612 *Journal of Mechanical Science and Technology (KSME Int. J.), Vol. 20, No. 5, pp. 612~620, 2006*

# Optimal Trajectory Generation for Biped Robots Walking Up-and-Down Stairs

**Ohung Kwon, Kweon Soo Jeon**
*Department of Precision Mechanical Engineering, Hanyang University,*
*17 Haengdang-Dong, Sungdong-Ku, Seoul 133-791, Korea*
**Jong Hyeon Park***
*School of Mechanical Engineering, Hanyang University,*
*17 Haengdang-Dong, Sungdong-Ku, Seoul 133-791, Korea*

This paper proposes an optimal trajectory generation method for biped robots for walking up-and-down stairs using a Real-Coded Genetic Algorithm (RCGA). The RCGA is most effective in minimizing the total consumption energy of a multi-dof biped robot. Each joint angle trajectory is defined as a 4-th order polynomial of which the coefficients are chromosomes or design variables to approximate the walking gait. Constraints are divided into equalities and inequalities. First, equality constraints consist of initial conditions and repeatability conditions with respect to each joint angle and angular velocity at the start and end of a stride period. Next, inequality constraints include collision prevention conditions of a swing leg, singular prevention conditions, and stability conditions. The effectiveness of the proposed optimal trajectory is shown in computer simulations with a 6-dof biped robot model that consists of seven links in the sagittal plane. The optimal trajectory is more efficient than that generated by the Modified Gravity-Compensated Inverted Pendulum Mode (MGCIPM). And various trajectories generated by the proposed GA method are analyzed from the viewpoint of the consumption energy : walking on even ground, ascending stairs, and descending stairs.

**Key Words :** Biped Robot, Optimal Trajectory, Genetic Algorithm, Stairs, Constraints, Energy Efficiency

## 1. Introduction

In addition to the need for improved technological development, more and more biped robots need to have the capability to adapt themselves to artificial environments since they must stay in artificial spaces and live together with human beings. The artificial elements include uneven surfaces, stairs, door thresholds and etc. If difficult problems associated with such man-made environments are solved, biped robots can live conveniently with human beings. Specially, as far as the walking motion of biped robots is concerned, the most difficult problem is to minimize energy consumption. In general, low energy gaits require lower power and lighter batteries and in turn make great reduction in weight of a biped robot.

Many researchers for the biped robot have studied the generation methods of energy-efficient trajectories using the kinematic structure and dynamic characteristics similar to a human (Peng and ONO, 2005). Since human beings walk with steady gaits minimizing the locomotion energy in everyday experience irrespectively of various locomotion conditions in man-made environments. If designing biped robots are captured with the

---

* Corresponding Author,
**E-mail** : jongpark@hanyang.ac.kr
**TEL** : +82-2-2220-0435; **FAX** : +82-2-2290-4634
School of Mechanical Engineering, Hanyang University, 17 Haengdang-Dong, Sungdong-Ku, Seoul 133-791, Korea. (Manuscript **Received** June 17, 2005; **Revised** February 27, 2006).

essential characteristics of the human walking motion, energy-efficient gaits result in the human-like walking motion. Choi et al. proposed a method to find optimal via-points data that minimize the sum of the deviation of velocities and accelerations and reduce the jerk using genetic algorithms (Choi et al., 1999). Cheng and Lin proposed a method to search the parameters of the controller and the gait. Thus, the design of the controller and the gait was formulated as a parameter search problem and a genetic algorithm was applied to improve the design (Cheng and Lin, 1995). Park and Choi proposed a method that minimizes the energy consumption by finding the optimal locations of the mass centers of the links, and the optimal trajectory of the legs (Park and Choi, 2004). Chevallereau et al. proposed a searching method for the optimal stride and period to generate optimal trajectories (Chevallereau et al., 1998). Here, these researchers were able to generate low energy gaits of simplified biped robots on even ground.

Despite these advances, it is necessary to minimize the locomotion energy for biped robots walking up and down stairs since the irregular ground conditions consume the greatest amount of all gaits. Shih proposed to synthesize an efficient walking pattern for ascending and descending stairs for a biped robot with 7 DOF (Shih, 1999). Hwang et al. simulated a humanoid robot walking up stairs using a virtual spring-damper contact model in order to precisely simulate a collision with friction between the foot and the ground (Hwang et al., 2003). However, these previous works were not analyzed from the viewpoint of the consumption energy with respect to walking up and down stairs.

This paper proposes a simple and fast-convergent optimization method to generate a trajectory that minimizes the energy consumption of a biped robot for ascending and descending stairs using a real-coded genetic algorithm. And a computed torque controller is applied to computer simulations for stable dynamic biped locomotion. In the saggital plane, a 6 degree of freedom biped robot model that consists of seven links is used. In order to approximate the walking gait, each joint angle trajectory is defined as a 4-th order polynomial of which coefficients are chromosomes. In various computer simulations, the proposed method is effective and stable, compared with the proposed GA method and a gait trajectory for biped robots based on the Modified Gravity-Compensated Inverted Pendulum Mode (MGCIPM) when the biped robot ascends stairs (Park and Kim, 1998). In this paper, we analyze and compare the energy efficiency of the three following cases: walking on even ground, ascending stairs, and descending stairs. Also, the energy efficiency is calculated by comparing the stair height of 5 cm with that of 10 cm.

This paper is organized as follows: The dynamics of a biped robot is described in Sec. 2. The constraints for ascending and descending stairs and proposed genetic algorithms for optimization are presented in Sec. 3 and 4, respectively. Section 5 describes computer simulation and comparisons of the energy efficiency, followed by conclusions in Sec. 6.

## 2. Dynamics of Biped Robot Model

The biped robot is modeled as an anthropomorphic planar mechanism system of seven links that connected by 6 actuated rotational joints as shown in Fig. 1. This model expresses the characteristics of human walking very well. The dy-

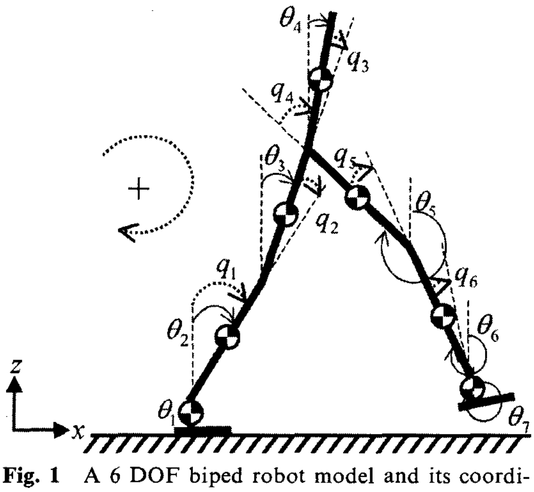

**Fig. 1** A 6 DOF biped robot model and its coordinates with absolute angles and relative angles

namic equations derived from the Lagrange's equation based on the kinetic and potential energies of each link can be written as:

$$\mathbf{M}(q)\ddot{q}+\mathbf{C}(q,\dot{q})+\mathbf{G}(q)=\boldsymbol{\tau} \tag{1}$$

where $q=[q_1\ q_2\ q_3\ q_4\ q_5\ q_6]^T\in\mathbb{R}^6$ is the vectors described with relative angles. And $\mathbf{M}(q)\in\mathbf{R}^{6\times6}$ is the inertia matrix, $\mathbf{C}(q,\dot{q})\in\mathbf{R}^{6\times1}$ is the vector of coriolis and centrifugal forces, $\mathbf{G}(q)\in\mathbf{R}^{6\times1}$ is the vector of gravity force, and $\tau\in\mathbf{R}^{6\times1}$ is the vector of torques on each joint.

A transformation matrix, $\mathbf{E}\in\mathbf{R}^{6\times7}$ is used to transform from relative angles $q$ to absolute angles $\theta\in\mathbb{R}^7$.

$$\theta=E^Tq,\ E=\begin{bmatrix}0&-1&1&0&0&0&0\\0&0&-1&1&0&0&0\\0&0&0&1&-1&0&0\\0&0&0&0&1&-1&0\\0&0&0&0&0&1&-1\\0&0&0&0&0&0&1\end{bmatrix} \tag{2}$$

It is assumed that the mass of a link is concentrated on the center of each link and the supporting foot is fixed on the ground. It is also assumed that a full locomotion cycle is divided into a single support phase and an exchange of supporting roles: the former is that one leg is in contact with the ground and the other leg swings forward. The latter is that the two legs simultaneously trade roles.

## 3. Constraints

### 3.1 Equality constraints

The foot of the swing leg must be on the stairs both in the beginning and in the end of a stride period since locomotion is periodic. Thus, the speed and the configurations of the left and right legs at the end of the stride become identical to those of the right and left legs, respectively, at the beginning of the stride. This is called the initial conditions. Therefore, the tip position $(x_{tip}, z_{tip})$ of a swing foot must satisfy the initial conditions.

Firstly, when the biped robot ascends the stairs, the equality constraints that are satisfied at the start and end of the stride are expressed as follows:

$$\text{At } t=0:\ x_{tip}(0)=-S,\ z_{tip}(0)=-H \tag{3}$$

$$\text{At } t=t_f:\ x_{tip}(t_f)=S,\ z_{tip}(t_f)=H \tag{4}$$

Also, when the robot descends the stairs, the equality constraints that are satisfied at the start and end of the stride are expressed as follows:

$$\text{At } t=0:\ x_{tip}(0)=-S,\ z_{tip}(0)=H \tag{5}$$

$$\text{At } t=t_f:\ x_{tip}(t_f)=S,\ z_{tip}(t_f)=-H \tag{6}$$

where $t_f$ is a stride period, $S$ is a stride, and $H$ is a stair height. With such conditions, the repeatability conditions with absolute angles must be satisfied.

$$\theta_i(0)=\theta_{8-i}(t_f)\quad(i=1,\cdots,7) \tag{7}$$

$$\dot{\theta}_i(0)=\dot{\theta}_{8-i}(t_f)\quad(i=1,\cdots,7) \tag{8}$$

### 3.2 Inequality constraints

The tip and toe of a swing foot must not penetrate the wall and face of the stairs during a locomotion period. Thus, its tip and toe trajectory must be satisfied with the collision prevention condition according to the structure of the stairs. When the biped robot ascends the stairs, its foot-tip must move over the face of the stairs and its foot-toe must avoid colliding with a wall of the stairs. Therefore,

$$\begin{cases}\text{if } -\frac{3S}{2}<x_{tip}(t)\le-\frac{S}{2} & \text{then } z_{tip}(t)>-H+\delta h\\ \text{if } -\frac{S}{2}<x_{tip}(t)<\frac{S}{2} & \text{then } z_{tip}(t)>\delta h\\ \text{if } \frac{S}{2}<x_{tip}(t)\le\frac{3S}{2} & \text{then } Z_{tip}(t)>H+\delta h\end{cases} \tag{9}$$

$$\begin{cases}\text{if } -H<z_{toe}(t)\le0 & \text{then } x_{toe}(t)<-\frac{S}{2}-\delta s\\ \text{if } 0<z_{toe}(t)\le H & \text{then } x_{toe}(t)<\frac{S}{2}-\delta s\\ \text{if } H<z_{toe}(t)\le 2H & \text{then } x_{toe}(t)<\frac{3S}{2}-\delta s\end{cases} \tag{10}$$

Similarly, when the biped robot descends the stairs, its foot-tip and foot-heel must not penetrate the wall and face of the stairs during a locomotion period. Therefore,

$$\begin{cases} \text{if } -\frac{3S}{2} < x_{tip}(t) \leq -\frac{S}{2} & \text{then } z_{tip}(t) > H + \delta h \\ \text{if } -\frac{S}{2} < x_{tip}(t) < \frac{S}{2} & \text{then } z_{tip}(t) > \delta h \\ \text{if } \frac{S}{2} < x_{tip}(t) \leq \frac{3S}{2} & \text{then } Z_{tip}(t) > -H + \delta h \end{cases} \tag{11}$$

$$\begin{cases} \text{if } -H < z_{heel}(t) \leq 0 & \text{then } x_{heel}(t) > \frac{S}{2} + \delta s \\ \text{if } 0 < z_{heel}(t) \leq H & \text{then } x_{heel}(t) > -\frac{S}{2} + \delta s \\ \text{if } H < z_{heel}(t) \leq 2H & \text{then } x_{heel}(t) > -\frac{3S}{2} + \delta s \end{cases} \tag{12}$$

where $\delta h$ and $\delta s$ denote the minimum allowable distances to keep its swing foot not to collide with the stairs.

In order to prevent knee joints from being fully stretched and then to avoid singular configurations, two additional constraints are applied to the knee joint, one for each leg.

$$\theta_2 - \theta_3 > \delta\theta \tag{13}$$

$$\theta_6 - \theta_5 > \delta\theta \tag{14}$$

where $\delta\theta$ is the maximum allowable angle to avoid a singular configuration.

Next, in order for a biped robot to walk stably, it is necessary to take account of the dynamic stability. In general, we use the concept of the Zero Moment Point (ZMP). The ZMP is defined as a point on the ground where the sum of all the moments of active forces is equal to zero. If the ZMP is inside the contact polygon between the foot and the ground, the biped robot walks stably. Thus, the nearer the ZMP exists from the center of the foot, the more stable the biped robot becomes. It can be computed by the following equation.

$$x_{ZMP} = \frac{\sum_{i=1}^{6} m_i(\ddot{z}_i + g) - \sum_{i=1}^{6} m_i \ddot{x}_i z_i}{\sum_{i=1}^{6} m_i(\ddot{z}_i + g)} \tag{15}$$

where $(x_i, z_i)$ is the position of the mass center of a link $i$ and $m_i(I_i)$ is the mass (inertia) of a link $i$. Therefore,

$$\| x_{ZMP} \| < \frac{\gamma}{2} \tag{16}$$

where $\gamma$ is the maximum allowable distance between the ZMP and the center of the foot.

## 4. Genetic Algorithms for Optimization

The trajectory of each joint angle during a stride is expressed as a 4th-order polynomial of time t instead of high order polynomial of time t in order to guarantee fast convergence. The coefficients of the polynomials representing all the joint angles are used as design variables.

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} \end{bmatrix} = \begin{bmatrix} 1 \\ t \\ t^2 \\ t^3 \\ t^4 \end{bmatrix} \tag{17}$$

where coefficient, $a_{i,j}(i=1, \cdots, 6, j=1, \cdots, 5)$, are design variables. In application, the total number of the design variables is reduced from 30 to 14 by the use of the equality constraints, Eq. (3) to Eq. (8). Since it increases the approach speed of GA. Therefore, the number of chromosomes used in the genetic algorithms is 14.

The performance index to be minimized is defined as

$$J(d) = \frac{1}{2} \int_0^{t_f} u^T Q u \, dt \tag{18}$$

where $u_i = \tau_i \cdot q_i$, $i = 1 \sim 6$ and $d \in \mathbb{R}^{14}$ denote the joint powers applied at each joint and the design variables, respectively. And, $Q = diag(\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6)$ is a positive definite matrix. Its elements, $\omega_{1 \sim 6}$, are the weighting factors that represent contributions by joint motors to the cost function.

The inequality constraints in Eq. (9) to (16) can be expressed as:

$$g_j(d) \leq 0 \ (j = 1, \cdots, n) \tag{19}$$

where a transformation method converts a constraint optimization problem into an unconstraint problem with a transformation function of

$$F(d, r) = J(d) + P(g_j(d), r) \tag{20}$$

where $r$ is a vector of penalty parameters and $P$ is a real valued function which of imposing the penalty is controlled by $r$. The form of penalty function $P$ depends on the transformation method used. The exterior penalty function method is used :

$$P(g(d), r)=\sum_{j=1}^{n} r_j[g_j^+(d)]^2 \tag{21}$$

where $g_j^+(d)=\max(0, g_j(d))$, and $r_j$ is a weight factor. The value of function $g_j^+(d)$ is zero if inequality is satisfied, $(g_j(d)<0)$, and it is positive if inequality is violated, i.e. $(g_j(d)>0)$.

Finally, optimal trajectories for a biped robot are obtained from the unconstraint function in Eq. (20) and the optimal method. The optimal method used in this paper is a real-coded genetic algorithm (Goldberg, 1989 ; Jin, 2002). Since the real- coded genetic algorithms are robust to ill-condition in the optimization function and they can always find global solutions from optimal problems defined in a very large domain with highly complex constraints.

The flow chart of the genetic algorithms is shown in Fig. 2. Firstly, genetic algorithms create each joint angle. And they evaluate whether the collision prevention condition, the singular prevention condition, and the stability condition are satisfied or dissatisfied. If they are dissatisfied with these conditions, GA operators regenerate each joint angle. The GA operators are composed of the reproduction of a gradient-like selection method, the crossover of a modified simple crossover method, and the mutation of a boundary mutation method. These processes are repeated until satisfactory joint angles are found or other a certain stop is met.

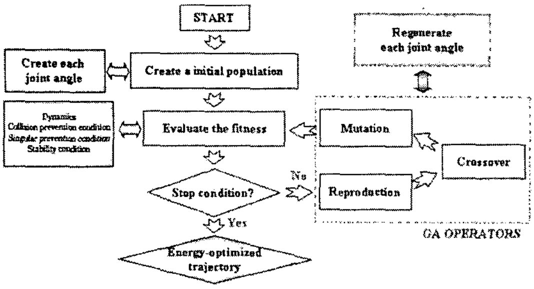

**Fig. 2** Flow chart of the proposed genetic algorithms

## 5. Simulations

The effectiveness of the proposed optimal trajectory for walking up-and-down stairs is shown in computer simulations. The parameters of the biped robot model are listed in Table 1. And the locomotion parameters used in the simulations are shown in Table 2. The stride length is 30 cm and the stair height is 5 cm.

Figures 3 and 4 show a stick diagram and each joint trajectory of the biped robot ascending the stairs, respectively. The parameters used in the proposed genetic algorithms are listed in Table 3. If the number of generations reaches the maximum number of generations set by the software or if the value of the cost function does not change for 100 conservative generations, the simulation is terminated. Without these conditions, it takes about 5,000 generations. But, under these conditions, it takes about 400 generations to obtain a reasonable result as shown in Fig. 5. These simulations indicate that the optimal locomotion trajectory is similar to that of a human for ascending the stairs. Especially, we know that the

**Table 1** Parameters of the biped robot model used in simulations

| Link No. | Length (m) | Mass (kg) |
|---|---|---|
| Link 1 | 0.1 | 1 |
| Link 2 | 0.4 | 5 |
| Link 3 | 0.4 | 4 |
| Link 4 | 0.5 | 6 |
| Link 5 | 0.4 | 4 |
| Link 6 | 0.4 | 5 |
| Link 7 | 0.1 | 1 |

**Table 2** Locomotion parameters used in simulations

| Parameters | Values |
|---|---|
| $S$ | 0.3 m |
| $H$ | 0.05 m |
| $\delta s$ | 0.001 m |
| $\delta h$ | 0.001 m |
| $\delta\theta$ | 0.001 rad |
| $\gamma$ | 0.19 m |
| $t_f$ | 1.0 sec |

**Table 3** Parameters used in the genetic algorithms

| Parameters | Values |
|---|---|
| Maximum Generations | 5,000 |
| Population | 30 |
| Chromosome Length | 14 |
| Crossover Ratio | 0.9 |
| Mutation Ratio | 0.02 |

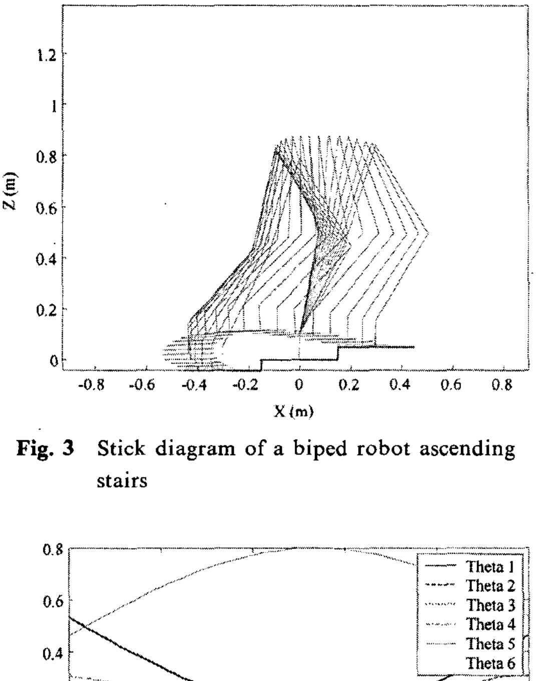

**Fig. 3** Stick diagram of a biped robot ascending stairs

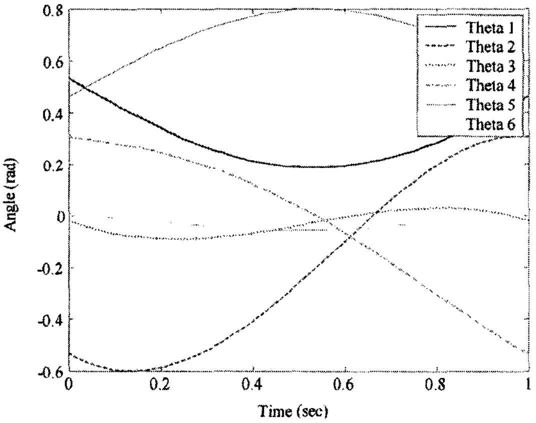

**Fig. 4** Joint angle on each joint when ascending stairs

robot foot moves somewhat backward at the beginning of a stride period in order to avoid colliding with the stairs. In Fig. 4, the joint angles at the beginning of a stride according to the repeatability condition are exchanged one to one with those at the end of a stride.

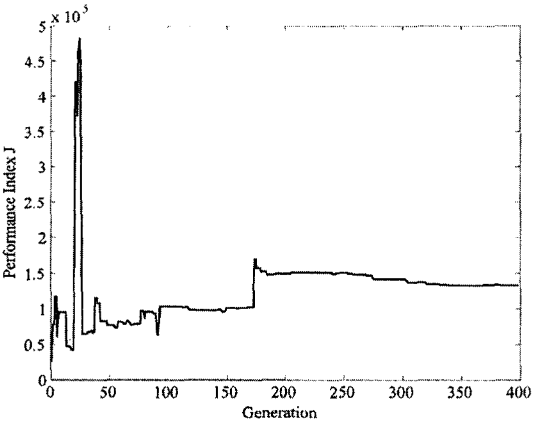

**Fig. 5** Performance index J when ascending stairs

Figures 6 and 7 are similarly a stick diagram and each joint trajectory of the biped robot ascending the stairs, respectively. This figure shows that more and more the optimal locomotion trajectory is similar to that of a human. The performance index converges a reasonable result after about 700 consecutive generations.

From the viewpoint of the consumption energy, the proposed optimal trajectory is compared with those generated by the MGCIPM in Appendix A. Figure 8 is a stick diagram of the biped robot ascending the stairs based on the leg trajectories generated by the MGCIPM. Figures 9, 10 and 11 show the joint powers when ascending the stairs, when descending stairs and when ascending the stairs by the MGCIPM, respectively. By comparison with these joint powers during a period, Figure 12 shows that the proposed optimal trajectory is more efficient than the MGCIPM when ascending the stairs. Thus, the consumption energy is reduced by 54%. In this Figure, the vertical direction is the sum of the square of each joint power at every time step on each joint. The dotted line and the solid line denote the power of the MGCIPM method and the power of the GA method, respectively.

The trajectories generated by the proposed GA method are analyzed according to three locomotion patterns : walking on even ground, ascending stairs, and descending stairs. Figure 12 shows that walking on even ground is the most efficient of the

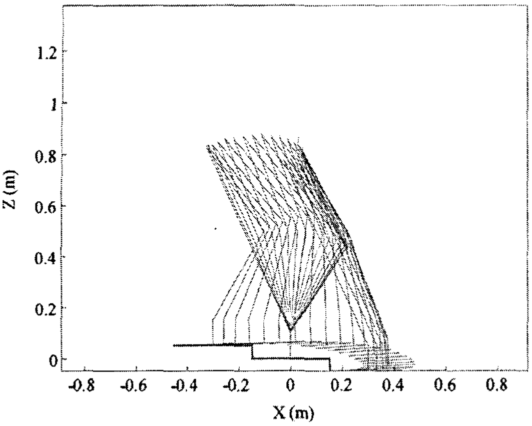

**Fig. 6** Stick diagram of a biped robot descending stairs

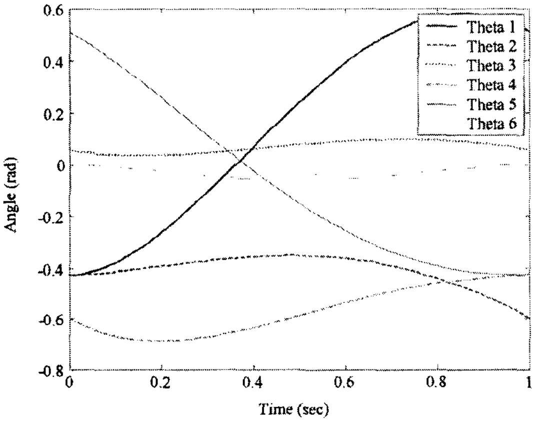

**Fig. 7** Joint angle on each joint when descending stairs

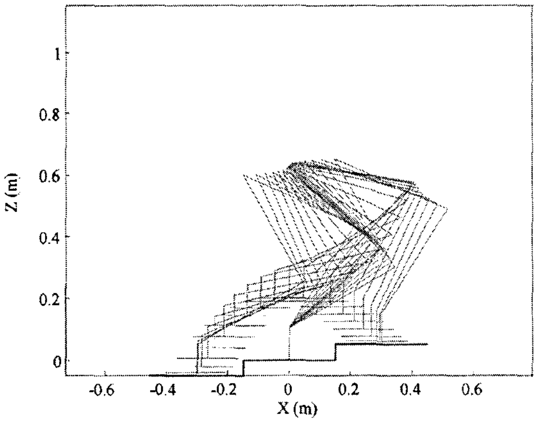

**Fig. 8** Stick diagram of a biped robot ascending stairs based on the leg trajectories generated by the MGCIPM

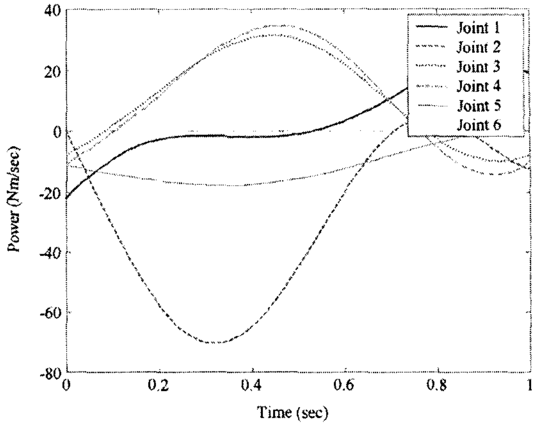

**Fig. 9** Joint power on each joint when ascending stairs

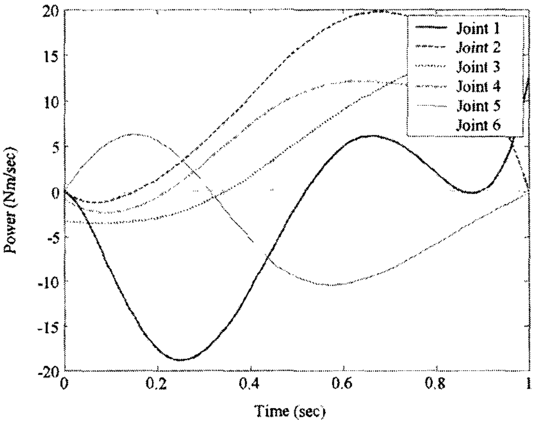

**Fig. 10** Joint power on each joint when descending stairs

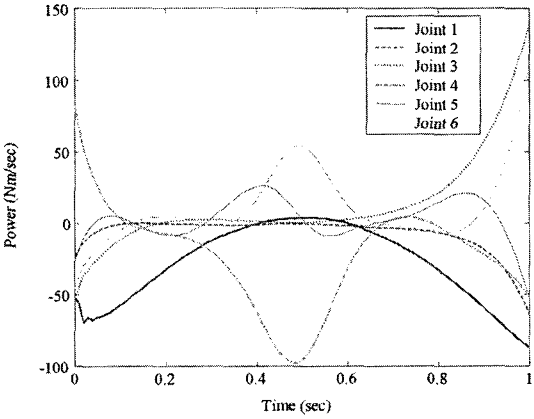

**Fig. 11** Joint power on each joint by the MGCIPM

three. Thus, in case of walking on even ground, the consumption energy is reduced by 92% and 54%, compared with ascending the stairs and descending the stairs, respectively. In this figure, the dotted line denotes the total consumption power when walking on even ground during a stride. The dash-dotted line denotes the total consumption power when ascending the stairs. And the solid line denotes the total consumption power when descending the stairs. Also, when descending the stairs, the consumption energy is greatly reduced by 82%, compared with ascending the stairs.

Figure 13 shows how much the energy according to the stair height is consumed when descending the stairs. In this figure, the dotted line and the solid line denote the total powers when the stair height is 5 and 10 cm, respectively. This figure indicates that the consumption energy in the stair height of 5 cm is reduced by 77%, compared with the stair height of 10 cm.

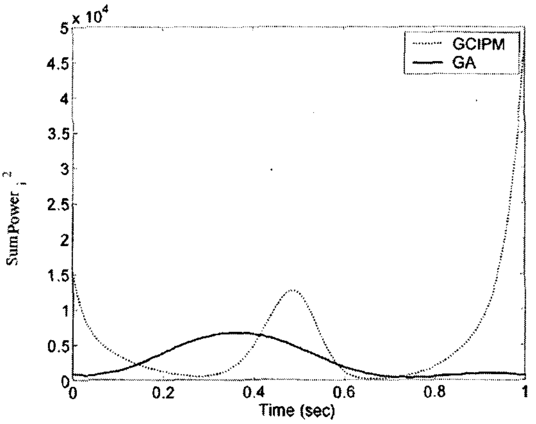

**Fig. 12** Power comparison between the MGCIPM method and the proposed GA method

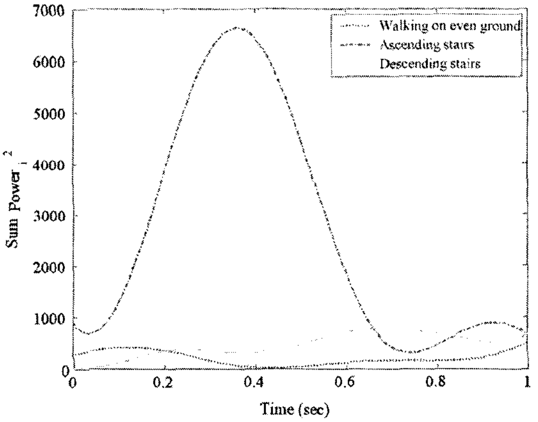

**Fig. 13** Power comparison with walking on even ground, ascending stairs, and descending stairs

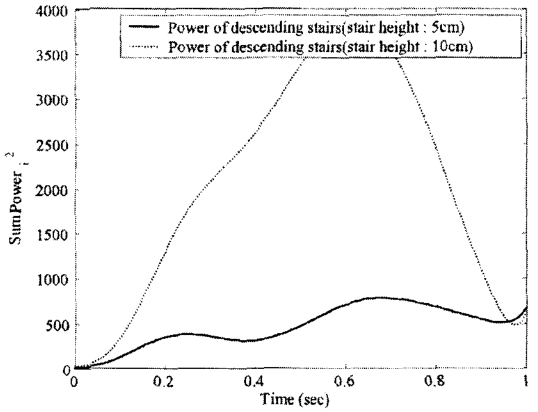

**Fig. 14** Power comparison between the stair height of 5 cm and 10 cm when descending stairs

## 6. Conclusion

This paper proposed the optimal trajectories to minimize the locomotion energy of the biped robot ascending and descending stairs by the real-coded genetic algorithms. According to the stair structure, equality and inequality constraint conditions are made to find out the energy-optimized trajectory. The computer simulations with a 6-DOF biped robot showed that the proposed optimal trajectory is effective and stable. Thus, the consumption energy is reduced by 54%, compared to the Modified Gravity-Compensated Inverted Pendulum Mode.

## Acknowledgments

This work was supported by Korea Science and Engineering Foundation Grant (F01-2003-000-00187-0).

## References

Cheng, M. Y. and Lin, C. S., 1995, "Genetic

Algorithm for Control Design of Biped Locomotion," *IEEE Int. Conf. on Systems, Man and Cybernetics*, pp. 1315~1320.

Chevallereau, C., Formal'sky, A. and Perrin, B., 1998, "Low Energy Cost Reference Trajectories for a Biped Robot," *IEEE Int. Conf. on Robotics and Automation*, pp. 1398~1404.

Choi, S. H., Choi, Y. H. and Kim, J. G., 1999, "Optimal Walking Trajectory Generation for a biped Robot Using Genetic Algorithm," *IEEE Int. Conf. on Intelligent Robots and Systems*, pp. 1456~1461.

Furusho, J. and Sano, A., 1990, "Sensor-based control of a nine-link biped," *Int. J. of Robotics Research*, Vol. 9, No. 2, pp. 83~98.

Goldberg, D. E., 1989, *Genetic Algorithm in Search, Optimization, and Machine Learning*, Addison Wesley.

Hwang, Y., Inohira, E., Konno, A. and Uchiyama, M., 2003, "An Order n Dynamic Simulator for a Humanoid Robot with a Virtual Spring-Damper Contact Model," *IEEE Int. Conf. on Robotics and Automation*, pp. 31~36.

Jin, G. G., 2002, *Genetic Algorithms and Their Applications*, Kyo Woo Sa, Korea.

Park, J. H. and Choi, M. S., 2004, "Generation of An Optimal Gait Trajectory for Biped Robots Using A Genetic Algorithm," *JSME International Journal, Series C*, Vol. 47, No. 2, pp. 715~721.

Park, J. H. and Kim, K. D., 1998, "Biped Robot Walking Using Gravity-Compensated Inverted Pendulum Mode and Computed Torque Control," *IEEE Int. Conf. on Robotics and Automation*, pp. 3528~3533.

Peng, C. and ONO, K., 2005, "Accuracy Analysis of Optimal Trajectory Planning Methods Based on Function Approximation for a Four-DOF Biped Walking Model," *Journal of Mechanical Science and Technology*, Vol. 19, No. 1, pp. 452~460.

Shih, C. L., 1999, "Ascending and Descending Stairs for a Biped Robot," *IEEE Transactions on Systems, Man, and Cybernetics-Part A : Systems and Humans*, Vol. 29, No. 3, pp. 255~268.

Tzafestas, S., Raibert, M. and Tzafestas, C., 1996, "Robust Sliding-mode Control Applied to a 5-Link Biped Robot," *Journal of Intelligent and Robotic Systems*, 15, pp. 67~133.

Zalzala, A.M.S. and Fleming, P.J., 1997, "Genetic algorithms in engineering systems," *IEE*.

## Appendix A

### Modified Gravity-Compensated Inverted Pendulum Mode (MGCIPM)

In general, the locomotion trajectories with respect to the center of mass and a swing foot are specified, based on the inverted pendulum model under the assumption that the ZMP exists inside the center of the supporting foot of a biped robot (Park and Kim, 1998). Here, they are modified and applied to ascend or descend stairs. Firstly, the foot trajectory of a swing leg with respect to the origin of the reference frame is defined using sinusoidal functions.

$$x_{tip}(t) = -S\cos(\omega_f t)$$

$$z_{tip}(t) = \frac{h_f}{2}[1-\cos(\omega_f t)] + 2Ht - H$$

Next, according to the position of the center of mass and the foot position, the trajectory of the center of mass with respect to the origin of the reference frame is derived.

$$x_{com}(t) = C_1 e^{\omega t} + C_2 e^{-\omega t} + \eta\cos(\omega_f t)$$

$$z_{com}(t) = H_z + Ht$$

where

$$\omega = \sqrt{\frac{g}{H_z}},\ \omega_f = \frac{\pi}{t_f}$$

and $H_z$ and $h_f$ are the height of the center of mass and the height of the maximum swing foot, respectively. And $C_1$, $C_2$, and $\eta$ are the coefficients related to the initial conditions.