Effects of a One-Way Clutch on the Nonlinear Dynamic Behavior of Spur Gear Pairs under Periodic Excitation

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Nonlinear behavior analysis was used to verify whether a one-way clutch is effective for reducing the torsional vibration of a paired spur gear system under periodic excitation. The dynamic responses were studied over a wide frequency range by speed sweeping to check the nonlinear behavior using numerical integration. The gear system with a one-way clutch showed typical nonlinear behavior. The oscillating component of the dynamic transmission error was reduced over the entire frequency range compared to a system without a one-way clutch. The one-way clutch also eliminated unsteady continuous jump phenomena over multiple solution bands, and prevented double-side contact, even with very small backlash. Installing a one-way clutch on both sides of the gear system was more effective at mitigating the negative effects of external periodic excitation and various parameter changes than a conventional gear system without a one-way clutch.

Key Words : Dynamic Transmission Error (DTE), Gear Pairs, One-Way Clutch, Nonlinear Behavior, Torsional Vibration, Periodic Excitation

1. Introduction

Many studies have examined gear dynamics to reduce noise and vibration. To analyze the dynamic behavior of a gear system, the nonlinearity of mesh stiffness and backlash must be considered. Several works have modeled gear systems, including complicating effects, such as nonlinear mesh stiffness and backlash (Blankenship and Kahraman, 1995 ; Parker et al., 2000 ; Theodossiades and Natsiavas, 2000), nonlinear mesh damping (Amabili and Rivola, 1997), sliding friction force (Vaishya and Singh, 2001), and shaft flexibility (Litak and Friswell, 2003). Tooth modification (Townsend, 1992), phasing in planetary gears (Parker, 2000), and optimizing boundary conditions (Cheon, 2003 ; Cheon and Parker, 2004) have been applied to reduce noise and vibration in gear systems.

Due to periodic engine pulsations in mechanical systems, such as in automobiles and airplanes, it is common for a gear system to receive periodic excitation from the input or output shafts. Even an electric motor has a torsional vibration due to electric and mechanical reasons.

One-way clutches are widely used in the serpentine belt systems of automobiles and heavy vehicles to mitigate the torsional vibration generated by periodic engine pulsations. A one-way clutch engages or disengages according to the relative angular speed between the driving and driven elements and acts as a vibration absorber (Zhu and Parker, 2005). Though there are several types of one-way clutches (wrap spring type, cam type, ball type, and sprag type), they perform basically the same function that of decoupling driving shaft rotations in the non-driven direction by its vibration from rotations of the driven shaft and allowing rigid body rotations only. Figure 1

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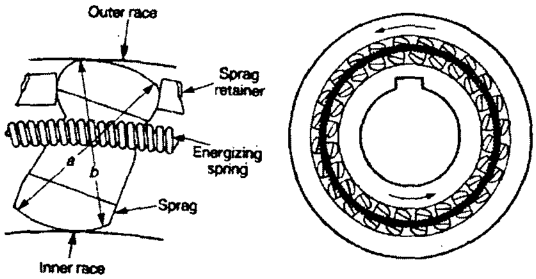

Fig. 1 Schematic diagram of a sprag type one-way clutch

shows the schematic diagram of a sprag type one-way clutch.

It is reasonable to think that a one-way clutch might reduce the vibration in a gear system, such as a serpentine belt system, by suppressing the torsional vibration of the gear system. This study focused on the dynamic behavior of spur gear pairs with a one-way clutch under periodic excitation through the input or output shaft. The dynamic responses were studied over a wide frequency range by speed sweeping to verify the nonlinear behavior, such as softening nonlinearity and jump phenoma, around the natural frequency. Since the oscillating part of the dynamic transmission error (DTE) is the main source of noise and vibration in gear systems (Smith, 1999), the oscillating components of the DTE were compared as the main dynamic response.

2. Mathematical Model

The nonlinear gear model shown in Fig. 2 (a) has been widely used by many researchers (Blankenship and Kahraman, 1995 ; Parker et al., 2000 ; Theodossiades and Natsiavas, 2000 ; Vaishya and Singh, 2001 ; Litak and Friswell, 2003). In this study, new model shown in Fig. 2(b) has been used. The system consisted of two gears mounted on input and output shafts through a one-way clutch. The one-way clutch was modeled as a nonlinear spring with discontinuous stiffness, i.e., zero stiffness for the disengaged clutch and finite linear stiffness for the engaged clutch. When the rotation of the driver exceeds that of gear 1, the clutch is engaged and clutch

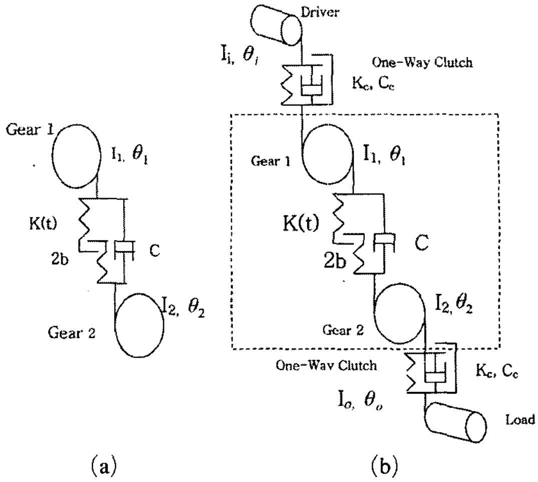

Fig. 2 Schematic diagram of a gear pair : (a) without one-way clutch, (b) with a nonlinear one-way clutch at both sides of the driving and driven shafts

torque is transmitted from the driver to gear 1. When the rotation of the driver is less than the rotation of gear 1, then the clutch is disengaged and no torque is transmitted (Zhu and Parker, 2005). The same occurs between gear 2 and the load.

The gears were perfect involute spur gears with no modifications. The torsional flexibility of both shafts was neglected, and the inertias of the input shaft and driver and those of the output shaft and load were lumped together, respectively.

Gears 1 and 2 had base circles of radii r_1 and r_2 and mass moments of inertia of I_1 and I_2, respectively ; I_i and I_o were the mass moments of inertia of the driver and load, respectively. The pair of gears was modeled using two disks coupled with a nonlinear mesh stiffness and mesh damping ; $k(t)$ and c were the mesh stiffness and damping coefficient of the gear pair, respectively. The total backlash was $2b$; θ_i, θ_o, θ_1, and θ_2 represented the vibrations of the driver, load, and gears 1 and 2 about the nominal rigid body rotation, respectively. K_c and C_c were the stiffness and damping coefficient of the one-way clutch, respectively.

The speed of the input shaft was assumed to be constant and controllable with a predetermined excitation component $\overline{\theta}_i$. The two gears and output shaft were assumed to show rotational displacement only. The equations of motion of the two gears and load are represented as

$$
\begin{aligned}
&I_1\ddot{\theta}_1+c[r_1\dot{\theta}_1-r_2\dot{\theta}_2]+k(t)\beta(t)=T_{c1}(t)\\
&I_2\ddot{\theta}_2-c[r_1\dot{\theta}_1-r_2\dot{\theta}_2]-k(t)\beta(t)=-T_{c2}(t)\\
&I_o\ddot{\theta}_o=T_{c2}(t)-T_o
\end{aligned}
\quad (1)
$$

Forcing terms $T_{c1}(t)$ and $T_{c2}(t)$ are the torques transferred by clutch 1 to gear 1, and by clutch 2 to the load, respectively. They are piecewise linear functions and are represented as

$$
\begin{aligned}
&T_{c1}(t)=\begin{Bmatrix} K_c(\theta_i-\theta_1)+C_c(\dot{\theta}_i-\dot{\theta}_1), \text{ when } \theta_i>\theta_1 \\ 0, \text{ when } \theta_i\leq\theta_1 \end{Bmatrix}\\
&T_{c2}(t)=\begin{Bmatrix} K_c(\theta_2-\theta_o)+C_c(\dot{\theta}_2-\dot{\theta}_o), \text{ when } \theta_2>\theta_o \\ 0, \text{ when } \theta_2\leq\theta_o \end{Bmatrix}
\end{aligned}
\quad (2)
$$

where T_o is the static load torque.

Both the driver and load might be subject to external excitation by a periodic engine pulsation, unbalance, misalignment, etc., with frequency $p\times\omega$, where p is an integer and ω is the frequency of shaft rotation. In this study, $p=1$ because the dominant frequency of the periodic excitation is usually $1\times\omega$. The excitation components of the driver and load were represented as

$$
\begin{aligned}
\overline{\theta}_i&=\Theta_i\sin\omega t\\
\overline{\theta}_o&=\Theta_o\sin\omega t
\end{aligned}
\quad (3)
$$

respectively.

In this study, both the external forces and the mesh stiffness and backlash were regarded as being nonlinear. The mesh stiffness variation $k(t)$ was the time-varying mesh stiffness obtained by assuming a rectangular wave (Kahraman and Blankenship, 1999).

$$
k(t)=k_o+\sum_{r=1}^{R}k_r\cos(2\pi rf_m t-\phi_r) \quad (4)
$$

where f_m is the mesh frequency ; k_o is the average mesh stiffness value and k_r and ϕ_r are the r^{th} Fourier coefficient and phase angle of $k(t)$, respectively. Here, $R=5$.

Gear backlash nonlinearity was modeled as a piecewise linear function.

$$
\beta(t)=\begin{Bmatrix} r_1\theta_1-r_2\theta_2-b, \text{ when } r_1\theta_1-r_2\theta_2>b \\ r_1\theta_1-r_2\theta_2+b, \text{ when } r_1\theta_1-r_2\theta_2<-b \\ 0, \text{ when } |r_1\theta_1-r_2\theta_2|\leq b \end{Bmatrix} \quad (5)
$$

The damping coefficients c of the tooth mesh and c_c of the clutch were calculated as

$$
\begin{aligned}
C&=2\zeta\sqrt{k_o/[(r_1^2/I_1)+(r_2^2/I_2)]}\\
C_c&=2\zeta_c\sqrt{K_cI_i}
\end{aligned}
\quad (6)
$$

where ζ and ζ_c are the damping ratios of the tooth mesh and clutch, respectively (Kasuba and Evans, 1981).

For comparison, the behavior of a gear pair system mounted on the shaft directly without a one-way clutch was also analyzed. The equations of motion of gears 1 and 2 without a one-way clutch can be expressed as

$$
\begin{aligned}
&(I_1+I_i)\ddot{\theta}_1+c[r_1\dot{\theta}_1-r_2\dot{\theta}_2]+k(t)\beta(t)=T_i\\
&(I_2+I_o)\ddot{\theta}_2-c[r_1\dot{\theta}_1-r_2\dot{\theta}_2]-k(t)\beta(t)=-T_o
\end{aligned}
\quad (7)
$$

where T_i is the static input torque.

The dynamic transmission error (DTE) was defined as $(r_1\theta_1-r_2\theta_2)$. The solutions were obtained using direct time domain numerical integration (fifth-order Runge-Kutta algorithm).

3. Parametric Study

The primary purpose of this work was to examine the nonlinear dynamic response of a single gear pair with a one-way clutch under external excitation through the input and output shafts. Table 1 shows the dimensions of an identical gear

Table 1 Dimensions of the gears

Teeth Number	50
Module	3 mm
Pressure Angle	20°
Face Width	0.02 m
Modulus of Elasticity	207×10^9 N/m^2
Density	7600 kg/m^3
Base Radius	0.07047 m
Backlash (2b)	400×10^{-6} m
Mass	2.8 kg
Mass Moment of Inertia	7.875×10^{-3} kg-m^2

pair used in previous research (Kahraman and Blankenship, 1999 ; Parker et al., 2000) for comparison.

The average mesh stiffness k_o was 462.1×10^6 N/m, and the contact ratio was 1.75. The damping ratio of the tooth mesh was $\zeta = 0.07$. The steady component of the input torque, T_i, was 150 N-m, and the load torque was $T_o = r_2/r_1 \times T_i$. The stiffness of the one-way clutch was selected as 15×10^3 (N-m/rad) to produce a torque equal to the input torque at a clutch displacement of 0.01 rad. The inertias of the driver and load were kept the same as those of the gears. The oscillating component of DTE was less than 3 μm in the no-clutch condition (Figs. 3 and 4), so the maximum values of the excitations, Θ_i and Θ_o, in equation (3) were adjusted to produce 3 μm tangential direction displacements.

To detect jump phenomena, an increasing and decreasing frequency sweep was executed at a constant ratio for a wide range across the first natural frequency. To get stable data after a speed change, 1×10^5 time step data were discarded before averaging was performed. The time step was 1×10^{-5} sec.

The oscillating dynamic transmission error (ODTE) component at a specific constant speed was defined as

$$\mathrm{ODTE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(DTE_{i+1} - DTE_i)^2} \qquad (8)$$

where N is the total number of time steps used for averaging, which was 1×10^4 in this study. The calculated natural frequency was around 2700 Hz.

Figure 3 shows the DTE response of the gear sets without external excitation under the no-clutch (NC) condition. The results predicted multiple resonances, softening nonlinearity, kink (onset of contact loss), and jump phenomena, which are typical in a nonlinear gear system, and generally agreed qualitatively with the results of Parker et al. (2000). A distinct softening nonlinearity occurred as the peak bent to the left. Primary resonance was evident for a mesh frequency of $f_m \approx f_n \approx 2{,}700$ Hz. The difference is thought to be due to the value of the shaft inertia and backlash

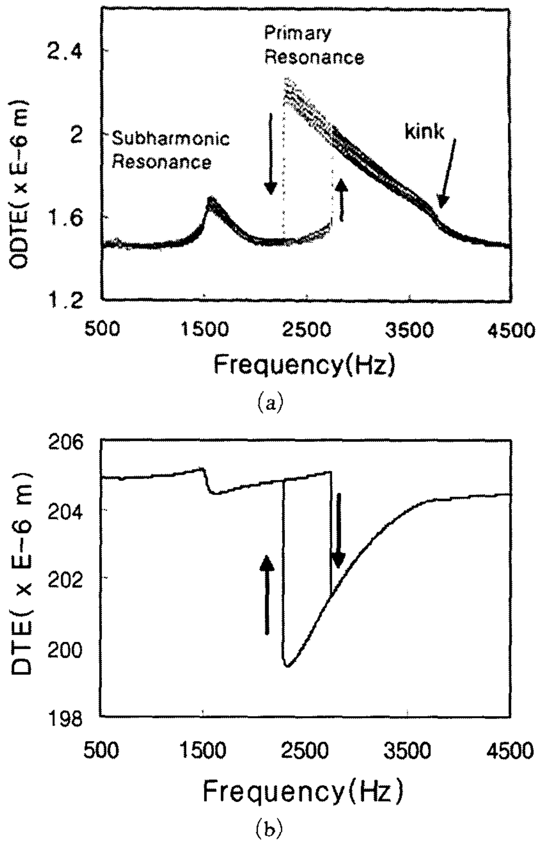

Fig. 3 DTE responses of a gear pair system without external excitation under the no-clutch (NC) condition : (a) ODTE component and (b) DTE

not mentioned in their study.

Figure 4 shows the DTE responses of the gear pair in condition NC with excitation at the input shaft (driver). The magnitude and overlap frequency range were almost the same as those without excitation. However, the responses were unsteady showing repeated jump phenomena over the overlap frequency range (multiple solution bands) during increasing and decreasing speed. This kind of unsteady behavior was due to the external excitation. This means that the external excitations in a gear system without a one-way clutch will induce more vibration.

These trends were similar under conditions with the excitation at the output shaft.

Figure 5 shows the displacement of the gear in condition NC during speed sweeping under vari-

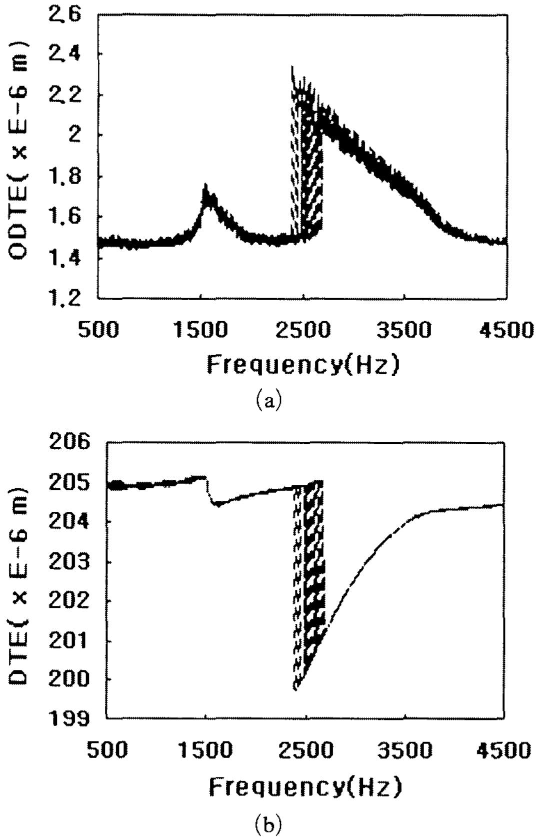

Fig. 4 DTE responses of the gear pair in condition NC with excitation at the input shaft

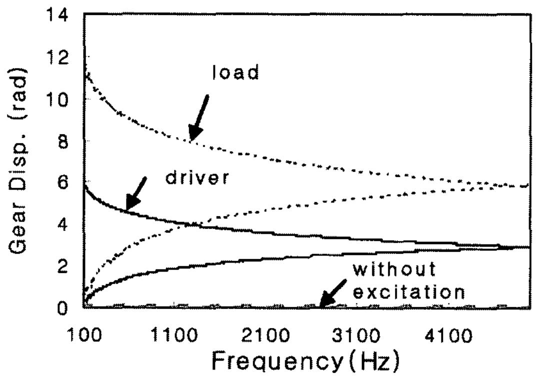

Fig. 5 Displacement of the gear in condition NC during speed sweeping under various excitation conditions: without excitation, driver excitation, and load excitation

ous excitation conditions: without excitation, driver (input shaft) excitation, and load (output shaft) excitation. The gear maintained forward

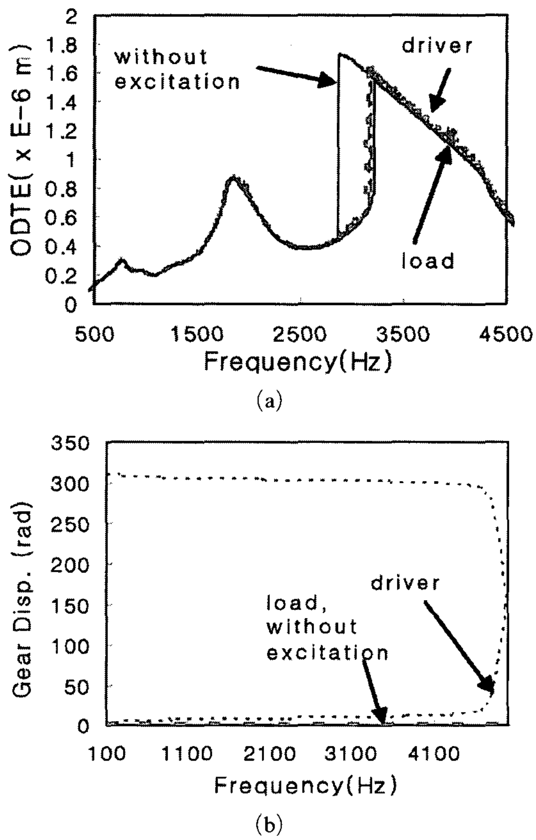

Fig. 6 DTE and displacement responses of condition with clutch at both shafts (CB) under various excitation conditions: (a) ODTE and (b) gear displacement

displacement, and increased with time. However, the total displacement was very small.

Figure 6 shows the DTE and displacement responses with a clutch at both shafts (CB) under various excitation conditions. The overlap range shifted to a higher frequency due to decreased inertia separated by the one-way clutch on both sides. The displacement of the gear continued to increase with time, and was greater than that of the NC condition. However, the DTE responses showed steady behavior over the overlap frequency range without any repeated jump phenomena in every excitation condition. This means that a one-way clutch is effective at mitigating the negative effects of external excitation. The overlap range was narrower in excitation conditions than in the condition of zero excitation.

Figure 7 shows the DTE responses in conditions NC and CB with external excitation at the driver. Over the entire frequency range, the ODTE components could be reduced if a one-way clutch were installed at both shafts.

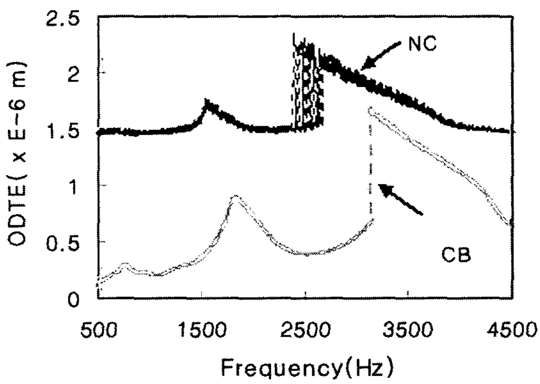

Fig. 7 DTE responses in conditions NC and CB with external excitation at the driver

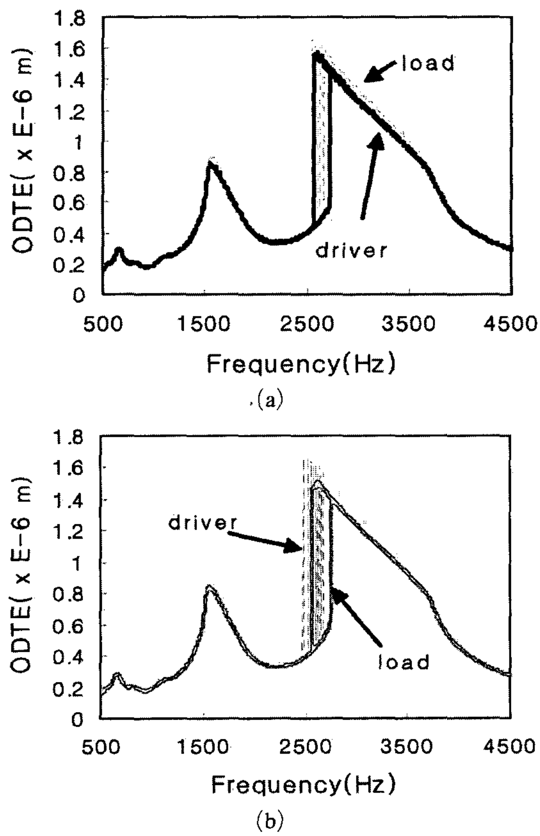

Fig. 8 DTE responses under excitation at the input and output shafts under different clutch conditions : (a) clutch at the driving shaft only and (b) clutch at the driven shaft only

Figure 8 shows the DTE responses under excitation at the driver and load in different clutch conditions : with the clutch at the driving shaft only (CI) and with the clutch at the driven shaft only (CO). The ODTE component could be reduced if a one-way clutch were installed at only one of these shafts. However, the responses showed unsteady behavior over the overlap range when there was excitation at the other side of the installed clutch. The dynamics of the one-way clutch has a dominant effect on the gear installed near the clutch.

Figure 9 shows the DTE and displacement responses under different excitations in the no-clutch condition, demonstrating the effects of load inertia. Keeping the inertia of the driver and two gears constant at the same value, the inertia of the load was varied from one-tenth to ten times that of the other elements. As the load inertia increased, the overlap band shifted to lower frequencies. The gear displacement increased as the load inertia decreased in the driver excitation condition, while the trends were reversed under the load excitation condition. The DTE and displacement responses depended dominantly on the load inertia in condition NC.

Figure 10 shows the DTE and displacement responses under different excitations in condition CB, demonstrating the effects of load inertia. The overlap range did not show any unsteady behavior, and did not shift much under various inertia variations. Installing clutches at both shafts effectively suppressed the negative effects of a large load inertia, which were dominant in condition NC. The ODTE component of an inertia ratio of ten increased dominantly at the 2^{nd} superharmonic frequency. The displacement of the gear increased considerably at specific frequency ranges during speed increases and decreases, especially when the load ratio was high. Contrary to condition NC, the displacement increased more under driver excitation than under load excitation.

Figure 11 shows the DTE response under conditions NC and CB with driver excitation for various backlashes, while keeping the other parameter values nominal. In condition NC, the ODTE components decreased with the backlash. When the backlash was very small (5 μm), a hardening nonlinearity due to double-side contact appeared, demonstrating that a torsional vibration greater than the backlash causes double-side contact.

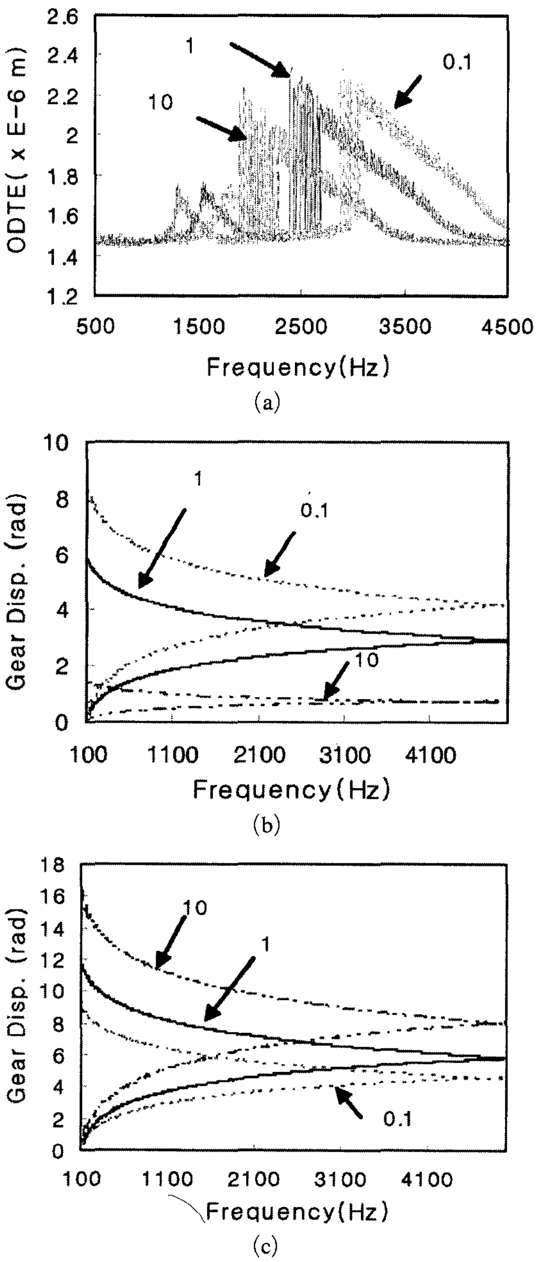

Fig. 9 DTE and displacement responses under different excitations in the no-clutch condition : (a) ODTE, (b) gear displacement under driver excitation, and (c) gear displacement under load excitation

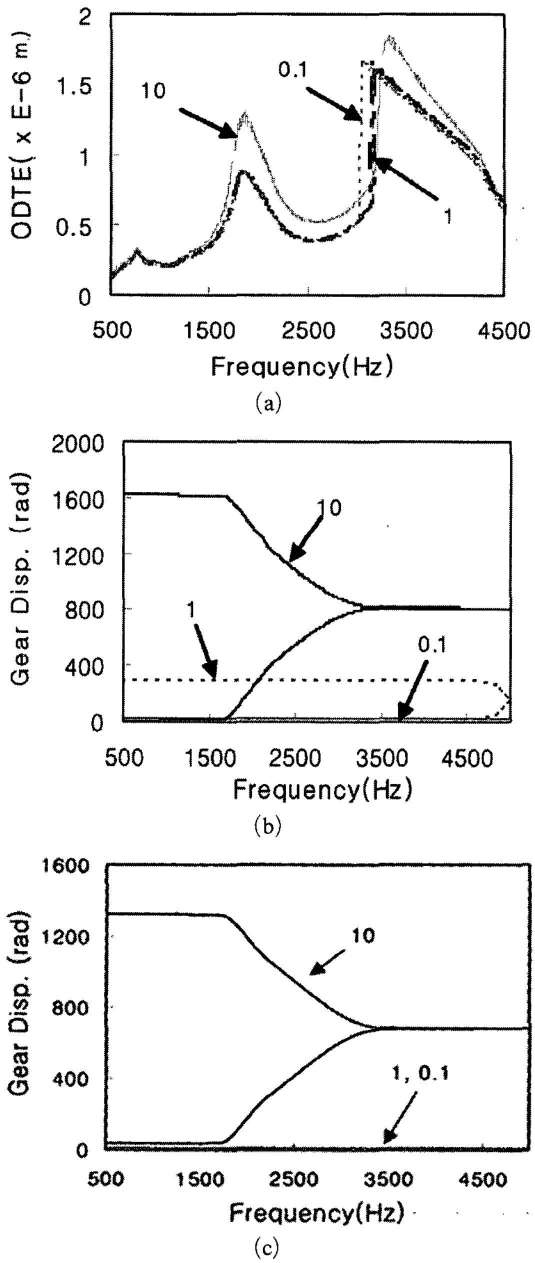

Fig. 10 DTE and displacement responses under different excitations in condition CB : (a) ODTE, (b) gear displacement under driver excitation, and (c) gear displacement under load excitation

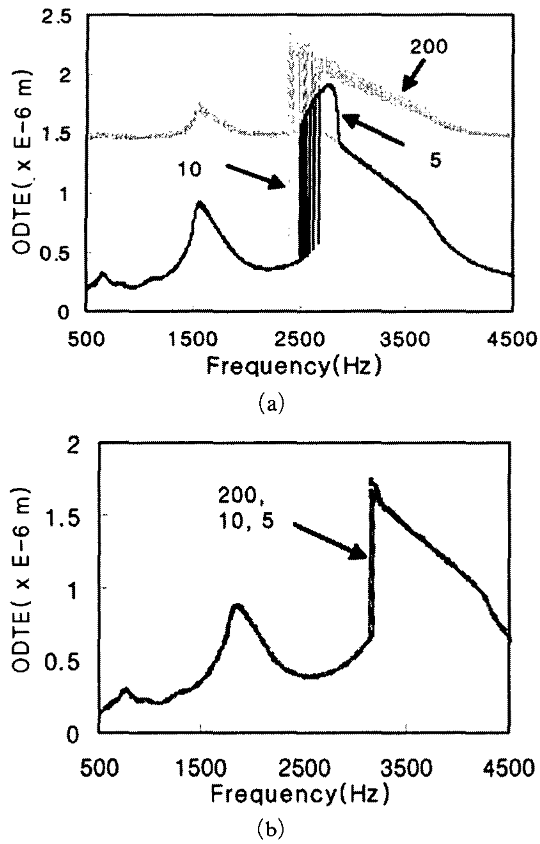

Fig. 11 DTE response under conditions NC and CB with driver excitation for various backlashes (b ; μm) : (a) NC and (b) CB

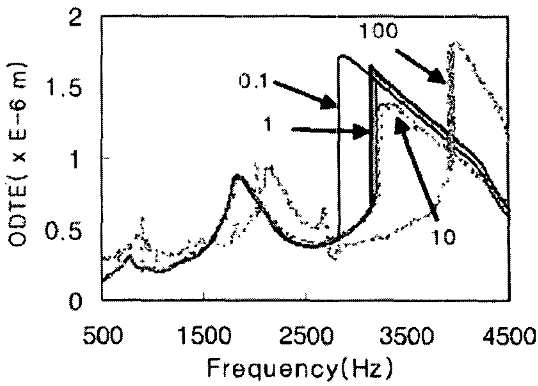

Fig. 12 DTE responses in condition CB with driver excitation for various clutch stiffnesses

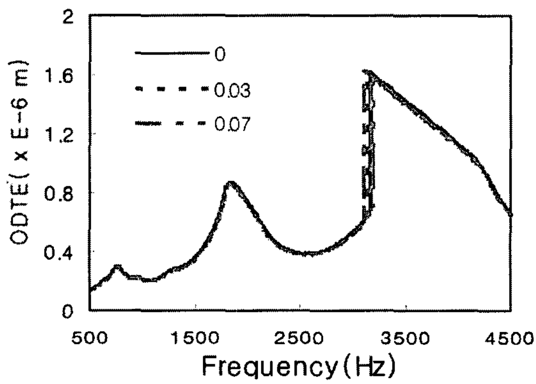

Fig. 13 DTE responses in condition CB under driver excitation for various clutch-damping ratios

By contrast, the DTE responses were barely affected by backlash in condition CB, and the double-side contact nonlinearity did not occur even when the backlash was very small. The clutches installed at both shafts seemed to be effective in preventing potential double-side contact.

Figure 12 shows the DTE responses in condition CB with driver excitation for various clutch stiffnesses, while keeping the other parameters at nominal values. The stiffness was normalized relative to the nominal value 1.5×10^4 N-m/rad. When the clutch stiffness exceeded a specific range, the response lost its typical nonlinear behavior and became very unsteady. The unsteady responses resulted from the impulsive torque induced by the large clutch stiffness. Hence, the clutch stiffness should be kept below a specific value to keep steady responses. These trends were similar under conditions CI and CO.

Figure 13 shows the DTE responses in condition CB under driver excitation for various clutch-damping ratios, while keeping the other parameter values nominal. There was no dominant difference among the responses for various clutch damping ratios. The DTE responses in condition CB depended little on clutch damping.

4. Conclusions

This study examined whether a one-way clutch is effective for reducing the torsional vibration of a gear system under external excitation using a paired gear model considering only rotational motion. A numerical integration method was used to solve the equations of motion.

The gear system with a one-way clutch showed typical nonlinear behavior, such as softening nonlinearity and jump phenomena, as in the system without a one-way clutch. However, the oscillating DTE component of the system with a one-way clutch was reduced over the entire frequency range. The one-way clutch was more effective at mitigating the negative effects of external excitation and variation in several parameters than a conventional gear system without a one-way clutch. Installing a one-way clutch on both sides of the gear system was more effective than installing a clutch at the input or output side only. The stiffness of the one-way clutch had a great effect on the behavior of the gear system, and must be kept below a specific value for a steady response. Nevertheless, clutch damping had little effect on the behavior of the gear system. All the analytical results presented in this study should be verified through further experimentation.

Acknowledgments

This paper was supported by Wonkwang University in 2005.

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