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ROBUST RELIABILITY DESIGN OF VEHICLE COMPONENTS WITH ARBITRARY DISTRIBUTION PARAMETERS

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ABSTRACT–This study employed the perturbation method, the Edgeworth series, the reliability optimization, the reliability sensitivity technique and the robust design to present a practical and effective approach for the robust reliability design of vehicle components with arbitrary distribution parameters on the condition of known first four moments of original random variables. The theoretical formulae of the robust reliability design for vehicle components with arbitrary distribution parameters are obtained. The reliability sensitivity is added to the reliability optimization design model and the robust reliability design is described as a multi-objection optimization. On the condition of known first four moments of original random variables, the respective program can be used to obtain the robust reliability design parameters of vehicle components with arbitrary distribution parameters accurately and quickly.

KEY WORDS : Vehicle components, Reliability optimization design, Reliability sensitivity analysis, Robust design, Arbitrary distribution parameters, Cases

1. INTRODUCTION

In the recent years much research has been done to quantify uncertainties in engineering systems and their combined effect on the reliability. Theoretically, these uncertainties are modeled as random variables governed by joint probability density or distribution functions. In practice, the exact joint probability density functions are often unavailable or difficult to obtain for reasons of insufficient data. Not infrequently, the available data may only be sufficient to evaluate the first few moments such as the mean, variance and correlations. Current design practice tends to account for the uncertainties by use of factors of safety and reliability. During the last four decades, reliability design (Zhang *et al.*, 1998; Haldar and Mahadevan, 2000; Zhang and Liu, 2002), reliability optimization design (Tu *et al.*, 1999; Lee *et al.*, 2003; Choi *et al.*, 2005; Zhang *et al.*, 2005c) and reliability sensitivity analysis technique (Wu, 1994; Zhang *et al.*, 2003; Zhang *et al.*, 2005a) have been described.

Quality engineering tools from Taguchi Method (Taguchi, 1993) has been successfully employed in the last few decades to develop robust products that will perform their intended functions with low sensitivity to variations of design variables. A robust design is, in general, considered to be one that is insensitive to variations of design variables. Such a design can be achieved by selecting suitable design variables to render the design performance insensitive to the various causes of variation. In recent years, the robust design and the robust reliability design have been used widely and developed well in industrial product design field (Liaw and DeVries, 2001; Sandgren and Cameron, 2002; Zhang *et al.*, 2005b).

In practical engineering, the aforementioned methods are very effective to improve product design with the consideration of the uncertainties, which include geometry parameters, material properties, loadings, etc. Unfortunately, in the industry reliability optimization, reliability sensitivity technique and robust design usually have been used separately in different design stages for different purposes. Therefore, It is essential to combine reliability optimization, reliability sensitivity and robust design and to develop a robust reliability design approach. The objective of this paper is to extend the concept of reliability optimization through the use of reliability sensitivity coupled to a robust reliability design method for making reliability of the product insensitive to the variations of the design variables.

A great number of methods presume that original random variables are normal distributions. When non-normal original random variables are involved, Rosenblatt transformation (Rosenblatt, 1952) and the Hasofer Lind-Rackwitz Fiessler (Hasofer and Lind, 1974; Liu and

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Kiureghian, 1991) are often used. However, the exact joint probability density function is often unavailable or difficult to obtain for reasons of insufficient data. Not infrequently, the available data may only be sufficient to evaluate the first few moments such as the mean, variance, the third moment and the fourth moment of the random variables. Under such a condition, it is difficult to employ the Rosenblatt transformation and the Hasofer Lind-Rackwitz Fiessler and to obtain the robust reliability design parameters without distribution function. Thus, an alternative computational method of robust reliability design with arbitrary distribution parameters is required. The paper discusses robust reliability design of vehicle components with arbitrary distribution parameters on the condition of known first four moments of original random variables. Using the perturbation method, the Edgeworth series, reliability optimization, reliability sensitivity technique and robust design, this paper proposes an efficient numerical method for robust reliability design of vehicle components with arbitrary distribution parameters. The robust reliability design method is formulated in this paper. The approach presented can be used to obtain information of robust reliability design for some vehicle components accurately and quickly.

2. PERTURBATION METHOD OF RELIABILITY DESIGN

Most design of engineering systems must be accomplished without the benefit of complete information; consequently, the assurance of performance can seldom be perfect. Moreover, many decisions that are required during the process of design are invariably made under conditions of uncertainty. Therefore, there is invariably some chance of nonperformance or failure and of its associated adverse consequences; hence, risk is often unavoidable.

The vector of random parameters $\boldsymbol{X}$ and the state function $g(\boldsymbol{X})$ are expanded as

$$X = X_d + \varepsilon X_p \tag{1}$$

$$g(X) = g_d(X) + \varepsilon g_p(X) \tag{2}$$

where ε is a small parameter. The part of equations (1) and (2) that expressed by subscript d is the certain part of the random parameters, and the part that expressed by subscript p is the random part, having a zero mean value in the random parameters. Obviously, it is necessary for the value of the random part to be smaller than the value of the certain part. Both sides of equations (1) and (2) are evaluated about the mean value of random variables as follows

$$E(\boldsymbol{X}) = E(\boldsymbol{X}_d) + \varepsilon E(\boldsymbol{X}_p) = \boldsymbol{X}_d = \overline{\boldsymbol{X}} \tag{3}$$

$$\mu_g = E[g(\boldsymbol{X})] = E[g_d(\boldsymbol{X})] + \varepsilon E[g_p(\boldsymbol{X})] = g_d(\boldsymbol{X}) \approx g(\overline{\boldsymbol{X}}) \tag{4}$$

Similarly, according to the Kronecker algebra (Vetter, 1973), the both sides of equations (1) and (2) are evaluated about the variance, the third moment and the fourth moment of the random variables and the state function as follows

$$\mathrm{Var}(\boldsymbol{X}) = E\{[X - E(\boldsymbol{X})]^{[2]}\} = \varepsilon^2 E[\boldsymbol{X}_p^{[2]}] \tag{5a}$$

$$\mathbf{C}_3(\boldsymbol{X}) = E\{[\boldsymbol{X} - E(\boldsymbol{X})]^{[3]}\} = \varepsilon^2 E[\boldsymbol{X}_p^{[3]}] \tag{5b}$$

$$\mathbf{C}_4(\boldsymbol{X}) = E\{[\boldsymbol{X} - E(\boldsymbol{X})]^{[4]}\} = \varepsilon^2 E[\boldsymbol{X}_p^{[4]}] \tag{5c}$$

$$\mathrm{Var}[g(\boldsymbol{X})] = \mathrm{E}\{[g(\boldsymbol{X}) - \mathrm{E}(g(\boldsymbol{X}))]^{[2]}\} = \varepsilon^2 \mathrm{E}\{[g_p(\boldsymbol{X})]^{[2]}\} \tag{6a}$$

$$\mathbf{C}_3[g(\boldsymbol{X})] = \mathrm{E}\{[g(\boldsymbol{X}) - \mathrm{E}(g(\boldsymbol{X}))]^{[3]}\} = \varepsilon^3 \mathrm{E}\{[g_p(\boldsymbol{X})]^{[3]}\} \tag{6b}$$

$$\mathbf{C}_4[g(\boldsymbol{X})] = \mathrm{E}\{[g(\boldsymbol{X}) - \mathrm{E}(g(\boldsymbol{X}))]^{[4]}\} = \varepsilon^4 \mathrm{E}\{[g_p(\boldsymbol{X})]^{[4]}\} \tag{6c}$$

where the Kronecker power is $\boldsymbol{P}^{[k]} = \boldsymbol{P} \otimes \boldsymbol{P}^{[k-1]} = \boldsymbol{P} \otimes \boldsymbol{P} \ldots \otimes \boldsymbol{P}$, and the symbol f represents Kronecker product which is defined as $(A)_{p\times q} \otimes (B)_{s\times t} = [a_{ij}\boldsymbol{B}]_{ps\times qt}$. .

By expanding the state function $g_p(\boldsymbol{X})$ to first-order approximation in a Taylor series of vector-valued functions and matrix-valued functions at a point $\mathrm{E}(\boldsymbol{X}) = \boldsymbol{X}_d$, which is on the failure surface $g_p(\boldsymbol{X}_d) = 0$, the expression of $g_p(\boldsymbol{X})$ is given

$$g_p(\boldsymbol{X}) = \frac{\partial g_d(\boldsymbol{X})}{\partial \boldsymbol{X}^T} \boldsymbol{X}_p \tag{7}$$

Substituting equation (7) into equations (6), we obtain

$$\sigma_g^2 = \mathrm{Var}[g(\boldsymbol{X})] = \varepsilon^2 \mathrm{E}\left[\left(\frac{\partial g_d(\boldsymbol{X})}{\partial \boldsymbol{X}^T}\right)^{[2]} \boldsymbol{X}_p^{[2]}\right] = \left(\frac{\partial g_d(\boldsymbol{X})}{\partial \boldsymbol{X}^T}\right)^{[2]} \mathrm{Var}(\boldsymbol{X}) \tag{8a}$$

$$\theta_g = \mathbf{C}_3[g(\boldsymbol{X})] = \varepsilon^2 \mathrm{E}\left[\left(\frac{\partial g_d(\boldsymbol{X})}{\partial \boldsymbol{X}^T}\right)^{[3]} \boldsymbol{X}_p^{[3]}\right] = \left(\frac{\partial g_d(\boldsymbol{X})}{\partial \boldsymbol{X}^T}\right)^{[3]} \mathbf{C}_3(\boldsymbol{X}) \tag{8b}$$

$$\eta_g = \mathrm{C}_4[g(\boldsymbol{X})] = \varepsilon^4 \mathrm{E}\left[\left(\frac{\partial g_d(\boldsymbol{X})}{\partial \boldsymbol{X}^T}\right)^{[4]} \boldsymbol{X}_p^{[4]}\right] = \left(\frac{\partial g_d(\boldsymbol{X})}{\partial \boldsymbol{X}^T}\right)^{[4]} \mathbf{C}_4(\boldsymbol{X}) \tag{8c}$$

where $\mathrm{Var}(\boldsymbol{X})$ is the variance matrix that include all variance and covariance of the random parameters, $\mathbf{C}_3(\boldsymbol{X})$ and $\mathbf{C}_4(\boldsymbol{X})$ are the third, the fourth central moments matrix that include all the third, the fourth central moments of the random parameters respectively. σ_g^2, θ_g and η_g are the variance, the third and the fourth central moments of the state function $g(\boldsymbol{X})$ respectively.

The reliability index is defined as

$$\beta = \frac{\mu_g}{\sigma_g} = \frac{\mathrm{E}[g(\boldsymbol{X})]}{\sqrt{\mathrm{Var}[g(\boldsymbol{X})]}} \tag{9}$$

It may be emphasized that the first-order approximation of μ_g and σ_g derived above must be evaluated at the mean values $(\mu_{X_1}, \mu_{X_2}, \ldots, \mu_{X_n})$. In some approximate sense, the reliability index may be direct used as a measure of reliability. If the distributions of the original random variables are normal, the distance from the "minimum" tangent plane to the failure surface may be used to approximate the actual failure surface, and the corresponding reliability, namely reliability, may be represented, as follows

$$R = \Phi(\beta) \tag{10}$$

where $\Phi(\ )$ is the standard normal distribution function. In the case where the state function $g(\boldsymbol{X})$ has non-normal distributions, Equation (10) is not valid. It is well known that the probabilistic characteristics of random variables could be described integrally with the probability density function or cumulative distribution function. Simultaneously, it could also be described with the moments of random variables. In reality, due to the lack of statistical data, the probability density function or cumulative distribution function of some original random variables are often unknown, and the probabilistic characteristic of these variables are often expressed using only statistical moments. On the condition of known first four moments of original random variables, the probability distribution function of the standardized variable is approximately expressed by the first four moments of original random variables using the Edgeworth series. Thus, information of robust reliability design for vehicle components with arbitrary distribution parameters can be obtained.

3. EDGEWORTH SERIES

For a state function $g(\boldsymbol{X})$, the standard forms can be expressed as

$$y = \frac{g(\boldsymbol{X}) - \mu_g}{\sigma_g} \tag{11}$$

The arbitrary distribution function of the standard random variable y that is approximately expressed by the standard normal distribution function using the Edgeworth series is addressed in (Cramer, 1964)

$$F(y) = \Phi(y) - \frac{\beta_1}{3!}\Phi^{(3)}(y) + \frac{\beta_2 - 3}{4!}\Phi^{(4)}(y) + \frac{10\beta_1^2}{6!}\Phi^{(6)}(y) \tag{12}$$

where the first four terms are available, and β_1, β_2 are the coefficient of skewness and the coefficient of kurtosis, respectively. $\Phi^{(i)}(\ )$ denotes ith differentiation of $\Phi(\ )$.

$$\beta_1 = \frac{\theta_g}{\sigma_g^3} \tag{13}$$

$$\beta_2 = \frac{\theta_g}{\sigma_g^4} \tag{14}$$

$$\Phi^{(i)}(y) = (-1)^{i-1} H_{i-1}(y)\varphi(y) \tag{15}$$

where $\varphi(\ )$ is the standard normal probability density function and $H_{i-1}(\ )$ is the Hermite polynomial

$$\begin{cases} H_{j+1}(y) = yH_j(y) - yH_{j-1}(y) \\ H_0(y) = 1, \quad H_1(y) = y \end{cases} \tag{16}$$

Thus the reliability R is represented as

$$\begin{aligned} R(\beta) &= P(g(\boldsymbol{X}) \geq 0) = P\left(\frac{g(\boldsymbol{X}) - \mu_g}{\sigma_g} \geq -\frac{\mu_g}{\sigma_g}\right) \\ &= 1 - P(y \leq -\beta) = 1 - F(-\beta) \end{aligned} \tag{17}$$

Due to the first four terms of the Edgeworth series are only used, sometimes the reliability $R>1$ may happen when Equation (17) is used to determine R. If $R>1$ appears, the amendatory expression from (Zhang *et al.*, 1998) is employed in this paper

$$R^*(\beta) = R(\beta) - \left\{\frac{R(\beta) - \Phi(\beta)}{\{1 + [R(\beta) - \Phi(\beta)]\beta\}^{\beta}}\right\} \tag{18}$$

According to reliability theory, the reliability R is between 0 and 1, namely, $0 \leq R \leq 1$. The amendatory expression (18) can ensure the reliability R to satisfy $0 \leq R \leq 1$ gradually and accurately.

4. RELIABILITY SENSITIVITY

It is of interest to establish the sensitivity from the system reliability analysis. The reliability sensitivity with respect to the mean value of the system parameters is approximately derived as follows

$$\frac{\mathrm{D}R(\beta)}{\mathrm{D}\bar{\boldsymbol{X}}^T} = \frac{\partial R(\beta)}{R\beta}\frac{R\beta}{\partial \mu_g}\frac{\partial \mu_g}{\mathrm{D}\bar{\boldsymbol{X}}^T} \tag{19}$$

where

$$\begin{aligned} \frac{\partial R(\beta)}{\partial \beta} &= \varphi(-\beta)\{1 - \beta[\frac{1}{6}\frac{\theta_g}{\sigma_g^3}H_2(-\beta) \\ &+ \frac{1}{24}\left(\frac{\eta_g}{\sigma_g^4} - 3\right)H_3(-\beta) + \frac{1}{72}\left(\frac{\theta_g}{\sigma_g^3}\right)^2 H_5(-\beta)] \\ &- \left[\frac{1}{3}\frac{\theta_g}{\sigma_g^3}H_1(-\beta) + \frac{1}{8}\left(\frac{\eta_g}{\sigma_g^4} - 3\right)H_2(-\beta) + \frac{5}{72}\left(\frac{\theta_g}{\sigma_g^3}\right)^2 H_4(-\beta)\right]\right\} \end{aligned} \tag{20}$$

$$\frac{\partial \beta}{\partial \mu_g} = \frac{1}{\sigma_g} \tag{21}$$

$$\frac{\partial \mu_g}{\mathrm{D}\bar{\boldsymbol{X}}^T} = \left[\frac{\partial \bar{g}}{\partial X_1}\frac{\partial \bar{g}}{\partial X_2}\cdots\frac{\partial \bar{g}}{\partial X_n}\right] \tag{22}$$

The reliability sensitivity with respect to the standard deviation of the system parameters is approximately derived as follows

$$\frac{\mathrm{D}R(\beta)}{\mathrm{DVar}(\boldsymbol{X})} = \left[\frac{\partial R(\beta)}{\partial \beta}\frac{\partial \beta}{\partial \sigma_g} + \frac{\partial R(\beta)}{\partial \sigma_g}\right]\frac{\partial \sigma_g}{\partial \mathrm{Var}(\boldsymbol{X})} \tag{23}$$

where

$$\frac{\partial \beta}{\partial \sigma_g} = \frac{\mu_g}{\sigma_g^2} \tag{24}$$

$$\frac{\partial R(\beta)}{\partial \sigma_g} = \varphi(-\beta)\left[\frac{1}{2}\frac{\theta_g}{\sigma_g^4}H_2(-\beta) + \frac{1}{6}\frac{\eta_g}{\sigma_g^5}H_3(-\beta) + \frac{1}{12}\frac{\theta_g^2}{\sigma_g^7}H_5(-\beta)\right] \tag{25}$$

$$\frac{\partial \sigma_g}{\partial \mathrm{Var}(\boldsymbol{X})} = \frac{1}{2\sigma_g}\left[\frac{\partial \bar{g}}{\partial \boldsymbol{X}} \otimes \frac{\partial \bar{g}}{\partial \boldsymbol{X}}\right] \tag{26}$$

Substituting the known conditions and the results derived earlier into equations (19) and (23), the reliability sensitivity and are obtained.

If the reliability computed by the Edgeworth series appear $R>1$, the results computed by the amendatory expression (18) are closer to that by Monte Carlo simulation than that by the Edgeworth series in interval [0.99, 1] that is usually used for reliability analysis in engineering computation practice. The distribution function curves derived from the amendatory expression (18) is monotonic in interval [0, 1]. Therefore, if the reliability computed by the Edgeworth series appear $R>1$, the reliability sensitivity that computed by the differentiation of the amendatory expression is more accurate than that computed by equations (19) and (23). (Sometime, the results computed by equations (19) and (23) are not right). If the results computed by the Edgeworth series appear $R>1$, the reliability sensitivity with respect to the reliability index is derived as follows

$$\frac{\partial R^*(\beta)}{\partial \beta} = \frac{\partial R(\beta)}{\partial \beta} + \left[\frac{\partial R(\beta)}{\partial \beta} - \varphi(\beta)\right]\frac{\beta(\beta-1)[R(\beta)-\Phi(\beta)]-1}{\{1+[R(\beta)-\Phi(\beta)]\beta\}^{\beta+1}} + \frac{[R(\beta)-\Phi(\beta)]\{1+[R(\beta)-\Phi(\beta)]\beta\}\ln\{1+[R(\beta)-\Phi(\beta)]\beta\}}{\{1+[R(\beta)-\Phi(\beta)]\beta\}^{\beta+1}} + \frac{[R(\beta)-\Phi(\beta)]\beta)}{\{1+[R(\beta)-\Phi(\beta)]\beta\}^{\beta+1}} \tag{27}$$

Substituting Equation (27) into $\partial R(\beta)/\partial\beta$ of equations (19) and (23), the reliability sensitivity $\mathrm{D}R/\mathrm{D}\bar{\boldsymbol{X}}^T$ and $\mathrm{D}R/\mathrm{DVar}(\boldsymbol{X})$ are obtained.

5. ROBUST RELIABILITY DESIGN

The robust reliability design of vehicle components is described as a multi-objective optimization, in which the minimum of the weight of vehicle components and the minimum of reliability sensitivity with respect to the mean value of design variables of vehicle components are taken as objective functions, while including a series of reliability and geometry constrains etc.. A typical robust reliability design problem can be formulated in the following form

$$\left.\begin{array}{l} \text{minimize } f(\boldsymbol{X}) = E\{f(\boldsymbol{X})\} \approx f(\bar{\boldsymbol{X}}) = \sum_{k=1}^{n} w_k f_k(\bar{\boldsymbol{X}}) \\ \text{subject to } R \geq R_0,\ q_i(\bar{\boldsymbol{X}}) \geq 0,\ (i=1,\ldots,l) \end{array}\right\} \tag{28}$$

where w_k are the weighting coefficients satisfying the following conditions

$$0 < w_k < 1 \text{ and } \sum_{k=1}^{n} w_k = 1 \tag{29}$$

The value of w_k is determined depending on the importance of each objective function. In the paper, two objective functions are given, $f_1(\bar{\boldsymbol{X}})$ is the area of vehicle components, and $f_2(\bar{\boldsymbol{X}})$ is reliability sensitivity with respect to the mean value of design parameter vector $\boldsymbol{x} = (x_1\ x_2\ \cdots\ x_m)^T$. R_0 is given reliability, is the inequality constraints.

6. NUMERICAL EXAMPLES

6.1. Robust Reliability Design of Semi-axle

Axle shafts are divided into three main groups, depending on the stresses to which the shaft is subjected: (a) fully floating, (b) three-quarters floating, and (c) semi-floating (in Figure 1). The fully floating shaft is generally fitted on commercial vehicles where torque and axle loads are greater. The construction of fully floating consists of an independently mounted hub that rotates on two bearings widely spaced on the axle housing. This arrangement relieves the shaft of all stresses except torsion; so the construction is very strong. Studs connecting the shaft to the hub transmit the drive and when the nuts on these studs are removed, the shaft may be withdrawn without jacking up the vehicle. The semi-floating shaft is suitable for light cars. A single bearing at the hub end is fitted between the shaft and the housing, so the shaft will have to resist all the stresses previously mentioned. To reduce the risk of fracture at the hub end (this would allow the wheel to fall off), the shaft diameter is increased. Any increase must be gradual, since a sudden change in cross-sectional areas would produce a stress-raiser and increase the risk of failure due to fatigue. The three-quarter floating shaft is defined the fully floating and semi-floating shaft, any alternative between the may be regarded as a construction which has a single bearing mounted between the hub and housing. The main

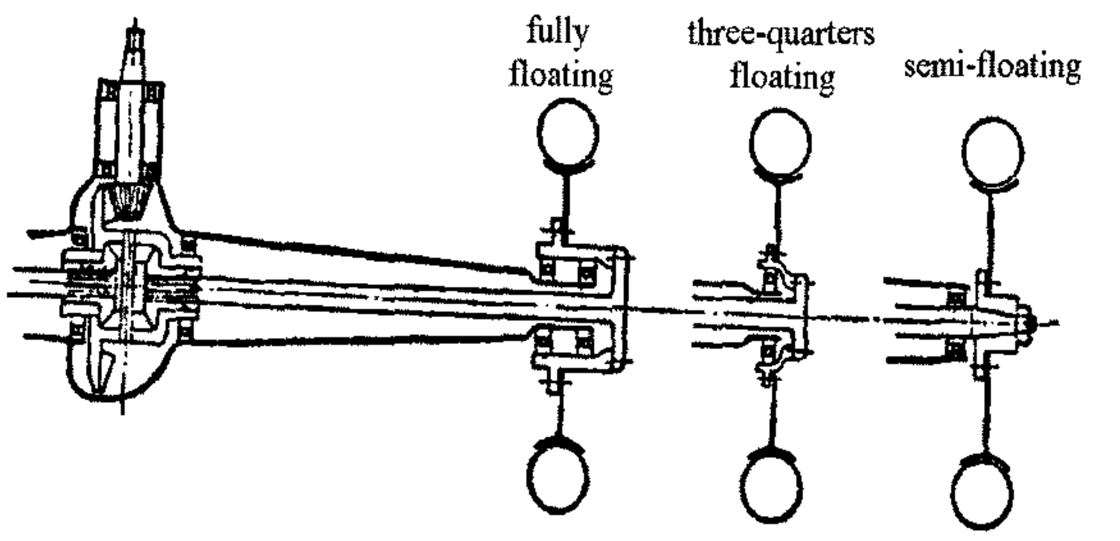

Figure 1. Structure of semi-axle.

shear stress on the shaft is relieved but all other stresses still have to be resisted.

On the basis of reliability theory, the state equation of the semi-axle is defined as

$$g(\mathbf{X}) = r - s \tag{30}$$

where r is material strength of semi-axle. The original random variable vector $\boldsymbol{X}$ is given by $X=(r\ T\ M\ d)^T$ respectively. where the mean matrix E(X), the variance matrix Var(X), the third central moment matrix $\mathbf{C}_3(X)$ and the fourth central moment matrix $\mathbf{C}_4(X)$ are known. Generally speaking, geometry parameters and material property are usually said to have normal distributions. But if the design variable has the arbitrary distribution, the proposed method could be applied.

The torsional moment T and the bending moment M on risk section are arbitrary distributed random variable with first four moments (T)=(1.1399×10^5 N·mm, 9.1904×10^3 N·mm, 9.038×10^11 (N·mm)^3, 3.9719×10^16 (N·mm)^4) and (M)=(1.4314×10^4 N·mm, 1.3011×10^3 N·mm, 2.5622×10^9 (N·mm)^3, 1.5845×10^13 (N·mm)^4) respectively. The material strength r is (μ_r, σ_r)=(820, 32) MPa.

(1) If the reliability R_0=0.999 is given, the design sizes, the minimum diameter d of the semi-axle computed by reliability optimization is as follows

$$d = 11.90469 \text{ mm}$$

According to the results computed by the reliability design optimization, the reliability index β, the reliability R and the reliability sensitivity $DR/D\bar{x}$ of the semi-axle, therefore, becomes

$$\beta=3.752273,\ R_E=0.9990106,\ R_{MCS}=0.99873,$$

$$DR/D\bar{x} = \frac{\partial R}{\partial \bar{d}} = 6.303 \times 10^{-3}$$

$$\varepsilon_R = \left|\frac{R_E - R_{MCS}}{R_{MCS}}\right| = \left|\frac{0.9990106 - 0.99873}{0.99873}\right| = 0.0281\%$$

where R_E is the reliability that computed by the Edgeworth series, R_{MCS} is the reliability that computed by Monte Carlo simulation (MCS) with 10^5 samples, e_R is the relative error of the reliability R.

(2) If the reliability R_0=0.999 is given, the design sizes, the minimum diameter d of the semi-axle at design point is extracted.

Find the minimum $f_1(x)$ of the area of the semi-axle and the minimum $f_2(x)$ of the reliability sensitivity $DR/D\bar{x}$ of the semi-axle respectively

$$f_1(x) = \frac{\pi}{4} x_1^2 \tag{31}$$

$$f_2(x) = \left|\frac{\partial R}{\partial x_1}\right| \tag{32}$$

where the design variables are $x=x_1=d$.

Subject to

$$R - R_0 \geq 0 \tag{33}$$

The initial values, d=15 mm, is given, and the solution for d of optimization is

$$d=12.3047 \text{ mm}$$

According to the results computed by the robust reliability design approach, the reliability index β, the reliability R and the reliability sensitivity $DR/D\bar{x}$ of the semi-axle, therefore, becomes

$$\beta=5.058967,\ R_E=0.9999885,\ R_{MCS}=0.99975,$$

$$DR/D\bar{x} = \frac{\partial R}{\partial \bar{d}} = 1.108 \times 10^{-4}$$

$$\varepsilon_R = \left|\frac{R_E - R_{MCS}}{R_{MCS}}\right| = \left|\frac{0.9999885 - 0.99975}{0.99975}\right| = 0.02386\%$$

On the basis of the above results, the bigger the reliability index β and the reliability R are, the less the values of the reliability sensitivities $DR/D\bar{x}$ are, the more robust the reliability of the semi-axle is. For this example, the two methods (this new method and the reliability design optimization) have been employed and the new method is very well for making the reliability insensitive to the variations of the design variables.

The relation curves between the reliability index β and the reliability R that computed by the Edgeworth series, the amendatory expression and Monte Carlo simulation with 10^5 samples of the semi-axle are illustrated in Figure 2 respectively. If the results computed by the Edgeworth series do not appear R>1, the relation curves in Figure 2 indicate the results computed by the Edgeworth series are closer to that by Monte Carlo simulation than that by the amendatory expression. If the results computed by the Edgeworth series do not appear R>1, the Edgeworth is selected; If the results computed by the Edgeworth series appear R>1, the amendatory expression is selected.

6.2. Robust Reliability Design of Rear-axle Housing

The differential in a rear-drive vehicle is housed in the rear-axle housing (in Figure 3), or carrier. Two main types of housing are in use: (a) split, split housing axles are formed in two halves and bolted together to contain

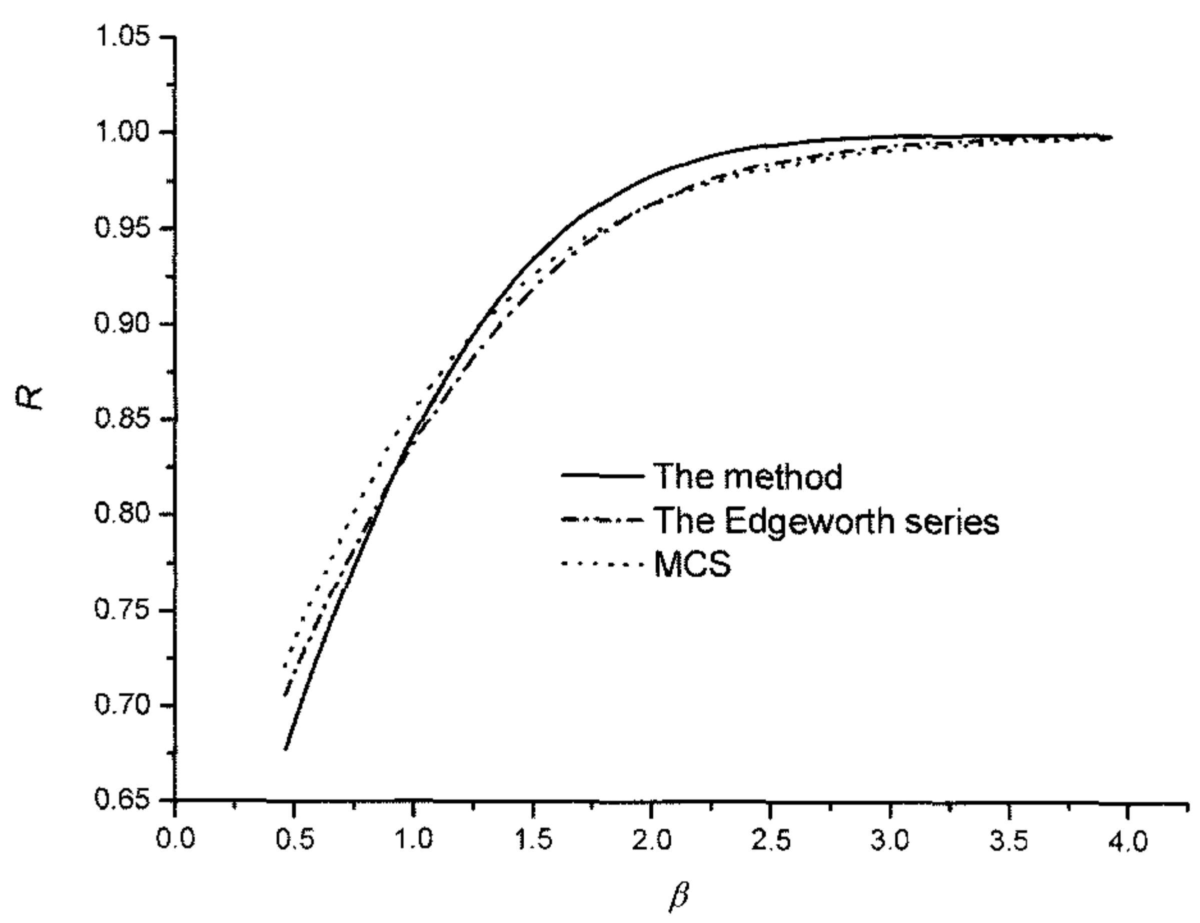

Figure 2. Relation between R and β of semi-axle.

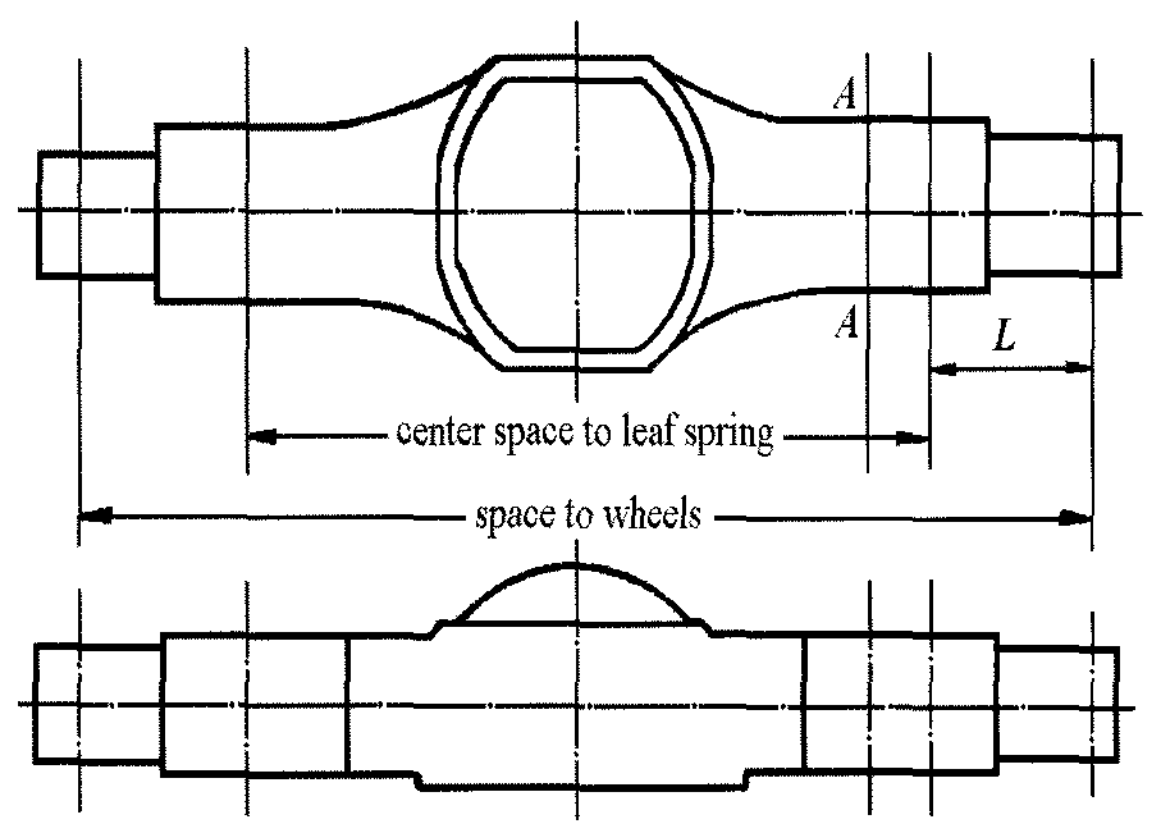

Figure 3. Structure of rear-axle housing.

the final drive and differential. (b) banjo, banjo axles are normally built up of steel pressings and welded together. The crown wheel assembly is mounted in a malleable iron housing that is bolted to the axle.

The integral axle housing is most commonly found on late-model cars and light trucks. A cast-iron carrier forms the center of the axle housing. Steel axle tubes are pressed into both sides of the carrier to form the housing. The housing and carries have a removable rear cover that allows access to the have a removable rear cover that allows access to the differential assembly. Because the carrier is not removable, the differential components must be removed and serviced separately. In addition to providing a mounting place for the differential, the axle housing also contains brackets for mounting suspension components such as control arms, leaf springs, and coil springs.

The rear-axle housing is generally loaded torsional moment and bending moment. The risk section is on two sides of leaf spring seats. The section is tubular numerously, and some section is quadrate with a circular hole.

The bending moment M on risk section is arbitrary distributed random variable with first four moments (M)= $(1.0976\times10^8$ N·mm, 1.0367×10^7 N·mm, -8.1367×10^{20} $(\text{N}\cdot\text{mm})^3$, 4.4569×10^{28} $(\text{N}\cdot\text{mm})^4)$. The material strength r is (μ_r, σ_r)=(433, 27.5) MPa.

(1) If the reliability R_0=0.999 is given, the design sizes, d, b, h, of the rear-axle housing with quadrate section computed by reliability optimization is as follows

b=122.7392 mm, h=143.3525 mm, d=102.739 mm

According to the results computed by the reliability design optimization, the reliability index β, the reliability R and the reliability sensitivity $DR/D\bar{x}^T$ of the rear-axle housing with quadrate section, therefore, becomes

$$\beta=3.014851,\ R_E=1.004416,\ R^*=0.9990001,$$
$$R_{MCS}=0.99658,$$

$$DR/D\bar{x}^T=\left[\frac{\partial R}{\partial \bar{b}}\frac{\partial R}{\partial \bar{h}}\frac{\partial R}{\partial \bar{d}}\right]=\begin{bmatrix}2.564\times10^{-4}\\4.790\times10^{-4}\\-2.228\times10^{-4}\end{bmatrix}^T,$$

$$\varepsilon_R=\left|\frac{R^*-R_{MCS}}{R_{MCS}}\right|=\left|\frac{0.9990001-0.99658}{0.99658}\right|=0.2428\%$$

where R^* is the reliability that computed by the amendatory expression.

(2) If the reliability R_0=0.999 is given, the design sizes, d, b, h, of the rear-axle housing with quadrate section at design point can be extracted.

Find the minimum $f_1(x)$ of the area of the rear-axle housing with quadrate section and the minimum $f_2(x)$ of the reliability sensitivity $DR/D\bar{x}^T$ of the rear-axle housing with quadrate section respectively

$$f_1(x)=x_1x_2-\frac{\pi}{4}x_3^2 \quad (34)$$

$$f_1(x)=\sqrt{\sum_{i=1}^{3}\left(\frac{\partial R}{\partial x_i}\right)^2} \quad (35)$$

where the design variables are $\boldsymbol{x}=[x_1\ x_2\ x_3]^T=[b\ h\ d]^T$. Subject to

$$R-R_0\geq0 \quad (36)$$

$$x_1-x_3\geq10 \quad (37)$$

$$x_2-x_3\geq10 \quad (38)$$

$$x_2-x_1\geq0 \quad (39)$$

The initial values, b=160 mm, h=164 mm, d=120 mm are given, and the solution for b, h, d of optimization are b=125.4279 mm, h=153.8173 mm, d=115.4277 mm

According to the results computed by the robust reliability design approach, the reliability index β, the reliability R and the reliability sensitivity $DR/D\bar{x}^T$ of the

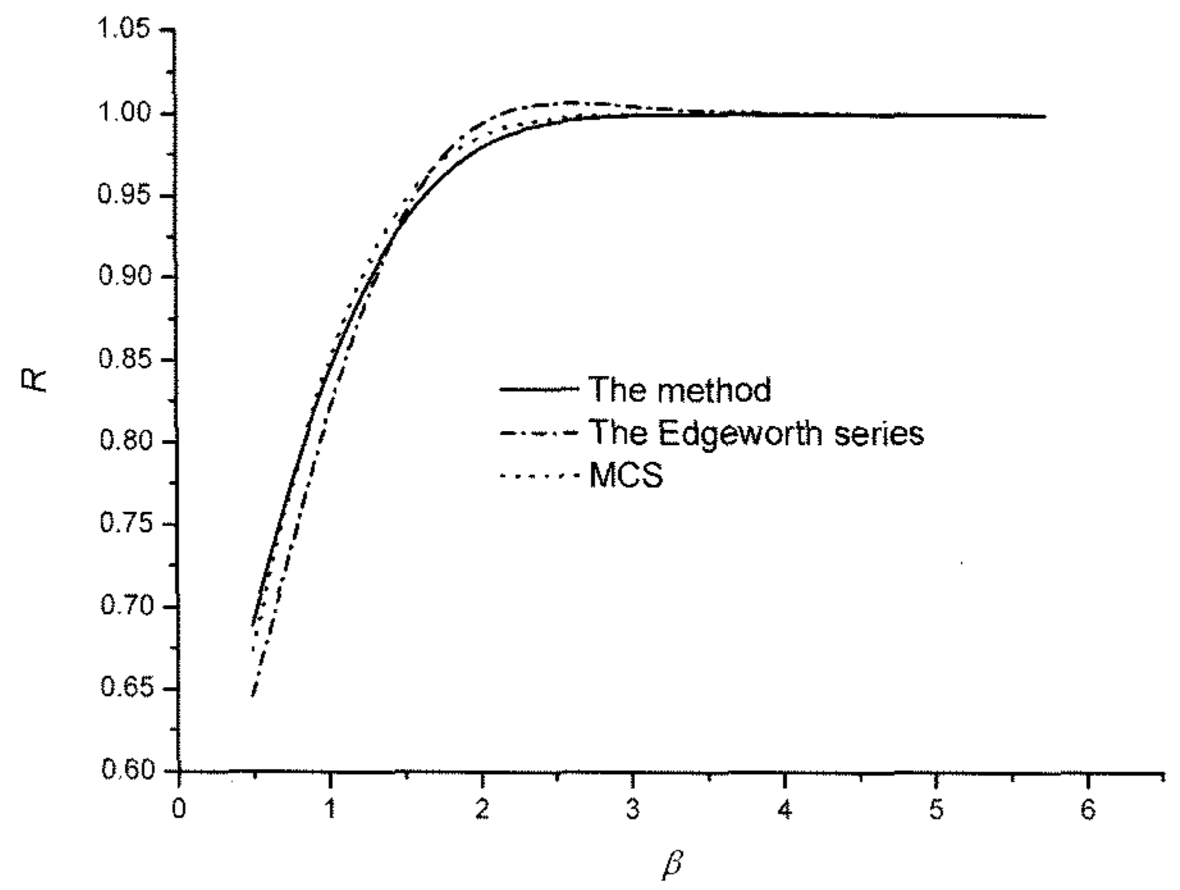

Figure 4. Relation between R and β of rear axle housing with quadrate section.

rear-axle housing with quadrate section, therefore, becomes

$$\beta=3.975766,\ R_E=1.000421,\ R^*=0.9999681,\ R_{MCS}=0.99936,$$

$$DR/D\bar{x}^T = \left[\frac{\partial R}{\partial \bar{b}}\frac{\partial R}{\partial \bar{h}}\frac{\partial R}{\partial \bar{d}}\right] = \begin{bmatrix} 9.748\times 10^{-6} \\ 1.772\times 10^{-5} \\ -9.772\times 10^{-6} \end{bmatrix}^T,$$

On the basis of the above results, the bigger the reliability index β and the reliability R are, the less the values of the reliability sensitivities $DR/D\bar{x}^T$ are, the more robust the reliability of the rear-axle housing is. For this example, the two methods (this new method and the reliability design optimization) have been employed and the new method is very well for making the reliability insensitive to the variations of the design variables.

The relation curves between the reliability index β and the reliability R that computed by the Edgeworth series, the amendatory expression and MCS with 10^5 samples of the rear-axle housing with quadrate section are illustrated in Figure 4 respectively. If the results computed by the Edgeworth series appear $R>1$, the relation curves in Figure 4 indicate the results computed by the amendatory expression are closer to that by MCS than that by the Edgeworth series in interval [0.99, 1] that is usually used for reliability analysis in engineering practice.

7. CONCLUSIONS

This paper probes into the method of robust reliability design for vehicle components with arbitrary distribution parameters. The problem is formulated as a multi-objective optimization problem. Two examples are used to illustrate robust reliability design formulation and demonstrate the approach. Reliability design and reliability optimization design utilizes reliability theory to deal with uncertainties, and robust reliability design attempts to make the reliability variations of vehicle components are insensitive to variations of design variables. Using the method, robust reliability design parameters of vehicle components can be obtained accurately and quickly, so it is an important exploration of robust reliability design research of vehicle components. The accuracy of the approach proposed is evaluated by comparing their results to that obtained from MCS. Based on the results, the method presented is an efficient and practical robust reliability approach of vehicle components. Similarly if the case studies are very complicated, the implicit limit-state functions, such as those defined by the large-scale finite element models, should be used to demonstrate the general applicability of the proposed method.

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