Finite Element Aided Design of Laminated and Sandwich Plates Using Reanalysis Methods

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Classical finite element programs are not well suited to the design of composite structures, because they are primarily analysis tools and need much time for the data input and as well as for the interpretation of the results. The aim of this paper is to develop a program which allows very fast analyses and reanalyses for design process, thanks to a fast reanalysis method with changes of data and conditions. Speed in the analysis is obtained by simplification of the analysed structure and limitations in its geometrical generality and improvements in numerical methods. The use of the program is made easy with interactive user-friendly facilities.

Key Words : Design, Finite Elements, Numerical Methods, Reanalysis, Composite Structures, Laminated Plates, Sandwich Plates

1. Introduction

The laminated and sandwich panels are often employed for aeronautic, automotive, civil engineering purposes. These panels are essentially composed of several material layers of varying thickness glued together. For example, in sandwich panels, while the material properties of the core generally provide the indispensable thermal insulation and low overall density, skin materials, according to their constitution, provide resistance to shock, inclemency, fire or simply an aesthetic effect. However, the elastic calculation of laminated and sandwich panels with the traditional theories of multilayer plates, shear or elasticity, remain complex as their formulation lead to a system of one or several differential equations.

Nowadays, the finite element method is frequently used to solve these equations and operates as a basic tool for the analysis process of structures. However, it may require a long and complex process, even for classical structures. For composite structures, the difficulties of finding pertinent and simple models for the material behaviour make finite element analysis even more difficult. Many general computation codes, such like NASTRAN, SAP or ANSYS, have been developed for industrial uses, mainly in the area of solid mechanics. These codes require intensive use of a computer and, consequently, often run on mainframes.

A review of existing programs for the design of composite structures shows that, generally, they are only analysis tools with no design facilities or efficient user-interface for data input or results in-

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terpretation. In structural design or optimisation, the procedures are generally iterative and require repeated analysis as the structure is progressively modified. In addition, the design of composite structures needs efficient facilities in order to overcome penalties such as heavy input data, long computational, output time due to the material properties and the multilayer structure. The inclusion of a finite element program in an optimisation process is frequently proposed as a way for design. In our opinion, this is generally impractical.

Recent reanalysis methods joined with an interactive user interface appear as one of the most realistic solutions to satisfy the designers of composite structures by countering the above drawbacks of classical finite element programs. As stated by Kane et al. (1990), the term reanalysis denotes any technique that allows for the subsequent analysis of a modified problem with less expenditure of computational resources than required to compute the response of the original problem. Generally, some information computed in the analysis of the original problem is reused in the analysis of the modified problem. Changes for design process can be graded in order of increasing difficulty as : applied forces, boundary conditions, material properties, geometry.

The use of reanalysis methods for the design of composite structures has been investigated in this paper. The present development illustrates an original reanalysis method for modification of applied forces and boundary conditions. To provide a piece of reference software on laminated and sandwich plate design in the same manner as the reference book of Timoshenko and Woinowsky-Krieger (1981) for homogeneous and isotropic plates, a specific program FEAD-LASP was developed. FEAD-LASP (Finite Element Aided Design for Laminated And Sandwich Plates) uses the original direct reanalysis method and provides a user friendly interface for the design of composite plates.

2. General Theories of Laminated and Sandwich Plates

Bibliographical research reveals three main theories for the study, in the elastic area of transversely loaded multilayer plates : 1) the traditional theory of thin orthotropic plates, 2) the plate theory with shear effects, 3) the theory of elasticity (in the case where a solution exists).

The adaptation of the classical isotropic plate theory to anisotropic materials is due to the work of Lekhnitskii (1963). The initially multilayer plate is replaced by an equivalent single ply model, based on the substitution of the real constitutive material by a fictitious equivalent material, the properties of which are determined from the laminated or sandwich properties. Displacements and deformations then obtained translate the average behaviour of a plate that becomes artificially homogeneous across its thickness. Only stresses require a ply by ply calculation. This theory is based essentially on two hypotheses : 1) normal planes remains normal to the neutral axis after application of loads, 2) displacements correspond only to the existence of bending moments.

This second method can be considered as a generalization of the previous theory. Some of the hypotheses assumed for thin plates can be preserved (σ_{zz} stress negligible through the thickness h, external and internal linearity, only transverse loadings). The effects of shear stresses are no longer neglected and generally, a parabolic law characterises the variation of the shear stress through the thickness. This theory, interesting and relatively simple in its literal application, nevertheless introduces some difficulties into the research of pertinent mathematical solutions. Finally, the resolution of plates non symmetric with respect to mid-plane appears rapidly inextricable with the increase in the number of layers.

The solution of the problem of the elasticity, in stress or displacement formulation, has been developed by Pagano (1970) in the case of multi-

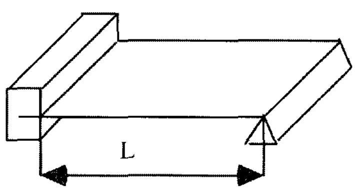

Fig. 1 Boundary conditions

layer plates. This theory has the advantage of providing exact theoretical results, without simplifying hypotheses as in the two previous theories. Although its major disadvantage is that it is not universal, it constitutes an applicable element of choice as compared to the other methods of calculation.

2.1 Comparison of "thin plate theory" and "theory of elasticity" using a sandwich panel under normal uniform pressure

The study focuses on a sandwich plate in cylindrical flexion. The input data, used to obtain the results, were the following :

Skins : thickness=h=33 mm, orthotropic material : Polyester resin and glass fibres

$E_x=E_y=5280$ MPa $G_{xy}=1955$ MPa $\nu_{xy}=\nu_{yx}=0.35$
$E_z=3970$ MPa $G_{yz}=G_{zx}=1320$ MPa $\nu_{zx}=\nu_{yz}=0.41$.

Core : thickness=120.4 mm, isotropic material : Polyurethane foam

$E=5$ MPa $\nu=0.22$.

Normal uniform pressure : q=0.1 MPa

2.2 Calculation of the maximum bending deflection

For a ratio S=L/h=4, the values of the maximum bending deflection at the centre of the plate are :

- classical thin plate theory : $w_{max}=0.57$ mm ;
- theory of elasticity : $w_{max}=13.21$ mm.

The gap between the two values is very large. Even for thinner sandwiches, this difference remains considerable.

2.3 Remarks

The classic theory appears, from the previous example, to be inefficient, especially for the calculation of bending deflections. The inability to translate the global behaviour of a sandwich panel is essentially due to the fact that the shear force has not been taken into account. The results provided by theory of the elasticity show that the shear force cannot be neglected in sandwich plates. However, the shear effects leads to the corresponding equations which becomes even more difficult. The use of the finite element method provides a solution to draw back this difficulty.

3. Finite Element Method and Reanalysis Method

3.1 *Finite element method*

The finite element method is today sufficiently well known (Zienkiewicz, 1971). The principle of the method is to replace a continuous structure by a discrete model (meshing process) composed of many structural elements of finite dimensions, called finite elements. These elements are connected to each other by a finite number of nodal points (or nodes). At these nodes, a certain number of displacement and possibly rotations representing degrees of freedom are defined. The resolution of the problem consists to determinate at all nodes either displacements or forces (displacement or stress formulation). The displacement method, described here, is based on the assumption that displacements in any finite element depend only on the nodal displacements of the element. The application of the minimum potential energy theorem on each element and the assembly of all elements result in a linear system of equations of the form :

$$\boldsymbol{K}\boldsymbol{u}=\boldsymbol{f} \tag{1}$$

where $\boldsymbol{K}$ is the stiffness matrix of the structure, $\boldsymbol{u}$ and $\boldsymbol{f}$ respectively the nodal displacement and total force vectors. From a mathematical point of view, the finite element method transforms the integral formulation of the equilibrium of the continuous structure into a set of discrete algebraic equations. That is especially the properties of the element that is necessary to define with care, in particular for composite structures. A solution for obtaining an efficient simple finite element for the analysis of laminated and sandwich structures led to the equivalent material model.

In order to obtain a simple and efficient finite element model for analysis of laminate and sandwich structures El Shaikh (1980) developed the equivalent material model, based on the substitution of real constitutive material by a fictitious equivalent material. The properties of the equivalent material are determined from laminate or sandwich properties. While this process allows use of the classical three-dimensional finite element in a slightly modified form, it avoids construction of special finite elements for the Mindlin-type laminated or sandwich plates and shells. Using elastic stiffness, which varies parabolically along the thickness, the modified three-dimensional element can reproduce exactly the behaviour of any of these structures, including coupling and shear effects. The fundamental result is that, with a convenient choice of material constants, a complete similarity between the Mindlin-type equations and some of the finite element equations can be achieved.

The theoretical analysis for the equivalent material model is easily implemented in finite element programs possessing a numerical integration and three-dimensional finite elements with variable numbers of nodes. It has been used with isoparametric Serendip elements. In the thickness, the three-point Gauss-Legendre integration was selected to take into account the variation of the material properties, and the classical rules of numerical integration were used in the two other directions.

It has been seen that the present method and numerical results are generally in excellent agreement, while reasonable agreement is found with the experimental results. It can be concluded then that this process easily converts a classical three-dimensional finite element program into an efficient laminated or sandwich program, using an equivalent material.

3.2 Comparison of "finite element method" with "theory of elasticity" using a sandwich panel under normal uniform pressure

We process here the same example already presented above. The following results were obtained :

$S=4$ and $q=1E^{-1}$ MPa :
- theory of elasticity :

  $w_{max}=13.21$ mm ;
- finite element method :

  $w_{max}=14.08$ mm (ANSYS).

$S=30$ and $q=1E^{-3}$ MPa :
- theory of elasticity :

  $w_{max}=25.1$ mm ;
- finite element method :

  $w_{max}=25.3$ mm (ANSYS).

For S=4 a largest error (7%) is generally observed on the bending deflection. This is due to the fact that for S<15, the exact shear stress distribution given by the theory of elasticity differs slightly from that imposed on the finite element method. The shear energy is then underestimated and the finite element method leads to an overestimation of the bending deflection of the structure.

3.3 Reanalysis Method

The design and optimisation of structures generally require iterative procedures and repeated analysis as the structure is progressively modified. In order to avoid the completely same analysis process at each iteration, many reanalysis techniques have been devised. Several of these techniques were reviewed by Arora (1976) in 1976.

The reanalysis methods are broadly classified as either direct (i.e. exact) or iterative in terms of solution process. These methods are formulated by using the force (or flexibility) variables, displacement (or stiffness) variables, or mixed variables. The direct methods give exact and closed-form solutions that have the same effect as solving the newly modified system of equations. Generally, the direct methods are efficient if the number of elements to modify is small. The reanalysis method presented hereafter is direct and formulated by using the displacement variables.

The reanalysis method initially proposed by Verchery (1990) permits to a reanalysis for simultaneous changes in kinematic constraints and applied forces, and gets a substantial gain in computation time. These kinematic constraints include boundary conditions, symmetry conditions, inextensibility or incompressibility constraints. The principle of this method is based on the fact that rigid body motions cause the singularities of the classical equilibrium system of discrete elastic structures. Therefore, a general solution can be expressed prior to any assignment of displacement boundary conditions. The solution for specified load and boundary conditions can then be determined by solving an associated small-size linear system. The theoretical method and its numerical evaluation are outlined hereafter.

In the traditional analysis method, the equation for the classical equilibrium system of discrete elastic structures is expressed as in Equation (1) above. This equation can only be solved by introducing the boundary conditions. Further, when the boundary conditions are changed, the entire process must start again from the beginning, solving frequently a very large linear system. This process is very time consuming. In order to reduce the computation time of classical analysis the following reanalysis process has been developed. The reanalysis process is decomposed in two steps. A first step is the computation of the flexibility matrix $\boldsymbol{S}$, which is a quasi-inverse of the stiffness matrix :

$$\boldsymbol{SK}=\boldsymbol{KS}=\boldsymbol{I}-\boldsymbol{RR}^T \tag{2}$$

This flexibility matrix is obtained through a regularization process, described theoretically by Verchery and numerically by Loredo (1993). Here $\boldsymbol{R}$ is the $n\times r$ matrix, the columns of which are the eigenvectors, chosen to be orthonormal, for the zero eigenvalues of the stiffness and flexibility matrices $\boldsymbol{K}$ and $\boldsymbol{S}$.

While this first step is general, the second step introduces all the data relevant to the kinematic constraints and loading conditions. The kinematic constraints (including the boundary conditions) are assumed to be independent and linear :

$$\boldsymbol{L}^T\boldsymbol{u}=\delta \tag{3}$$

in which $\boldsymbol{L}$ is a $p\times n$ matrix with rank p, and δ are p prescribed values. These kinematic constraints develop reactive forces $\boldsymbol{f}_r$ expressible in terms of p reaction parameters ϕ :

$$\boldsymbol{f}_r=-\boldsymbol{L}\phi \tag{4}$$

The total forces $\boldsymbol{f}$ are the sum of these reaction forces and the given forces $\boldsymbol{f}_d$. The solution for the displacement u can be expressed in the full form :

$$\boldsymbol{u}=\boldsymbol{S}(\boldsymbol{f}_d-\boldsymbol{L}\phi)+\boldsymbol{R}\omega \tag{5}$$

The rigid body motions ω and reaction parameters ϕ are obtained for a discriminant system :

$$\begin{bmatrix} -L^TSL & L^TS \\ R^TL & 0 \end{bmatrix}\begin{Bmatrix}\phi \\ \omega\end{Bmatrix}\begin{Bmatrix}\delta-L^TSf_d \\ R^Tf_d\end{Bmatrix} \tag{6}$$

The order $p+r$ of this discriminant system is much less than the original system (Equation 1), especially for problems with large degrees of freedom and small number of constraints. An advantage of this reanalysis method is that it can be applied to local changes in material and geometrical properties (Huang and Verchery, 1992). With this method, the reanalysis under various boundary conditions is made easier. This can be applied to aid design, which requires fast reanalysis processes for full efficiency or to simplify the solutions for contact and elastic crack propagation problems, which otherwise require long iteration methods. It can also be useful for substructuring large or repetitive structures.

We introduce here the "active degrees of freedom" and "definitive degrees of freedom" concepts that allow the previously described method to be faster again. Their principles consist of a reduction of the matrix computation by reducing the degrees of freedom (D.O.F.) and rigid body motions parameters to a minimal set completely determined by the studied structure. Sufficient information is kept to permit the solution.

An "active degree of freedom" represents any degree of freedom that receives either a kinematic or a loading constraint. The necessary number of degrees of freedom to solve the discriminant system for a particular study is equal to the active degrees of freedom. This number is inferior to the number (n) of the initial set. Most of the matrices belonging to the discriminant system can be "contracted", without loss of information. The discriminant system becomes :

$$\begin{bmatrix} -\bar{\boldsymbol{L}}^T\bar{\boldsymbol{S}}\bar{\boldsymbol{L}} & \bar{\boldsymbol{L}}^T\bar{\boldsymbol{R}} \\ \bar{\boldsymbol{R}}^T\bar{\boldsymbol{L}} & 0 \end{bmatrix}\begin{Bmatrix}\phi \\ \bar{\omega}\end{Bmatrix}=\begin{Bmatrix}\delta-\bar{\boldsymbol{L}}^T\bar{\boldsymbol{S}}\bar{\boldsymbol{f}}_d \\ \bar{\boldsymbol{R}}^T\bar{\boldsymbol{f}}_d\end{Bmatrix} \tag{7}$$

with $\bar{\boldsymbol{L}}=\boldsymbol{H}^T\boldsymbol{L}$, $\bar{\boldsymbol{S}}=\boldsymbol{H}^T\boldsymbol{SH}$, $\bar{\boldsymbol{R}}=\boldsymbol{H}^T\boldsymbol{R}$, $\bar{\boldsymbol{f}}=\boldsymbol{H}^T\boldsymbol{f}_d$.

$\boldsymbol{H}$ represents here the $n \times a$ contracting matrix with $a < n$.

The $\bar{S}$ matrix shows that only the corresponding rows or columns of the compliance matrix S have to be computed in the regularization process. This variant of the reanalysis method is very helpful at significantly reducing the computation time of the two steps of the reanalysis process.

A designer may eventually decide that some kinematic constraints, that correspond to a symmetry of the problem for example, remain constant during the full design process. The corresponding degrees of freedom then become "definitive," that is to say that the corresponding rows or columns that they represent in the matrices will not be computed but eliminated in the same manner as in the classical way. The sizes of the initial matrices $\boldsymbol{R}$, $\boldsymbol{K}$ and S will decrease and perhaps the matrix $\boldsymbol{R}$ of the rigid body motions may disappear completely. The gain in computation time for this method is then very large, but, no more changes can be made within the set of these "definitive" kinematic constraints.

In order to evaluate the performance of the proposed reanalysis methods the numerical operations were counted. An operation is defined as a multiplication plus an addition, and only the major sequences of operation are counted. These are only approximate comparisons because the integer arithmetic for address computations may vary among different algorithms. In some cases these operations may consume a substantial portion of the total computation time. The number of operations required for initial formulation of the global stiffness matrix, $\boldsymbol{K}$, has not been calculated as it is beyond the scope of this paper. The following study compares only the computation times for the two steps of the reanalysis method, the regularization process and the resolution of the discriminant system. A personal computer with a 387 arithmetic coprocessor and a 33 MHz clock was employed for evaluation of the corresponding time measurements.

The number of the currently elementary operations, that corresponds to the regularization process for the FEAD-LASP program (n=390 D.O.F.), reaches approximately 28 million. This number corresponds to a 7 minute duration and an $O(n^3/2)$ complexity for the algorithm.

For the 16-node Serendip, thick plate finite element employed in the FEAD-LASP program, with 3 degrees of freedom per node, the "in place" matrix inversion algorithm, used in FEAD-LASP, proves to be faster than the Loredo algorithm (Loredo, 1993), usually employed at this step, particularly when the size of the problem does not exceed 600 degrees of freedom (Fig. 2). The gain is approximately 5 million operations or 1.25 minutes. The main reasons for the gain are as follows. The algorithm of Loredo is principally based on the band shape of the K stiffness matrix, but does not consider the symmetry of the matrices. The algorithm used in FEAD-LASP does account for the symmetry of the matrices. For square symmetric matrices the savings in numerical operations may be significant. If the band width variation within the body of the stiffness matrix is taken into account, the number of operations is reduced. Above the limit of 600 D.O.F., the algorithm of Loredo works faster because of its $O(2n^2b + nb^2)$ complexity, where b represents the half-band width of the K stiffness matrix of the structure. The effects of the "active" and "definitive" D.O.F. are not included in this comparison. We have considered that all D.O.F. were "active" and none were "definitive".

Using the 16 node Serendip element, the difference between the number of elementary operations needed by the two resolution processes can be evaluated by the following expression :

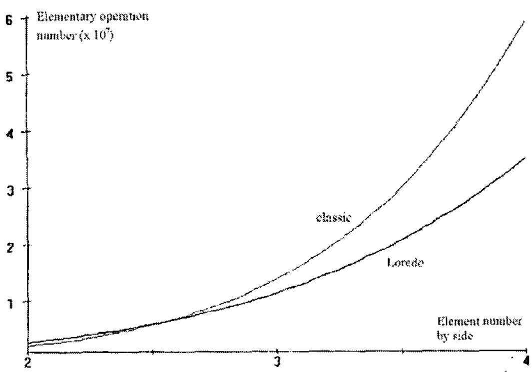

Fig. 2 Comparison between the algorithm of FEAD-LASP and Loredo (until 390 D.O.F.)

$$\text{Difference}=\frac{1544}{3}p+579pm-7636-1572m+1496m^2+12798m^3+2916m^4-\frac{1}{6}p^3+144pm^2-\frac{9}{2}p^2 \quad (8)$$

where p represents the kinematic constraints and m the number of finite elements along the edge of a square plate. Fig. 3 shows the favourable and unfavourable zones for this method as compared to the classical method. Notice that the favourable zone corresponds mainly to a weak number of kinematic constraints compared to the total number of degrees of freedom. The maximal computation gain, compared to the classic method, can reach two minutes for $m=6$ (800 D.O.F.) and $p<200$, with a calculation time of only 30 seconds for a reanalysis. The effects of the "active" and "definitive" D.O.F. are not included in this comparison.

3.4 Remarks

Several micro-computers and work-stations were used for the numerical part of this study. It was found that personal computers as well as low cost work-stations were well suited for a fast numerical treatment, i.e. only a few minutes for the first analysis, and reanalysis in seconds. Moreover, it was seen that, even in the case of very low-cost micro-computers, the total time was reasonable in a design process. The numerical accuracy of the results agrees with other finite element codes and the software as already demonstrated (Eyraud et al., 1995).

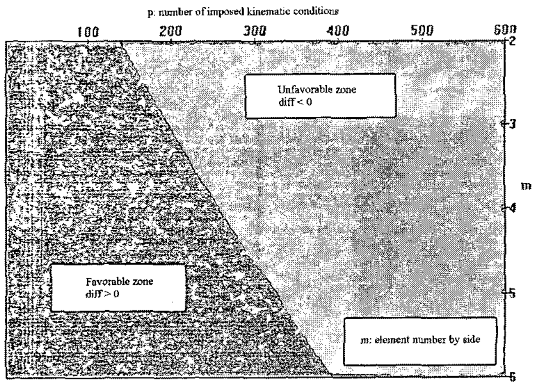

Fig. 3 Favourable and unfavourable zones for the FEAD-LASP reanalysis method compared to the classical method (for the iterative process)

4. Validation of FEAD-LASP Program

4.1 General description of FEAD-LASP program

To evaluate the proposed efficient and general reanalysis methods, which are not based on multiple automatic optimisations used in any universal finite element program but entirely piloted by the user, a specific finite element program was created, called FEAD-LASP (Finite Element Aided Design of Laminated And Sandwich Plates). Developed for laminated and sandwich plates, this program permits both analysis and design of composite plates in linear elasticity. The structure of FEAD-LASP is schematised in Fig. 4,

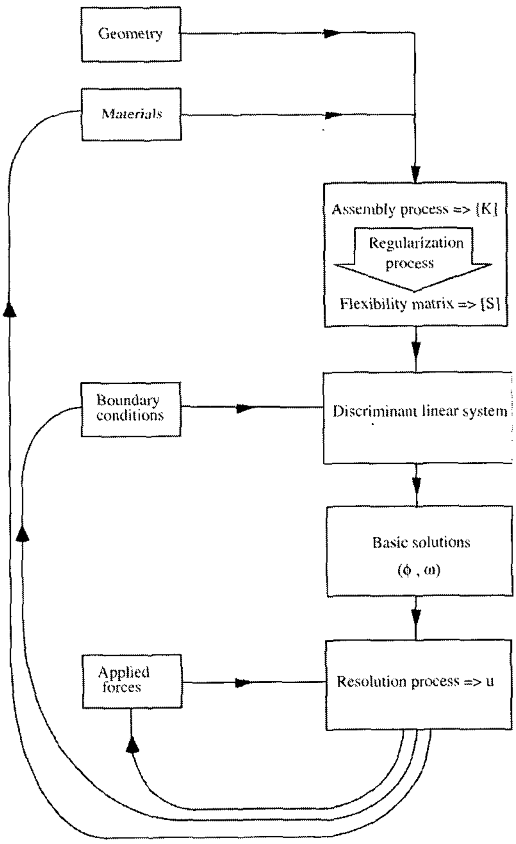

Fig. 4 General description of FEAD-LASP software

in which kinematic constraints are limited to boundary conditions.

The main objectives of this program were: 1) to obtain good compromise between the computation time and the treatment capacities of the program, 2) to significantly decrease of the computation time in order to reduce the waiting time of the users, 3) to provide a fully interactive package, in the sense of being self-explanatory and user-friendly. This resulting software consists of a Multiple Documents Interface (MDI) whose menu is centred around three main sections: input of data, resolution and results exploitation (Fig. 5).

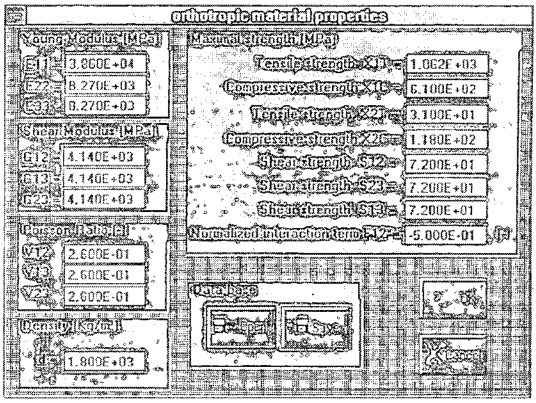

Fig. 5 Dialog box for the input of orthotropic material properties of the FEAD-LASP software

The data input begins with the geometrical definition of the structure. Rectangular composite and sandwich panels (in bending and in-plane deformations) with pre-defined regular meshes are employed, covering a wide field of practical needs. Three new menu items are then proposed to the direct or ply by ply laminate (or sandwich) definition. Appropriate and evolutive data bases are available for this task (Fig. 6). We selected a generalization stress-strain law, which describes efficiently the behaviour of both laminated plates with shear effects and sandwich plates. It takes into account all the specific features of these structures based on the extended laminated plate theory.

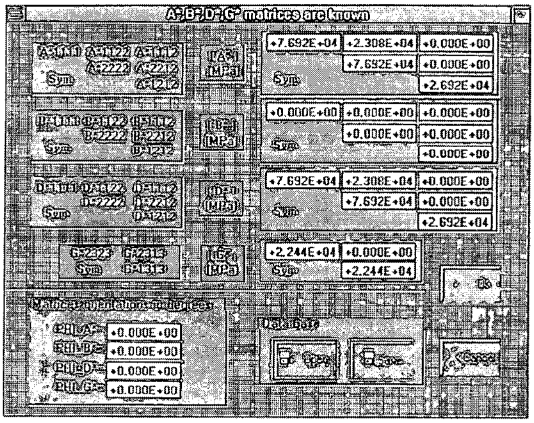

Fig. 6 Dialog box with a data base for the generalized stress-strain law used in FEAD-LASP software

Using the normalised stiffness (Tsai and Hahn, 1980), the generalized stress-strain law is written as:

$$\begin{Bmatrix} N \\ M \\ Q \end{Bmatrix} = \begin{bmatrix} hA^* & \frac{h^2}{2}B^* & 0 \\ \frac{h^2}{2}B^* & \frac{h^3}{12}D^* & 0 \\ 0 & 0 & hG^* \end{bmatrix} \begin{Bmatrix} \varepsilon^0 \\ \kappa \\ \gamma \end{Bmatrix} \tag{9}$$

where $\boldsymbol{N}$, $\boldsymbol{M}$ and $\boldsymbol{Q}$ are the in-plane forces, bending moments and shear forces; ε^0, κ and γ are the membrane strains, curvatures and shearing strains; $\boldsymbol{A}^*$, $\boldsymbol{B}^*$, $\boldsymbol{D}^*$ and $\boldsymbol{G}^*$ are the normalised in-plane, coupling, bending and shear stiffness. As we proceed with global stiffness, the definition problem of the stacking sequence may be treated separately (Verchery, 1990; Kandil and Verchery, 1989).

A method for the calculation of the generalized shear stiffness, which does not require the so-called shear correction factors, has also been incorporated, for both sandwich and laminated plates (Cheikh Saad Bouh, 1992; Pham Dang and Verchery, 1978). Boundary conditions are pre-defined for the standard cases (clamped and simply supported plates), but could be customised for particular cases. Uniform pressures, punctual loadings and natural weight, corresponding to the applied forces, can be customised as well. The menu "solver" launches the first analysis and the possible ulterior reanalyses. The resolution is based on the reanalysis method previously described. With this method, reanalyses under various boundary conditions and loadings are made easier and quicker. Two graphic post-treatments are dedicated to the results exploitation. The first one allows the 3D visualisation of the deformed structure and gives nodal information (displacements, reaction forces, strains and stresses). The second draws the displacement, strain and stress isovalues on the plate.

In view of the features currently provided by system environments such as Macintosh or Microsoft Windows, an interface adapted to the research field was designed, presenting similar facilities. This interface is user-friendly through the use of windows, icons, pop-up menus and dialog boxes technique. An in-line hypertext help is provided, as well, to assist users who have little or no knowledge of the internal structure and methods of the program. The quasi-multitasks environment was, as much as possible, employed to keep the dialog between the software and the user and permit a communication with others codes. It thus becomes possible to quasi-simultaneously compute an analysis and visualise results of the previous one. The interactivity of the solver is principally obtained through the use of the reanalysis method and various specific routines developed to reduce the computation time.

4.2 Implementation of FEAD-LASP program

The FEAD-LASP program uses a 16-node thick plate finite element for analysis. It is based on a kinematic and material similarity between laminated and sandwich structures and the three dimensional finite element with two nodes in the thickness and a fictitious equivalent material with properties varying along the thickness. This process applied to the finite element method allows use of the classical finite elements in a slightly modified form and avoids resorting to special finite elements.

During the program development, a dimensional analysis, based on the fact that a regular mesh was used, showed that only about 10% of the global matrix coefficients in a non-dimensional form are needed to generate the global matrix. Consequently, these quantities were computed and stored as data in the program. These data are common to all types of plates, so the assembly process is reduced to the multiplication of different data blocks by dimensional and materials properties. The analysis computational time (assembly process and solving) was compared with a classical finite element program. This comparison showed the high efficiency of the FEAD-LASP program compared with the method used in the classical finite element program. The FEAD-LASP program is written in C/C++ object oriented language to better manage the computer system facilities. Dynamic memory allocation and resources files have been employed to manage the memory capacity in an optimal manner and permit the translation of the program into various foreign languages. To run the program satisfactorily, a graphic display and an arithmetic coprocessor are required. The minimal required free storage capacity is about 4-Mbyte hard disc drive and 4-Mbyte RAM memory. A superior configuration is preferable and will of course provide better performances.

4.3 Validation

Two types of materials were used to build two laminated multilayer plates (laminate 1 and laminate 2) and one sandwich plate.

4.3.1 Material 1 (M1) : ply material

Material 1 is an orthotropic composite material made of an epoxy resin reinforced with unidirectional graphite fibres, with the following engineering constants and elastic stiffness :

E_{11}	=175 000 MPa
$E_{22}=E_{33}$	=7 000 MPa
$G_{12}=G_{13}$	=3 500 MPa
G_{23}	=1 400 MPa
$\nu_{12}=\nu_{13}=\nu_{23}$	=0.25
C_{1111}	=176 174.5 MPa
C_{1122}	=2 349 MPa
$C_{2222}=C_{3333}$	=7 498 MPa
C_{2233}	=1 898 MPa
$C_{1212}=C_{1313}$	=3 500 MPa
C_{2323}	=2 349 MPa

4.3.2 Material 2 (M2) : core material

Material 2 constitutes the core of the sandwich plate. It has the following engineering constants and stiffness (Whitney (Loredo, 1993)) :

$E_{11}=E_{22}$	$=280$ MPa
E_{33}	$=3\ 500$ MPa
$G_{23}=G_{13}$	$=420$ MPa
G_{12}	$=112$ MPa
$\nu_{13}=\nu_{23}$	$=0.02$
ν_{12}	$=0.25$
C_{1112}	$=77.2$ MPa
$C_{1111}=C_{1122}$	$=301.1$ MPa
C_{1133}	$=94.6$ MPa
C_{3333}	$=3\ 547.3$ MPa
$C_{1313}=C_{2323}$	$=420$ MPa
C_{1212}	$=112$ MPa.

4.3.3 Laminate 1 (without coupling)

This cross-ply laminate is composed of three plies of equal thickness (0.3 mm) with a [0/90/0] stacking sequence (Fig. 7). Using the classical laminated plate theory, the following normalised (Verchery, 1990) values (in MPa) are obtained for A^*, B^*, D^*, d^* ($d_F{}^*$ corresponds to the d^* matrix computed with FEAD-LASP program, including shear correction factors):

$$[A^*]=\begin{bmatrix}119298 & 1754 & 0\\ & 63157 & 0\\ \text{Sym} & & 3500\end{bmatrix}\qquad [B^*]=[0]$$

$$[D^*]=\begin{bmatrix}169200 & 1754 & 0\\ & 13255 & 0\\ \text{Sym} & & 3500\end{bmatrix}$$

$$[d^*]=\begin{bmatrix}2800 & 0\\ 0 & 2100\end{bmatrix}\qquad [d_F{}^*]=\begin{bmatrix}1559 & 0\\ 0 & 1790\end{bmatrix}$$

4.3.4 Laminate 2 (with coupling)

Laminate 2 is a 16-ply antisymmetric plate with coupling (Fig. 7). The thickness of each ply is equal to 0.2 mm. The corresponding A^*, B^*, D^* and d^* matrices (in MPa) are :

$$[A^*]=\begin{bmatrix}70609 & 22373 & 0\\ & 70609 & 0\\ \text{Sym} & & 24118\end{bmatrix}$$

$$[B^*]=\begin{bmatrix}-6551 & 1289 & -2632\\ & 3975 & -2632\\ \text{Sym} & & 1289\end{bmatrix}$$

$$[D^*]=\begin{bmatrix}72580 & 22370 & -987\\ & 68640 & -987\\ \text{Sym} & & 24118\end{bmatrix}$$

$$[d^*]=\begin{bmatrix}2450 & 0\\ 0 & 2450\end{bmatrix}\qquad [d_F{}^*]=\begin{bmatrix}1667 & 0\\ 0 & 1667\end{bmatrix}$$

4.3.5 Sandwich Plate

This symmetric sandwich is composed of a thick core (18 mm in thickness) made of the material 2 and two facings (1 mm each) of the material 1 (Fig. 8). The values (in MPa) of the matrices A^*, B^*, D^* and d^* are calculated using a special mixed variational theory developed for sandwich structures (Pham Dang, 1976):

$$[A^*]=\begin{bmatrix}17813 & 243 & 0\\ & 971 & 0\\ \text{Sym} & & 450\end{bmatrix}\qquad [B^*]=[0]$$

$$[D^*]=\begin{bmatrix}47743 & 536 & 0\\ & 2143 & 0\\ \text{Sym} & & 1039\end{bmatrix}$$

$$[d^*]=[d_F{}^*]=\begin{bmatrix}419 & 0\\ 0 & 417\end{bmatrix}$$

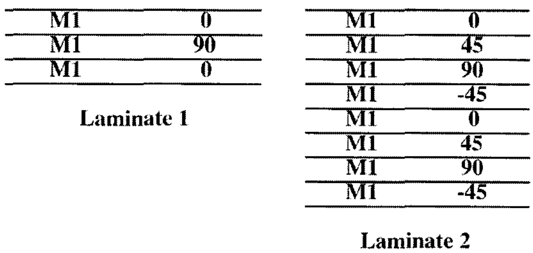

Fig. 7 Two laminated plate types used in the study

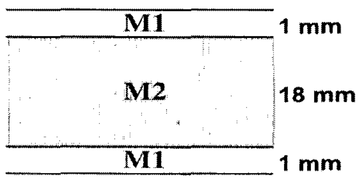

Fig. 8 Sandwich plate configuration

4.4 Application examples

Three examples have been studied for the performance evaluation of the different types of finite elements. For all examples, a clamped square plate has been considered. The deflection of the central point has been compared for the different finite elements. In a quadrant of the square plate, a 4× 4 mesh was used for the finite element analysis (Fig. 9). For the laminated plate with coupling, the totality of the plate has been meshed using 8× 8 elements except in the case of the FEAD-LASP program where only 4×4 elements were used. This accounts for an error for approximately 12% in the FEAD-LASP analysis.

4.4.1 Thick and thin composite plates (Laminate 1)

The analysis has been done for two types of loading conditions : concentrated force at the centre of the plate and uniform pressure. Table 1 gives the results for the plate under concentrated load in two cases : thick plate (L/H=11 and thin plate (L/H=222). Table 4 gives the results for the uniformly distributed load (L/H=222).

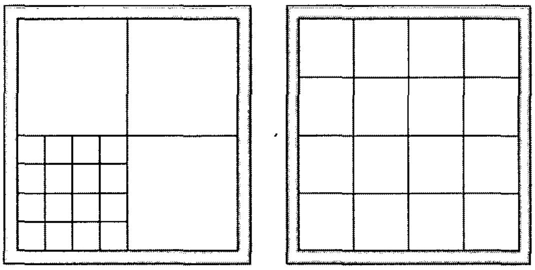

Fig. 9 Mesh generation for the quadrant of a square plate

4.4.2 Moderately thick plate with coupling (Laminate 2)

In this case, we used the laminate 2 with the aspect ratio L/H equal to 62. The results are given in Table 2 for concentrated load and Table 4 for the uniformly distributed load.

4.4.3 Thick sandwich plate

The comparison of the central deflection values for this example is given in Table 3 (concentrated load) and Table 4 (uniformly distributed load).

In these examples, various finite elements for plates were presented and their validity was studied for the analysis of laminated and sandwich plates. From results obtained here, they following conclusions can be made.

(1) The transverse shear effect for thick laminated and sandwich plates has an important influence.

(2) The Ahmad type element and the modified volumetric finite element have globally the same

Table 1 Comparison of the central deflection for the clamped plate under concentrated load (laminate 1)

	FINITE ELEMENT TYPES				
L/H	DKQ	Ahmad	FEAD-LASP	3D-Quadratic	3D-Cubic
11	0.12	0.60	0.71	0.53	0.53
222	2.71	2.52	2.22	2.18	2.48

Table 2 Comparison of the central deflection for the clamped plate under concentrated load (laminate 2)

	FINITE ELEMENT TYPES				
L/H	DKQ	Ahmad	FEAD-LASP	3D-Quadratic	3D-Cubic
62	4.87	4.91	4.29	4.90	5.09

Table 3 Comparison of the central deflection for the clamped sandwich plate under concentrated load

	FINITE ELEMENT TYPES				
L/H	DKQ	Ahmad	FEAD-LASP	3D-Quadratic	3D-Cubic
20	1.60	3.62	3.30	3.28	3.33

Table 4 Comparison of the central deflection for the clamped plate under uniform loading

	FINITE ELEMENT TYPES			
Material Types	DKQ	Ahmad	FEAD-LASP	3D-Quadratic
Laminate 1 (L/H=222)	0.44	0.43	0.42	0.43
Laminate 2 (L/H=62)	1.10	1.10	0.95	1.09
Sandwich (L/H=20)	2.710.22	0.52	0.47	0.47

Table 5 Bending deflections of a [(0/90°)4] laminated plate under different loadings and boundary conditions with the corresponding computation time for the reanalyses

Distributed load q=1 Mpa	Deflection (mm)	Reanalysis (s)	Classical analysis (s)
Clamped	0.67	45	65
Supported	1.47	20	48
Concentrated load p=1 MN	Deflection (mm)	Reanalysis (s)	Classical analysis (s)
Clamped	5.27	44	64
Supported	7.12	19	47

behaviour and can be used for composite plates with convenient accuracy, especially the volumetric element can have some advantages because of its simplicity.

(3) The estimation of the constitutive law is of primary importance for the laminated and sandwich plates. The classical laminated plate theory does not apply to sandwich structures.

(4) The preceding conclusions are valid for the FEAD-LASP program for both thin and thick plates.

The other main advantage of FEAD-LASP is that it permits changes in kinematic constraints and loadings in a very simple and quick manner. It then becomes possible to compute the bending deflection of the plate when it is clamped or under increasing loads, without repeating the entire data input process. Table 5 below shows the results of these reanalyses in terms of bending deflection and corresponding computation time (ratio L/h=10, 4×4 mesh, 387 DX 33 Mhz coprocessor).

5. Conclusion

This paper discussed the general theories used in the analysis of transversely loaded sandwich and laminated plates. Methods of reanalysis were examined and an original method was developed. A general description of the resulting program, FEAD-LASP, has been presented along with the methods of implementation and a validation of the results. The link between pre- and post-processors (data input and output processing) and the analysis stage (finite elements) to form a design process was described and the attractive features of the software, obtained by significant reductions in computation time and user friendly package, were emphasised.

The simplicity and the generality of the explicit reanalysis method incorporated in FEAD-LASP allows anticipation of a modular development of the program or the creation of new software that would consider a variety of geometrical structures or permit the use of other element types.

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