Sustained Oscillation of an Inverter-Fed Induction Motor Drive System and its Stabilization

Hongmei Li† and Takashi Hikihara*

Abstract - The sustained oscillation of rotor speed is often experienced in PWM inverter induction motor (IM) drive systems. In this paper the oscillation is investigated from the point of view of Hopf bifurcation theory. The sufficient and necessary conditions for existence of limit cycle are introduced to determine the bifurcation set in the stator voltage versus stator frequency plane. According to the conditions it is clarified that the bifurcation set inherently exists in the instable operation of IM. Moreover, it is numerically shown that the V/f curve can be adjusted to stabilize the sustained oscillation of rotor speed.

Keywords: sustained oscillation, inverter, IM drive system, Hopf bifurcation, stabilization

1. Introduction

Due to the affordable price, the rugged construction of IM, and the introduction of solid-state inverters, the constant volt per frequency (V/f) mode operation of IM has become popular. The great majority of variable speed drives, in operation today, are of this type. However undesirable sustained oscillation of rotor speed in IM drive systems are often found in steady state with low frequency and light load [1]-[3]. There are a few numerical analysis and experimental validation with relation to the oscillation [4]-[7]. However, the study on physical mechanism and control of sustained oscillation are still under research [8]-[9].

IM has nonlinear characteristic and magneto-mechanical coupling problem, so that the research on the sustained oscillation of rotor speed induces questions: what is the cause of the instability and what dominate the nonlinearity of IM. On the standpoint, we aim to analyze the sustained oscillation in IM drive system based on Hopf bifurcation theory and put forward an effective method to restrict the bifurcation.

The contents of the paper are summarized as follows. In Section II, practical assumptions for IM are introduced. Then, with the help of the d-q transformation of variables, an abbreviated mathematical model is derived for IM drive system. In Section III, the nonlinear dynamics of IM are numerically discussed. In Section IV, it is confirmed that there exist 1-3 equilibrium points for rotor angular velocity. The sufficient and necessary conditions are obtained for existence of limit cycle according to the estimation of the bifurcation set in the stator voltage versus stator frequency plane. Main result is the bifurcation set is physically explained by the appearance of limit cycle through Hopf. In Section V, V/f curve method is put forward to stabilize limit cycle of IM drive system.

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2. Mathematical Model of IM System

By making use of d-q transformation of variables, a basic abbreviated mathematical model of IM system is obtained as follows:

$$[V_m \quad 0 \quad 0 \quad 0]^T = [Z][i_{1d} \quad i_{1q} \quad i_{2d} \quad i_{2q}]^T \ , \tag{1}$$

$$[Z] = \begin{bmatrix} r_1 + pX_{11}/\omega_b & -\omega X_{11}/\omega_b & pX_{12}/\omega_b & -\omega X_{12}/\omega_b \\ \omega X_{11}/\omega_b & r_1 + pX_{11}/\omega_b & \omega X_{12}/\omega_b & pX_{12}/\omega_b \\ pX_{12}/\omega_b & s\omega X_{12}/\omega_b & r_2 + pX_{22}/\omega_b & -s\omega X_{22}/\omega_b \\ s\omega X_{12}/\omega_b & pX_{12}/\omega_b & s\omega X_{22}/\omega_b & r_2 + pX_{22}/\omega_b \end{bmatrix}$$

and

$$2Hp\omega_2/\omega_b + B\omega_2/\omega_b + T_l = X_{12}(i_{1q}i_{2d} - i_{1d}i_{2q}) \ , \tag{2}$$

where V_m denotes stator voltage transformed d-q coordinate, r_1 stator resistance, and r_2 rotor resistance, respectively. Then at ω_b , the rated maximum stator angular frequency, X_{11} denotes stator reactance, and X_{22} rotor reactance. Moreover s depicts slip, ω electrical angular velocity of applied stator voltage, and ω_2 electrical angular velocity of rotor. Mechanically, H denotes inertial constant, B friction coefficient, and T_l load torque. Electrically, i_{1d} represents instantaneous stator d-axis current, i_{1q} instantaneous stator q-axis current, i_{2d} instantaneous rotor d-axis current, and i_{2q} instantaneous rotor q-axis current. Here p implies d/dt.

Introducing the following linear transformation for summing up the variables:

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} = \begin{bmatrix} X_{11} & 0 & X_{12} & 0 & 0 \\ 0 & X_{11} & 0 & X_{12} & 0 \\ X_{12} & 0 & X_{22} & 0 & 0 \\ 0 & X_{12} & 0 & X_{22} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_{1d} \\ i_{1q} \\ i_{2d} \\ i_{2q} \\ \omega_2/\omega_b \end{bmatrix}, \quad (3)$$

Then expressions (1), (2), and (3) become

$$\begin{cases} py_1 = \omega_b r_1 B_r y_1 - \omega_b r_1 B_m y_3 + \omega y_2 + \omega_b V_m, \\ py_2 = \omega_b r_1 B_r y_2 - \omega_b r_1 B_m y_4 - \omega y_1, \\ py_3 = -\omega_b r_2 B_m y_1 + \omega_b r_2 B_s y_3 + \omega y_4 - \omega_b y_4 y_5, \\ py_4 = -\omega_b r_2 B_m y_2 + \omega_b r_2 B_s y_4 - \omega y_3 + \omega_b y_3 y_5, \\ py_5 = -BM\omega_b y_5 - \omega_b M T_l + \omega_b M B_m (y_1 y_4 - y_2 y_3), \end{cases} \quad (4)$$

$$M = 1/(2H\omega_b),\ B_r = X_{22}/(X_{12}^2 - X_{11}X_{22})$$
$$B_s = X_{11}/(X_{12}^2 - X_{11}X_{22}),\ B_m = X_{12}/(X_{12}^2 - X_{11}X_{22})$$

After using transformation, equation (4) forms an abbreviated mathematical model to study the dynamic performance of IM drive system.

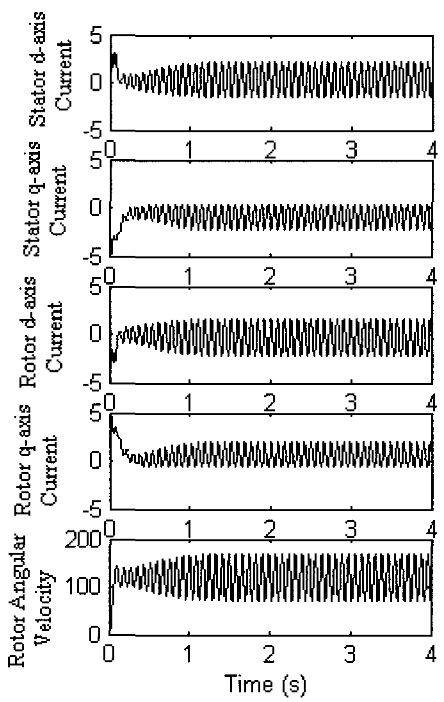

Fig. 1 Dynamic response of IM system

3. Numerical Simulation

IM drive system is simulated according to equation (4). The machine parameters are given in per unit as follows:

r_1=0.025, r_2=0.008, X_{11}=4.1, X_{22}=4.1, X_{12}=4.0, H=0.1, B=0.02, T_l=0, V_m=120/377, $\omega_b = 377$, $\omega = 120$.

Fig. 1 shows a result from dynamic response of IM system. The numerical calculation is performed by MATLAB.

Then the system becomes unstable because the sustained rotor angular velocity oscillation is evident in Fig. 2. These results show that the mathematical model of IM includes the nonlinearity that can show the sustained oscillation. Therefore, we can discuss the bifurcation set based on the numerical and theoretical consideration of the model.

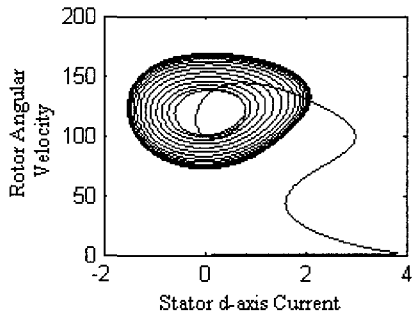

Fig. 2 Limit cycle of i_{1d} - ω_2 plane

4. Hopf Bifurcation

4.1 Calculation of Equilibrium Point

At first, let py_1, py_2, py_3, py_4, and py_5 be zero in equation (4). Through some algebraic manipulations on (4), a third-order nonlinear equation with respect to y_5 is obtained.

Assuming no-load and keeping B at sufficiently small, the equilibrium points for y_5 are figured out by

$$y_5{*}(1) = \omega/\omega_b,$$
$$y_5{*}(2) = \omega/\omega_b + r_2 B_s \omega / \sqrt{\omega_b^2 r_1^2 B_r^2 + \omega^2} + (\omega_b/\omega)(1 + \omega_b r_1 B_r / \sqrt{\omega_b^2 r_1^2 B_r^2 + \omega^2}) r_1 r_2 (B_s B_r - B_m^2),$$
$$y_5{*}(3) = \omega/\omega_b - r_2 B_s \omega / \sqrt{\omega_b^2 r_1^2 B_r^2 + \omega^2} + (\omega_b/\omega)(1 - \omega_b r_1 B_r / \sqrt{\omega_b^2 r_1^2 B_r^2 + \omega^2}) r_1 r_2 (B_s B_r - B_m^2),$$

1-3 equilibrium points for y_5 exist. It implies that the parameters of IM are designed undesirable, so that the IM possibly becomes unstable at certain operations even if it is fed by an ideal three-phase ac power source.

4.2 Conditions of Hopf Bifurcation

IM drive systems have many parameters. According to their change, the structural change of solutions appears. These changes are called bifurcation, and a set of the critical values that concerns the bifurcation is called a bifurcation set.

A nonlinear continuous-time IM system is described by a deferential equation (4) as follows:

$$\dot{y} = f(y,u), \tag{5}$$

with equilibrium points y*,

$$y^* : f(y^*,u) = 0, \tag{6}$$

Let A(u)=D_yf(y*,u) be Jacobian matrix of f(y,u) at the equilibrium point. Assume that A(u*) has a single pair of purely imaginary eigenvalues $\lambda(u^*) = \pm j\omega$, and no other eigenvalues with zero part, and furthermore,

$$d\,\mathrm{Re}(\lambda(u))/du\big|_{u=u^*} \neq 0, \tag{7}$$

Under these conditions, the Hopf bifurcation theorem depicts that there is a limit cycle at (y*, u*).

Assume the characteristic polynomial of A(u*) is expressed by:

$$f(\lambda) = q_0\lambda^5 + q_1\lambda^4 + q_2\lambda^3 + q_3\lambda^2 + q_4\lambda + q_5,\quad q_0 = 1 \tag{8}$$

The conditions for the existence of a pair of pure imaginary eigenvalues related to A(u*) can be formulated by the following way [10]-[11]:

$$H_q = \begin{bmatrix} q_1 & q_0 & 0 & 0 & 0 \\ q_3 & q_2 & q_1 & q_0 & 0 \\ q_5 & q_4 & q_3 & q_2 & q_1 \\ 0 & 0 & q_5 & q_4 & q_3 \\ 0 & 0 & 0 & 0 & q_5 \end{bmatrix},$$

$$D_1 = q_1 > 0,\quad D_2 = \det\begin{pmatrix} q_1 & q_0 \\ q_3 & q_2 \end{pmatrix} > 0,$$

$$D_3 = \det\begin{pmatrix} q_1 & q_0 & 0 \\ q_3 & q_2 & q_1 \\ q_5 & q_4 & q_3 \end{pmatrix} > 0,\quad D_4 = \det\begin{pmatrix} q_1 & q_0 & 0 & 0 \\ q_3 & q_2 & q_1 & q_0 \\ q_5 & q_4 & q_3 & q_2 \\ 0 & 0 & q_5 & q_4 \end{pmatrix},$$

$$D_5 = \det H_q = 0,$$

The seeking the appearance of pure imaginary eigenvalues leads us easily to find the limit cycles even if the system dimension is high. In the following discussion, our estimation is based on the abovementioned method.

2.3 Hopf Bifurcation Set

Hopf bifurcation set is determined and V/f curve illustrated in Fig. 3 at u=V_m. Hopf bifurcation and induced limit cycle oscillation of rotor speed is indicated in Fig. 1 and 2 for $\omega = 120$ and V_m=120/377.

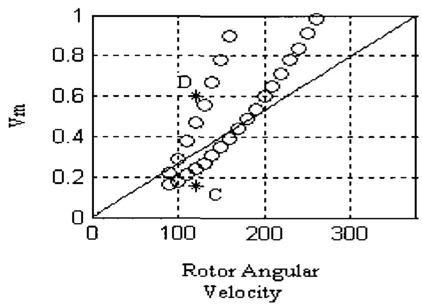

Fig. 3 Hopf bifurcation set on ω_2 - V_m plane (The blue line denotes V/f curve and circles Hopf bifurcation set)

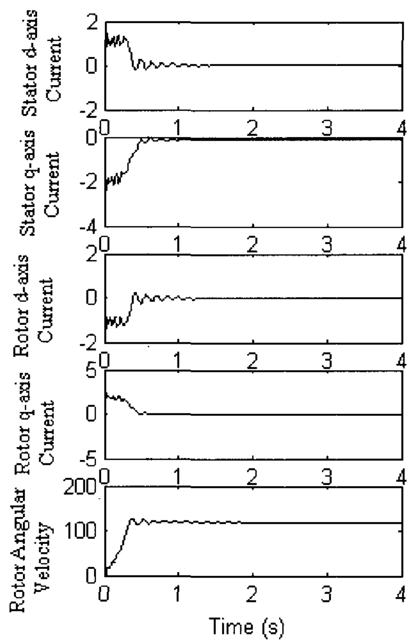

Fig. 4 Dynamic response for point C

5. Stabilization of IM Drive System

In Fig. 3, the IM has a limit cycle between $\omega = 90$ to $\omega = 180$. From a viewpoint of practical application, the setting of V/f has a possibility to stabilize the IM drive system and remove the sustained oscillation of rotor speed. As for the normal design of the IM drive system, the parameters are selected from the outside of the region that causes the limit cycles. However, the desirable parameters of the system are close to the region for practical use, the stabilization by selecting V/f is meaningful. The estimation by Hopf bifurcation theorem gives us the information how we can find the possible V/f parameters.

In order to confirm the analytical idea, here, numerical investigations are performed. An IM under test has the parameters indicated in the Section III.

For $\omega = 120$, V/f curve is set to avoid the Hopf bifurcation set. That is points C and D are located in the range of stability in Fig. 3.

Fig. 4 and 5 indicate the IM dynamic response for points C and D. The IM is stabilized and the limit cycle oscillation of rotor speed disappears.

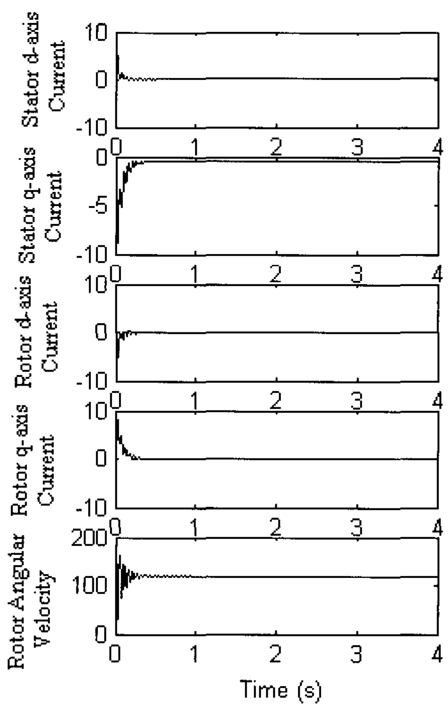

Fig. 5 Dynamic response for point D

6. Conclusion

The sustained oscillation of rotor speed in IM drive system is substantially caused by the nonlinearity of IM. It is clarified that the unstable state of IM drive system shows a limit cycle of rotor speed and the estimation of Hopf bifurcation set can explain the appearance of the limit cycle. The method for estimating the Hopf bifurcation set is also explained. At last, it is proposed that V/f curve can be set to avoid the limit cycle. Through the numerical simulation, we validate the effectiveness of the method.

In the further research, the stabilization of unstable equilibrium point under the load operation will be a fruitful problem even in the practical use.

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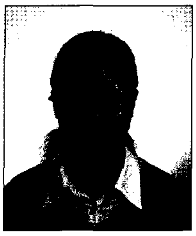

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