

Neural Robust Control for Perturbed Crane Systems

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In this paper, we present a new control methodology for perturbed crane systems. Nonlinear crane systems are transformed to linear models by feedback linearization. An inverse dynamic equation is applied to compute the system PD control force. The PD control parameters are selected based on a nominal model and are therefore suboptimal for a perturbed system. To achieve the desired performance despite model perturbations, we construct a neural network auxiliary controller to compensate for modeling errors and disturbances. The overall control input is the sum of the nominal PD control and the neural auxiliary control. The neural network is iteratively trained with a perturbed system until acceptable performance is attained. We apply the proposed control scheme to 2- and 3-degree-of-freedom (D.O.F.) crane systems, with known bounds on the payload mass. The effectiveness of the control approach is numerically demonstrated through computer simulation experiments.

Key Words : Crane Control Systems, Neural Network, System Perturbation

1. Introduction

Mechanical cranes are widely used in industry to move heavy objects. The control goal of the crane is to place an object in a desired position within a given time interval and with prescribed error bounds. Research in crane system modeling and control has been extensive and has recently led to the implementation of sophisticated crane systems. In particular, elaborate control strategies for several kinds of crane systems have been developed and successfully implemented in industry.

Yu et al. used a time-scale separation control method for an overhead crane system, in which a linearized model was used to describe the error dynamics (Yu et al., 1995). Yashida et al. proposed a saturating control approach based on a guaranteed control cost for a linearized crane model (Yoshida and Kawabe, 1992). An approximate crane system modeling was investigated to build exact model information and to design an adaptive controller by Martindale et al. (1995). Moustafa and Ebeid developed a nonlinear dynamic model for an overhead crane system and applied a linear feedback control based on a linearized state space equation (Moustafa and Ebeid, 1988). Lee studied nonlinear modeling of an overhead crane system using a new swing-angle definition and an anti-swing control methodology for decoupled linearized dynamic characteristics (Lee, 1998). More advanced researches for nonlinear dynamics of crane systems

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were also addressed more recently. Fantoni et al. proposed a passivity-based controller for an underactuated crane system using an energy-based nonlinear control scheme (Fantoni et al., 2000). Fang et al. investigated an energy-based control approach for an overhead system, in which additional nonlinear terms were injected to the controller to increase coupling between the gantry and the payload position (Fang et al., 2001). The approach resulted in a significant improvement in the transient response.

Most available crane control methodologies are typically based on a linearized system model and few researchers have addressed the issues of controller robustness or adaptation. Model inaccuracies or perturbations are inevitable in practice and result in unknown perturbations from the nominal crane model. However, some knowledge regarding the perturbation is known in practice. Based on this information, we must design a robust controller for the crane. However, uncertain or incompletely known perturbations complicate the design and implementation of the controller.

We propose a corrective control design for perturbed crane systems using a neural network in addition to nominal PID control. First, PD control is derived for nominal plant dynamics using inverse dynamics and feedback linearization (Hunt and Meyer, 1983). The PD control parameter selection is designed for the nominal plant model and may perform poorly for the perturbed system. Next, we design a neural network auxiliary controller to correct errors due to model perturbations. The control input is given by the sum of the neural control and the nominal PD control. The neural network is iteratively trained with perturbed models of the crane system, in which system parameters are arbitrarily varied within their known bounds. In this paper, we assume a payload mass change as system perturbation. This assumption is realistic since the load mass for a crane is not known a priori. For evaluation of the proposed control scheme, 2- and 3-D.O.F. crane systems are simulated and the control performance is compared to the nominal control for perturbed systems with no correction for perturba-

tion.

The remainder of this paper is organized as follows. In Section II, the crane controller design using feedback linearization and a neural network is presented. 2- and 3-D.O.F. crane models are introduced and the proposed control scheme is applied to them in Section III and IV, respectively. Several simulation results are provided and discussed in Section V. Finally, our conclusion is given in Section VI.

2. Controller Design of Crane Systems

In this Section, we derive a controller design based on feedback linearization for crane systems. We assume that the equation of motion for the crane is

$$M(q)\ddot{q} + V(q, \dot{q})\dot{q} + G(q) = f \quad (1)$$

where $M \in \mathbb{R}^{n \times n}$ is the inertia matrix, $V \in \mathbb{R}^{n \times n}$ is the centripetal-Coriolis matrix, $G \in \mathbb{R}^n$ is a gravity term, $f \in \mathbb{R}^n$ is an input vector, and $q \in \mathbb{R}^n$ is a state vector. For the Lagrangian dynamics of Eq. (1) (Slotine and Li, 1991), we simply apply a feedback linear transformation to the system equation to compute the input vector. The inverse dynamics provide an expression for the input vector

$$f = M(q)u + V(q, \dot{q})\dot{q} + G(q) \quad (2)$$

where a velocity state vector \dot{q} is assumed to be measured by sensor systems in practice. In Eq. (2), u is a new control input vector which leads to the linear model $\ddot{q} = u$. We select the PD control structure

$$u = K_p e + K_d \dot{e} \quad (3)$$

where K_p is the proportional control matrix and K_d is the derivative control matrix, e is the error vector and \dot{e} is its derivative vector. Assuming a constant reference vector, its derivative vector is zero. Substituting the control u of Eq. (3) to the linear dynamic equation $\ddot{q} = u$ gives the nominal closed-loop dynamics

$$\ddot{q} + K_d \dot{q} + K_p q - K_p r = 0 \quad (4)$$

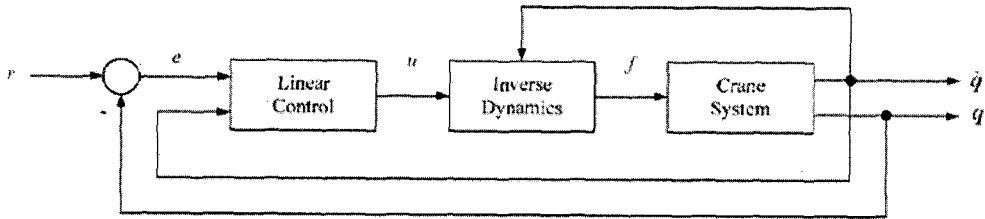


Fig. 1 Feedback linearization control for crane systems

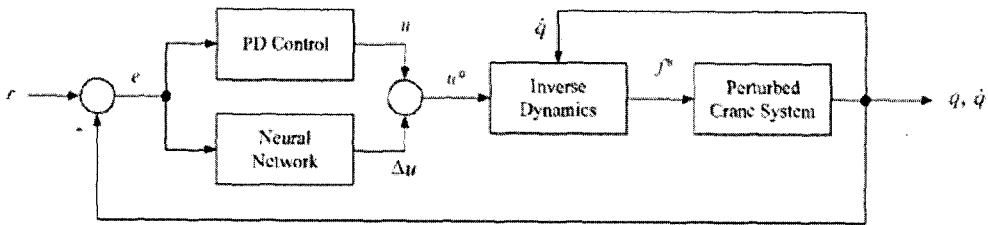


Fig. 2 Robust neural control for crane systems

where r is the reference vector. We select the control matrices K_p and K_d to assign the eigenvalues for Eq. (4) that provide the desired time response. Fig. 1 depicts the feedback linearization scheme for crane systems.

2.1 System perturbation

The input vector in Eq. (2) is derived under assumption that a system dynamic model in Eq. (1) is exact. In practice, however, this assumption is rarely valid due to modeling errors, environmental changes, etc. A controller constructed using a nominal model can therefore perform poorly in practice and compensation for model perturbations is needed. Adding a corrective control input vector Δu to Eq. (2), we express the input to the perturbed system as

$$f^* = M(q)(u + \Delta u) + V(q, \dot{q})\dot{q} + G(q) \quad (5)$$

Although perturbations are generally unknown prior to implementation, some information such as upper and lower bounds on parameter values is typically available. In this paper, we assume known perturbation bounds and use them to design the corrective control Δu through soft computation. Fig. 2 illustrates our neural network perturbation control for crane systems. The total control input is the sum of the nominal input and the corrective input from the neural network.

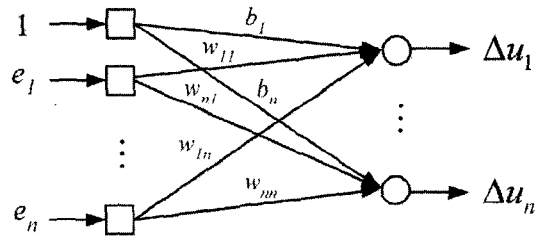


Fig. 3 A neural network controller

2.2 Neural network control

We use a neural network for the calculation of the corrective control input Δu . The neural network is iteratively trained to minimize a specified objective function under a perturbed system model in which perturbation value is changed within known bound for each training period. The proposed neural network is composed of a single-layer perceptron, shown in Fig. 3.

In Fig. 3, input patterns are realizations of the system error vector and output signals are the corrective input, which is expressed as

$$\Delta u_j = \varphi \left(\sum_{i=1}^n w_{ji} e_i + b_j \right), \quad j=1, \dots, n \quad (6)$$

where φ is an activation function, and w_{ji} and b_j denote the weight and bias, respectively. Network training is selecting optimal values of weights and biases using an appropriate optimization method given an objective function. We define the objective function as

$$J = \min_{w,b} \frac{1}{2} \left\{ \sum_{i=1}^n e_i^2 \right\} \quad (7)$$

and adjust the weights and biases using the gradient descent algorithm

$$w_{ji}(k+1) = w_{ji}(k) - \eta \frac{\partial J}{\partial w_{ji}} \quad (8)$$

$$b_j(k+1) = b_j(k) - \eta \frac{\partial J}{\partial b_j} \quad (9)$$

where k denotes the discrete iteration number and $i, j=1, \dots, n$. By using the chain rule to calculate partial derivatives in the right side of Eq. (8) and Eq. (9), we obtain

$$w_{ji}(k+1) = w_{ji}(k) + \eta e_j \left(\frac{\partial q_j}{\partial u_j} \right) e_i \quad (10)$$

$$b_j(k+1) = b_j(k) + \eta e_j \left(\frac{\partial q_j}{\partial u_j} \right) \quad (11)$$

where we linearly approximate the system Jacobian, for simplicity, by a change in the system output and input (Guez et al., 1988). Finally the adjustment rules of the weights and biases are given by

$$w_{ji}(k+1) = w_{ji}(k) + \eta e_j \left(\frac{q_j(k) - q_j(k-1)}{u_j(k) - u_j(k-1)} \right) e_i \quad (12)$$

$$b_j(k+1) = b_j(k) + \eta e_j \left(\frac{q_j(k) - q_j(k-1)}{u_j(k) - u_j(k-1)} \right) \quad (13)$$

3. 2-D.O.F. Crane System

We first consider a standard 2-D.O.F. crane system which is composed with both translation and rotational portions. The system model is shown in Fig. 4.

The motion equations of this system are

$$(m_t + m_p) \ddot{x} + b\dot{x} + m_p L \ddot{\theta} \cos(\theta) - m_p L \dot{\theta}^2 \sin(\theta) = f \quad (14)$$

$$(m_t + m_p) \ddot{\theta} + b\dot{\theta} + m_p L \ddot{x} \cos(\theta) - m_p L \dot{x}^2 \sin(\theta) = f \quad (15)$$

where m_t is a crane mass, m_p is a payload mass, L is a loop length, g is a gravity constant, f is a force scalar applied to a crane system, and the coordinates x and θ are the crab position and angular position of the line, respectively. In practice, the crane mass is essentially constant, but a

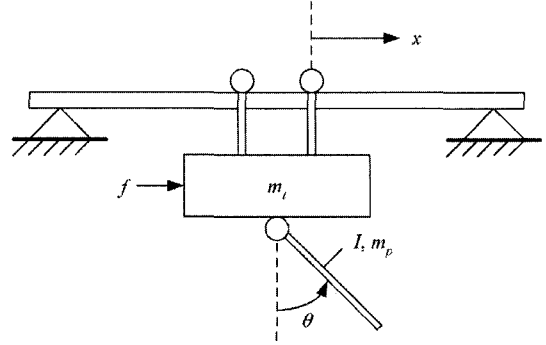


Fig. 4 A 2-D.O.F. crane system

payload mass changes within a limited range as the load of the crane changes.

3.1 Controller design

Based on Eq. (3), PD control for this system is

$$u = K_p \begin{bmatrix} e_x \\ e_\theta \end{bmatrix} + K_d \begin{bmatrix} \dot{x} \\ \dot{\theta} \end{bmatrix} \quad (16)$$

where $e_x = x - r_x$ in which r_x is a reference position and $e_\theta = \theta$ due to a zero reference angle. We simply define diagonal parameter matrices as $K_p = \text{diag}(k_{p_x}, k_{p_\theta})$ and $K_d = \text{diag}(k_{d_x}, k_{d_\theta})$. By applying PD control, an input scalar formed in an inverse dynamic equation is expressed as

$$f = (m_t + m_p) u_x + m_p L \cos(\theta) u_\theta + b\dot{x} - m_p L \dot{\theta}^2 \sin(\theta) \quad (17)$$

where $u_x = k_{p_x} e_x + k_{d_x} \dot{x}$ and $u_\theta = k_{p_\theta} \theta + k_{d_\theta} \dot{\theta}$. The applied force including the corrective control terms is

$$f^* = (m_t + m_p) (u_x + \Delta u_x) + m_p L \cos(\theta) (u_\theta + \Delta u_\theta) + b\dot{x} - m_p L \dot{\theta}^2 \sin(\theta) \quad (18)$$

where Δu_x and Δu_y are computed by the neural network shown in Fig. 3 as stated in Section II. For this case, the two input signals of the network are e_x and e_θ and the output signals are Δu_x and Δu_θ .

4. 3-D.O.F. Crane System

In this section, we consider a 3-D.O.F. under-actuated overhead crane with two external inputs (Fang et al., 2003). Fig. 5 depicts the system

model and its dynamic motion equation is

$$M \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\theta} \\ \ddot{\phi} \end{bmatrix} + V \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ m_p g L \sin(\theta) \\ 0 \end{bmatrix} = \begin{bmatrix} f_x \\ f_y \\ 0 \\ 0 \end{bmatrix} \quad (19)$$

In Eq. (19), the inertia matrix M is

$$M = \begin{bmatrix} m_p + m_r + m & 0 & m_p L \cos(\theta) \sin(\phi) & m_p L \sin(\theta) \cos(\phi) \\ 0 & m_p + m_c & m_p L \cos(\theta) \cos(\phi) & -m_p L \sin(\theta) \sin(\phi) \\ m_p L \sin(\theta) \sin(\phi) & m_p L \cos(\theta) \cos(\phi) & m_p L^2 + I & 0 \\ m_p L \sin(\theta) \cos(\phi) & -m_p L \sin(\theta) \sin(\phi) & 0 & m_p L^2 \sin^2(\theta) + I \end{bmatrix} \quad (20)$$

and the centripetal-Coriolis matrix V is given by

$$V = \begin{bmatrix} 0 & 0 & -m_p L \sin(\theta) \sin(\phi) \dot{\theta} + m_p L \cos(\theta) \cos(\phi) \dot{\phi} & m_p L \cos(\theta) \cos(\phi) \dot{\theta} - m_p L \sin(\theta) \sin(\phi) \dot{\phi} \\ 0 & 0 & -m_p L \sin(\theta) \cos(\phi) \dot{\theta} - m_p L \cos(\theta) \sin(\phi) \dot{\phi} & -m_p L \cos(\theta) \sin(\phi) \dot{\theta} - m_p L \sin(\theta) \cos(\phi) \dot{\phi} \\ 0 & 0 & 0 & -m_p L^2 \sin(\theta) \cos(\theta) \dot{\phi} \\ 0 & 0 & m_p L^2 \sin(\theta) \cos(\theta) \dot{\phi} & m_p L^2 \sin(\theta) \sin(\theta) \dot{\theta} \end{bmatrix} \quad (21)$$

where m_c is the cart mass, the input forces f_x and f_y acted on the cart and the rail. The system coordinates are : x , the position along x -axis, and y , the position along the y -axis. θ is a payload angle with respect to the vertical, and ϕ is the projection of the payload angle along the x -axis. Other notations in Eqs. (19) ~ (21) are identical to those of the 2-D.O.F. crane system model in Eq. (14) and Eq. (15). We make several assumptions regarding the 3-D.O.F. crane. First, the payload and cart are linked by a rigid and massless connector. Second, the state variables and their derivatives are measured. Third, the cart mass and the rod length are known exactly. Fourth, friction

at the ball joint which links the payload to the cart is ignored, and this joint does not rotate relative to the link rod. Finally, a bound of θ is $-\pi < \theta < \pi$.

4.1 Controller design

As in Eq. (16), a PD control vector with diagonal parameter matrices is selected and the control input is

$$u = \begin{bmatrix} u_x \\ u_y \\ u_\theta \\ u_\phi \end{bmatrix} = \begin{bmatrix} k_{p_x} & 0 & 0 & 0 \\ 0 & k_{p_y} & 0 & 0 \\ 0 & 0 & k_{p_\theta} & 0 \\ 0 & 0 & 0 & k_{p_\phi} \end{bmatrix} \begin{bmatrix} e_x \\ e_y \\ e_\theta \\ e_\phi \end{bmatrix} + \begin{bmatrix} k_{d_x} & 0 & 0 & 0 \\ 0 & k_{d_y} & 0 & 0 \\ 0 & 0 & k_{d_\theta} & 0 \\ 0 & 0 & 0 & k_{d_\phi} \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix} \quad (22)$$

where $e_x = x - r_x$, $e_y = y - r_y$, $e_\theta = \theta$, and $e_\phi = \phi$ for zero reference angles. The input forces applied to the crane are computed using the control vector of Eq. (19) as

$$f_x = (m_p + m_r + m_c) u_x + m_p L \cos(\theta) \sin(\phi) u_\theta + m_p L \sin(\theta) \cos(\phi) u_\phi \quad (23)$$

$$f_y = (m_p + m_c) u_y + m_p L \cos(\theta) \cos(\phi) u_\theta - m_p L \sin(\theta) \sin(\phi) u_\phi \quad (24)$$

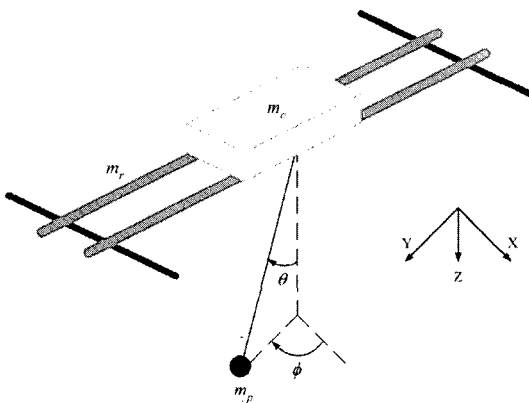


Fig. 5 A 3-D.O.F. crane system

The perturbation control inputs are

$$f_x = (m_p + m_r + m_c) (u_x + \Delta u_x) + m_p L \cos(\theta) \sin(\phi) (u_\theta + \Delta u_\theta) + m_p L \sin(\theta) \cos(\phi) (u_\phi + \Delta u_\phi) \quad (25)$$

$$f_y = (m_p + m_c) (u_y + \Delta u_y) + m_p L \cos(\theta) \cos(\phi) (u_\theta + \Delta u_\theta) - m_p L \sin(\theta) \cos(\phi) (u_\phi + \Delta u_\phi) \quad (26)$$

where the perturbation control inputs are computed by a neural network whose inputs are e_x , e_y , e_θ , and e_ϕ , and outputs are Δu_x , Δu_y , Δu_θ , and Δu_ϕ .

5. Simulation Examples and Results

We simulated the two crane systems presented in Section II and III with nominal PD control and corrective neural network control using MATLAB®. Three simulation examples for each crane system were performed: First, the nominal systems were simulated applying PD controls whose parameters are selected from several experiments using nominal models. Second, we used PD control with crane models in which the payload mass is arbitrarily changed resulting in unacceptable control performance. Finally, we apply neural network correction together with nominal PD control to the cranes and compare the results to nominal control. Example I is a simulation example for the 2-D.O.F. crane system and Example II is for a 3-D.O.F. system.

Example I-1: In this example, a nominal model of the 2-D.O.F. crane with PD control is simulated. Crane system parameter values are $m_p =$

160[kg], $m_c = 23$ [kg], $L = 2.5$ [m], and $I = 1.5$ [kg·m²]. We use a control time interval of [0, 20] sec and a reference (desired) position $r_x = 10$ [m]. Thus, the control goal is that the crane reaches the reference position within this time interval. Iterative simulations showed that the best control performance was achieved with the PD control parameter values $k_{p_x} = 1.23$, $k_{p_\theta} = 0.51$, $k_{d_x} = 2.18$, and $k_{d_\theta} = 0.28$. Fig. 6 shows the crane position and angle trajectories and the control input trajectory. The position response has a small peak overshoot around 3.5 sec, but settles at the desired position within the [0, 20] sec interval and the control performance is satisfied. In the loop angle response, there is an undershoot as well as an overshoot in the transient response and the system settles after about 6.2 sec. Thus, the performance of the control system is satisfactory and the control parameters are suitable for the nominal system.

Example I-2: We simulated the perturbed system with PD control used in Example I-1. As system perturbation, a payload mass of 5,000[kg] is added, i.e. $m_p = 5,160$ [kg], which is the maximum allowed for this system. A simulation scenario identical to Example I-1 is used. Fig. 7 illustrates the system responses and input. As expected, we observe from the results that the performances is unsatisfactory, and the position and angle dynamics do not settle within the specified time interval. Moreover, the control input magnitude is on the average larger than that of Example I-1. Consequently, the control parameters which were selected for the nominal model are inappropriate

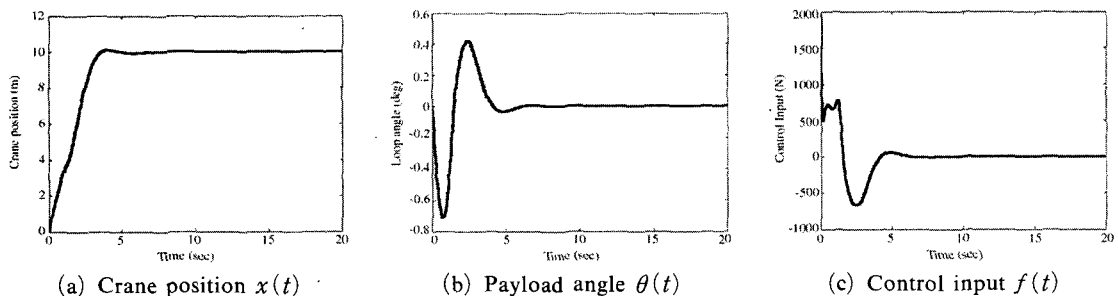


Fig. 6 System trajectories and control input (Example I-1)

for the perturbed system.

Example I-3: In this Example, a neural network is constructed as described in Section II and the perturbed system with a neural control along with a nominal PD control is simulated under same simulation environment. The initial values of the weights and biases were randomly selected via a uniform distribution in $[-0.5, 0.5]$ and a bipolar sigmoid activation function is used. The network is iteratively trained with the perturbed system in which the payload mass is changed within the range $[160, 5,160]$ kg for each training period. The trajectories of the crane position and loop angle as well as control inputs for PD control

and the neural control are plotted in Fig. 8. The results demonstrate that the control performance is improved and the position and angle dynamics settle to the desired levels within the prescribed time interval. Regarding the control inputs, the magnitude of the PD control term first increases to a large positive value around the peak time, but the neural control has a relatively small negative value. We interpret this as corrective action by the neural network to improve the control performance.

Example II-1: From this example, the 3-D.O.F. crane system is simulated as in Example 1. The system parameter values are the same as the 2-

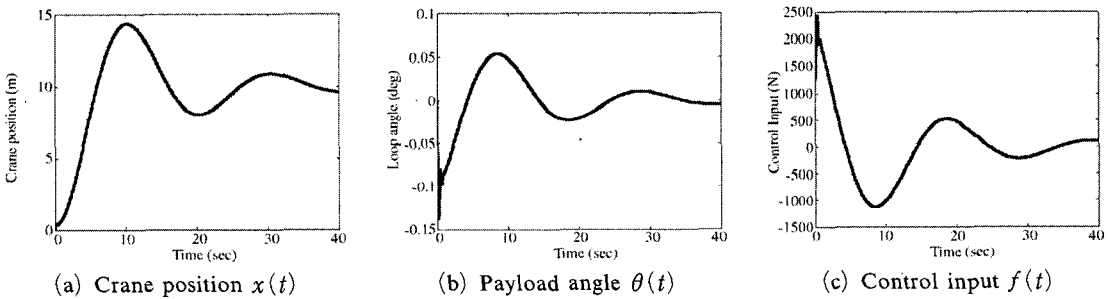


Fig. 7 System trajectories and control input (Example I-2)

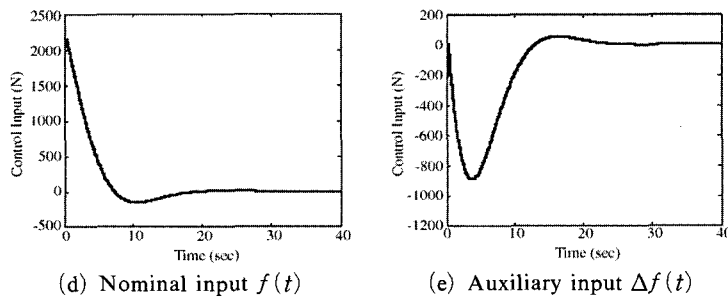
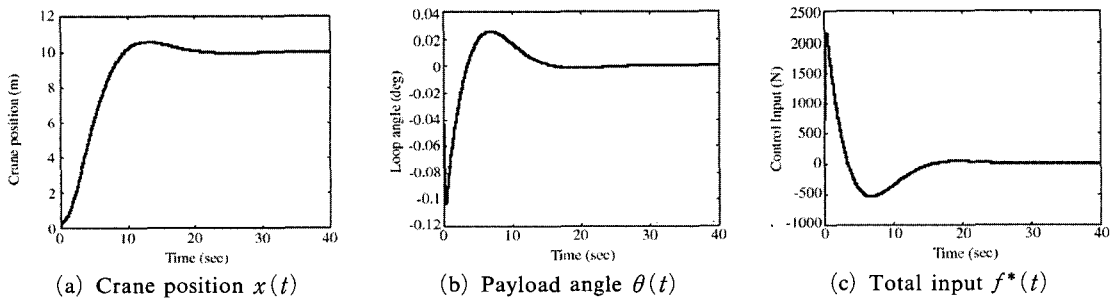


Fig. 8 System trajectories and control inputs (Example I-3)

D.O.F. system and the rail mass is $m_r=190$ [kg]. The reference positions are $r_x=10$ [m] and $r_y=3$ [m]. The PD control parameter matrices are chosen as $K_p=diag\{0.1, 0.25, 0.5, 0.35\}$ and $K_d=diag\{0.35, 0.5, 0.8, 0.73\}$ for the nominal system. Fig. 9 shows the position and angle responses for the nominal system with PD control. The two position responses have about 20[%] and 17[%] overshoot in the transient period, respectively, but both reach the reference level in about 30 sec. The two angle responses have acceptable transient responses and converge to the steady-state region.

Example II-2 : The crane model is simulated with a perturbed payload mass of $m_p=5,160$ [kg] and with nominal PD control. The other parameter

values and PI control values are set as Example II-1. Simulation results, shown in Fig. 10, indicate unsatisfactory performance. Clearly, the control parameter values must be retuned to meet the design specifications.

Example II-3 : The perturbed system is composed of neural network control together with a nominal PD control. The neural network is designed and trained similarly to and with the same settings as for the 2-D.O.F. crane. The simulation results of Fig. 11 illustrate the crane positions and angles in which the control performances are evidently improved. The overshoots and the settling times for the responses are considerably diminished and the oscillations and the magnitudes are correspondingly reduced in the angle

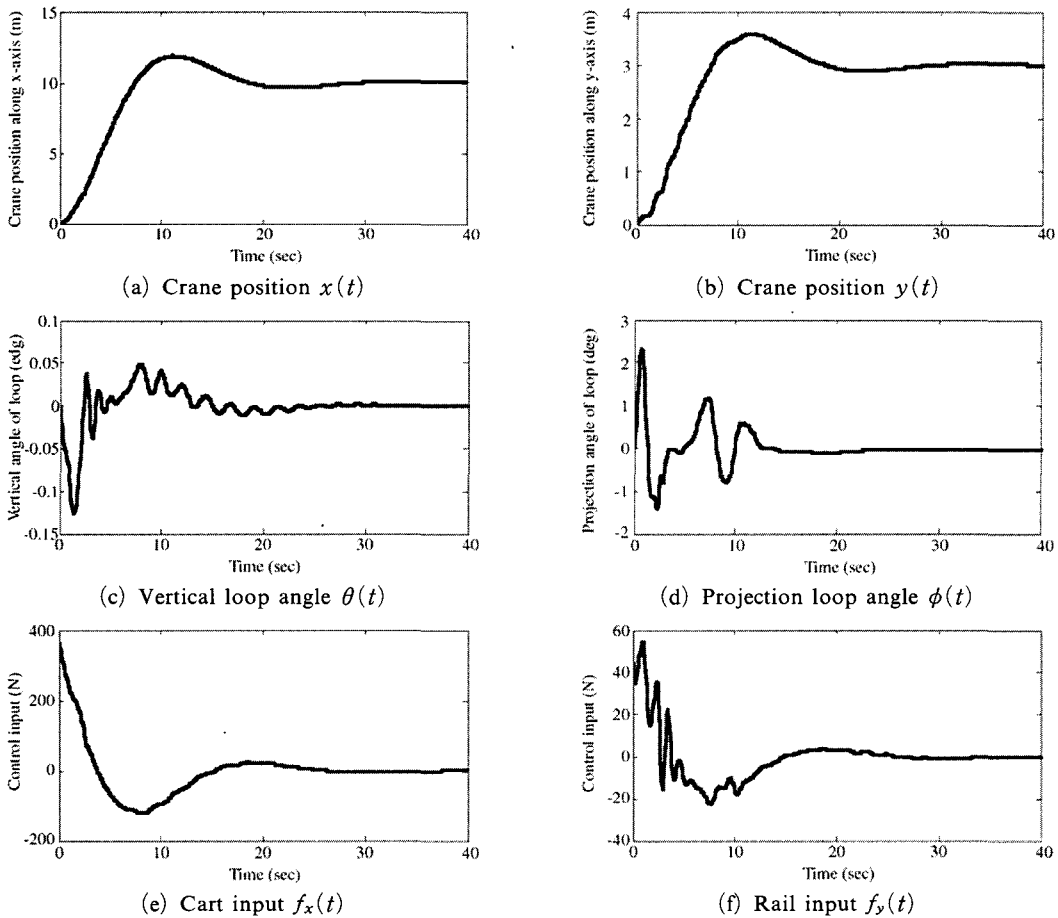


Fig. 9 System trajectories and control inputs (Example II-1)

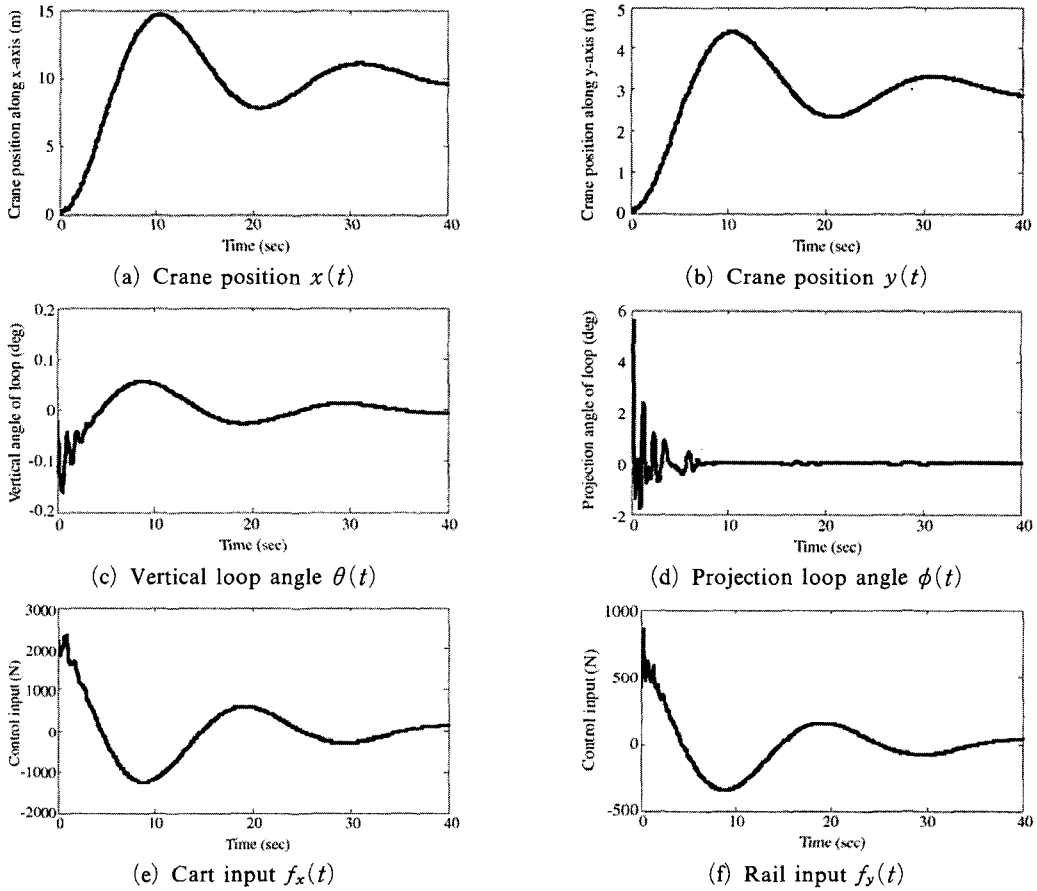


Fig. 10 System trajectories and control inputs (Example II-2)

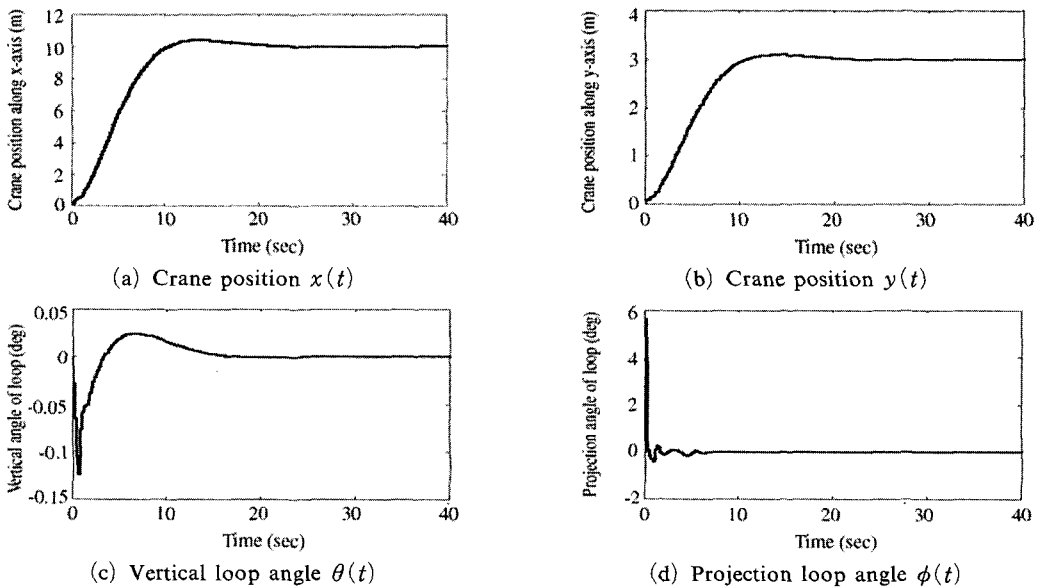


Fig. 11 System trajectories (Example II-3)

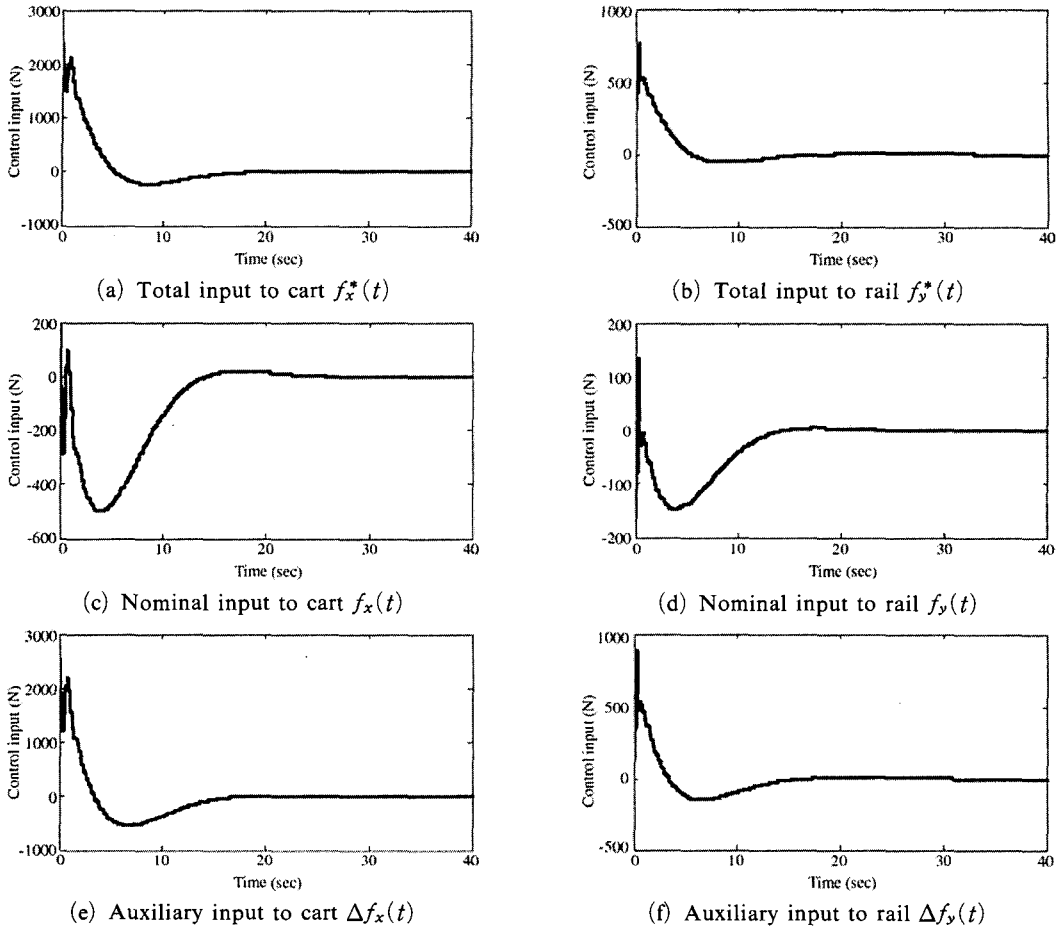


Fig. 12 Control inputs (Example II-3)

dynamics. Time histories of control input dynamics are plotted in Fig. 12. We finally conclude from the simulation results that the corrective neural network control significantly improves the control performance for the perturbed system.

6. Conclusions

We propose a new robust control for perturbed crane systems using a neural network. We obtain PD control for the nominal crane model, then design a neural network to compensate for model perturbations. The control input to the perturbed system is composed of a PD control and a corrective neural control input. 2- and 3-D.O.F crane systems in which the payload mass was perturbed were simulated with the proposed control and numerically analyzed. From the analysis,

we conclude that the corrective control component is required for the perturbed systems to meet design specifications. With corrective control the transient performance of the cranes was considerably improved. The overshoots for the crane positions and the payload angles responses were reduced and their settling times were significantly diminished. In future work, we will consider more general perturbations in the crane model and develop neural control correspondingly. We will also examine the stability of the perturbed control system using Lypunov stability theory (Khalil, 1996).

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