
8	Algebra: Brackets	Ch 5. Similarities
9	Arithmetic: Fractions and Percentages	
10	Probability – Two Events	
11	Angles, Bearings and Maps	
	Y8B	
12	Formulae	
13	Money and Time	
14	Straight Line Graphs	
15	Polygons	
16	Circles and Cylinders	
17	Units of Measure	
18	Speed, Distance and Time	
19	Similarity	
20	Questionnaires and Analysis	

Comparing the two textbooks, it seems that, the MEP textbooks are typical "Definition-based curriculum" and the PEP textbooks are typical "Theorem-based curriculum" (Figure 13).

As shown in Figure 13, the MEP textbooks have more chapters in year 8 with topics jumping around. In most chapters, students only need to learn a few basic mathematics concepts or procedures and then use them to solve some routine exercises.

However, in traditional Chinese mathematics textbooks, the situation changes a lot. There are only a few chapters in a single textbook, and the adjacent chapters are usually closely related so that solving a problem often demands the use of some former knowledge. All topics in each chapter are dealt with in depth and theoretically and needed more time to learn and teach. Besides the formal definitions of mathematics concepts, there are usually many theorems in textbooks, especially in Geometry Textbooks (for example, there are total 84 theorems in Geometry Book II). These theorems not only require students to understand, but also can be counted as arguments for mathematics proofs or "theorem-based reasoning" (Chin & Tall 2000, 2001). For example, let's look back at the Chinese sample test item (Sample B) displayed at the beginning of this paper. In order to solve that single problem, students must apply at lest nine theorems. So we can say, the Chinese mathematics textbooks are typical theorem-based curricula (Bao 2002a).

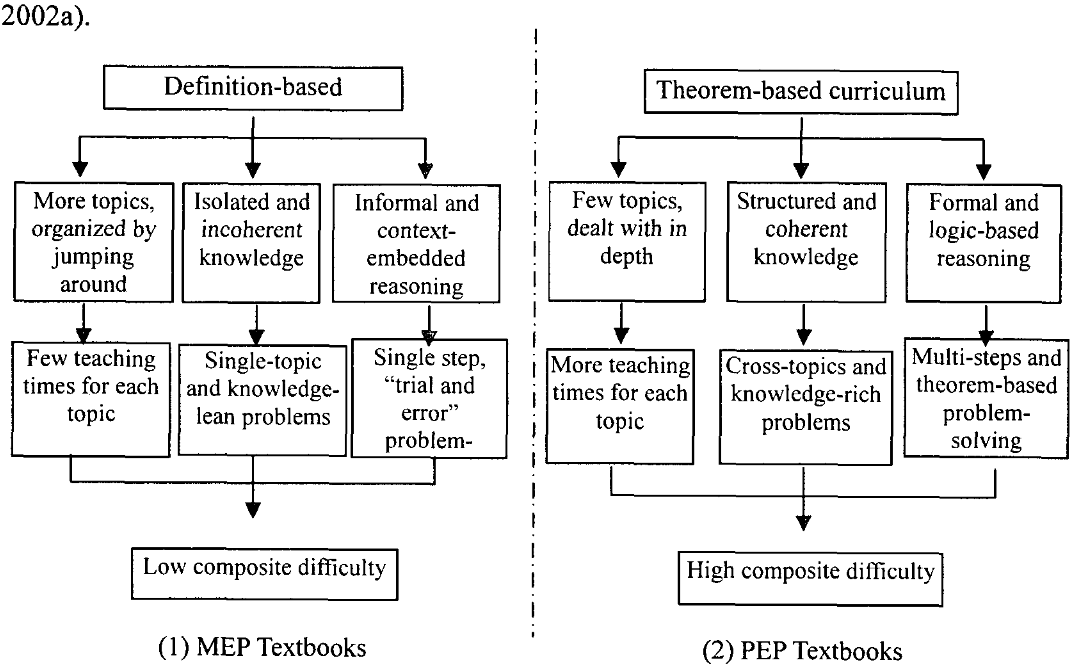

Figure 13: Characteristics of Chinese and British mathematics textbooks

The different styles of curricula in China and UK further lead to the differences of curriculum purpose. The Chinese mathematics curriculum aims at mastery of basic knowledge, concepts, and subject-specOific thinking skills tied to well-organized frameworks of systemized curriculum topics. Meanwhile, the main reason of teaching and learning mathematics in UK is the applications of mathematics in daily life and work place (Bao 2004).

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