THE JOURNAL OF THE ACOUSTICAL SOCIETY OF KOREA VOL.25, NO.2E 2006. 6 pp. 79~84

Multi-Termination Technique for the Measurement of Characteristic Impedance and Propagation Constant of Sound Absorbing Materials Using an Impedance Tube

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(Received Jun 7 2006; Accepted Jun 14 2006)

Abstract

Acoustic characteristics of a sound absorbing material can be identified, if the characteristic impedance and propagation constants are known, which have generally been determined experimentally. One easy method determining these two essential parameters is to measure the one dimensional wave characteristics in the impedance tube. In this study, the effects of backing conditions on the impedance tube measurement have been examined using several pairs of generally used end conditions. The results showed that the measured values are similar for most pairs of end conditions: however, it was observed that the measured characteristic impedance for different thickness did not agree well for some pairs. In this work, the multi termination method, using three or more known backing conditions, was suggested to reduce such random errors, which are mostly caused by the test procedure. Employing three terminations as a set, comprised of a rigid end, an end with porous material, and an end with a backing cavity, it was demonstrated that improved measured results could be obtained for an open cell PU foam varying widely with three different thicknesses.

Keywords: *Sound absorbing material, Bulk propagation characteristics, Multi-termination method*

I. Introduction

Knowledge on the acoustical properties of a sound absorbing material is important in predicting the end result at various practical applications for noise control and architectural acoustic tuning. However, the theoretical calculation of the acoustical properties is not easy because of the microscopic complexity of materials. Thus, in general, the acoustic characteristics have been usually obtained experimentally in the macroscopic viewpoint. Among the parameters that represent such macroscopic characteristics, the characteristic impedance and the propagation constant are most fundamental ones, from which other acoustical properties can be easily obtained. There have been two kinds of research works for obtaining the characteristic impedance and propagation constant: One is the analytical or semi-empirical method using the physical parameters such as porosity, flow resistivity, elastic constants, etc. [1-5] and the other is the experimental method [6-13].

The experimental methods can be classified into three categories; (1) direct method, (2) transfer matrix method, and (3) method using the transfer functions. Scott[6] presented a direct method that employed a probe microphone through a thick porous sample and measured the decay of sound pressure and the variation of phase with the change of positions. In this method a sufficiently thick sample was required, which is sometimes

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impractical, and the disturbances caused by the microphone probe raised a problem in application to some types of materials [7]. Method using the transfer matrix was tried recently by Song and Bolton [13]. The measurement may be performed quickly owing to the reciprocal nature of sound transmission through a homogeneous material.

Methods using the transfer functions are based on a pair of measurements of normal surface impedance [8-12], either changing the thickness or end conditions. In those methods, sound propagating in a test tube is decomposed into right- and left-going components by using multiple microphones: the surface-normal impedance can be calculated from the ratio between the complex amplitudes of two wave components. After obtaining the two surface normal impedances, characteristic impedance and propagation constants of the porous material can be estimated. According to the changing conditions employed, each method has been given a name, such as 'two-thickness method', 'two-cavity method', or 'two-load method'. Detailed historical reviews and discussions about various methods are available in some articles [12, 13].

In this study, currently existing impedance measurement methods using the transfer functions was compared and a technique resulting a best result was suggested. In the measurement, four types of commonly used backing conditions were selected and five pairs out of them were used as the 'two loads'. The sensitivity of measured results to backing conditions was investigated and, in addition, the practical measurement procedures and conditions were examined in search of various causes of errors other than the change in backing conditions. As a result, random errors due to test procedure were identified and the multi-termination method was suggested in order to minimize the errors.

II. Impedance tube measurement technique

The basic theory and the general calculation procedure of impedance tube measurement using the surface-normal impedance can be briefly summarized as follows. When the two different backing conditions are given for an identical sample as shown in Fig. 1, the measured surface-normal impedances for each backing condition, $Z_{n,i}$, is given by [2]

$$Z_{n,i} = Z_c \frac{-jZ_{b,i}\cot(\gamma L)+Z_c}{Z_{b,i}-jZ_c\cot(\gamma L)}\ , \quad (\mathrm{i} = 1,2) \tag{1}$$

where $Z_{b,i}$ denotes the surface impedance for the ith backing condition. Here, Z_c and γ mean the characteristic impedance and the propagation constants to be obtained for the test sample with thickness L, respectively.

In the measurement, the surface-normal impedances are to be calculated by using the wave decomposition technique [14]. Rewriting Eq. (1) in terms of γ yields

$$\frac{Z_{n,i}+Z_c}{Z_{n,i}-Z_c}\cdot\frac{Z_{b,i}-Z_c}{Z_{b,i}+Z_c} = e^{2j\gamma L}\ . \quad (\mathrm{i} = 1,2) \tag{2}$$

Here, the right hand side is expressed only by a function of both γ and L, which are assumed to be invariable with the change of backing conditions. Consequently, the following identity should be satisfied:

$$\frac{Z_{n,1}+Z_c}{Z_{n,1}-Z_c}\cdot\frac{Z_{b,1}-Z_c}{Z_{b,1}+Z_c} = \frac{Z_{n,2}+Z_c}{Z_{n,2}-Z_c}\cdot\frac{Z_{b,2}-Z_c}{Z_{b,2}+Z_c}\ . \tag{3}$$

Solving Eq. (3) for Zc yields

$$Z_c = \left[\frac{Z_{n,1}Z_{n,2}(Z_{b,1}-Z_{b,2})-Z_{b,1}Z_{b,2}(Z_{n,1}-Z_{n,2})}{(Z_{b,1}-Z_{b,2})-(Z_{n,1}-Z_{n,2})}\right]^{1/2}, \tag{4}$$

and γ can be obtained from Eq. (2) as

$$\gamma = \frac{1}{2jL}\left(\frac{Z_{n,i}+Z_c}{Z_{n,i}-Z_c}\cdot\frac{Z_{b,i}-Z_c}{Z_{b,i}+Z_c}\right). \quad (\mathrm{i} = 1,2) \tag{5}$$

As can be noticed in Eq. (4), the calculated results will be greatly influenced by the similarity between the two backing impedances, $Z_{b,i}$. Moreover, for very highly resistive materials, in which the measured surface-normal impedances are relatively insensitive to backing conditions, the results may be lack of confidence. However, for most of porous sound absorbing materials, this kind of measurement technique has been utilized very extensively in spite of the aforementioned source of error.

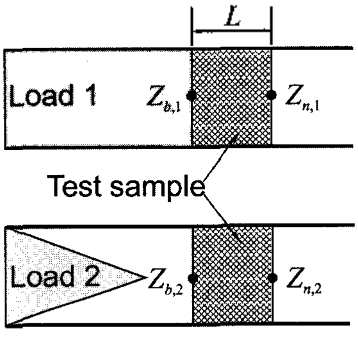

Fig. 1. Example layout of the specimen and terminations used for the measurement of bulk acoustic parameters based on the two-load technique.

III. Two-load method

3.1. Experimental setup

Figure 2 shows the schematic of the measurement setup for implementing the two-load method, which employs two different backing conditions in the impedance tube.

In this work, the inner diameter of the tube was 45 mm and the separation distance between microphones determined the applicable frequency range. Jang and Ih [15] showed that the multiple microphone technique for the in-duct measurement of acoustic parameters could tremendously reduce the random errors stemming from the test procedure and the measurement setup. They also found that, by using three or more uniformly spaced microphones, the effective frequency range could be widened in comparison with the two-microphone method. In this study, three phase-calibrated microphones were used with the microphone spacing of 35 mm and 170 mm for high- and low-frequency range, respectively. Effective measurement ranges were 100-900 Hz for the low-frequency range and 250-4900 Hz for the high-frequency range. In the work reported here, only the high-frequency results were presented for the comparison purpose.

The material measured for demonstration was the polyurethane foam of flow resistivity being 109 kRayls/m and of 15, 20, and 25 mm in thickness. Note that the samples were not extraordinarily thick. Three samples having the same thickness were tested, several times for each, and the averaged results were taken. In should be mentioned that the foam-like materials are sensitive to the boundary condition or the mounting condition of the sample. From this reason, random errors or data scatter due to edge leakages may appear big in the result [16-18].

Table 1. List of pairs of backing conditions employed in experiments.

Set No.	Load 1	Load 2
1	Rigid end	Absorbing end
2	Rigid end	Open pipe (135 mm long)
3	Rigid end	Closed cavity (20 mm long)
4	Rigid end	Closed cavity (40 mm long)
5	Closed cavity (20 mm long)	Closed cavity (40 mm long)

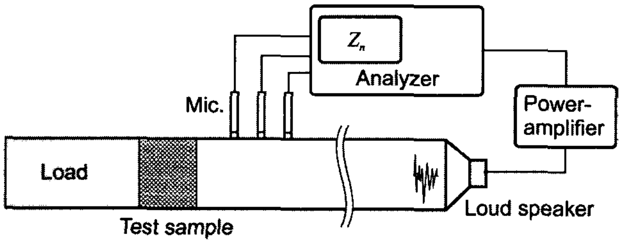

Fig. 2. Schematic of test setup using the three-microphone technique.

3.2. Backing conditions

Four types of backing conditions were selected for applying the two-load technique: rigid wall, absorbing material with known impedance, open pipe of 135 mm in length, and closed cavity with either 20 mm or 40 mm in depth. Five combinations of two end conditions among them were used in the experiment as listed in Table 1. Note that the depths of backing cavities behind the sample were chosen so as not to locate the critical frequencies within the frequency range of interest [11], whereas the length of the open pipe was chosen arbitrarily.

3.3. Measured results

Figure 3 shows the measured surface-normal impedances for all five backing conditions used, which are normalized by the characteristic impedance of air, with ρ_0 and c_0 being the density and the speed of sound of air, respectively. Note that the results are values at the frontal surface of each sample looking into the sound source.

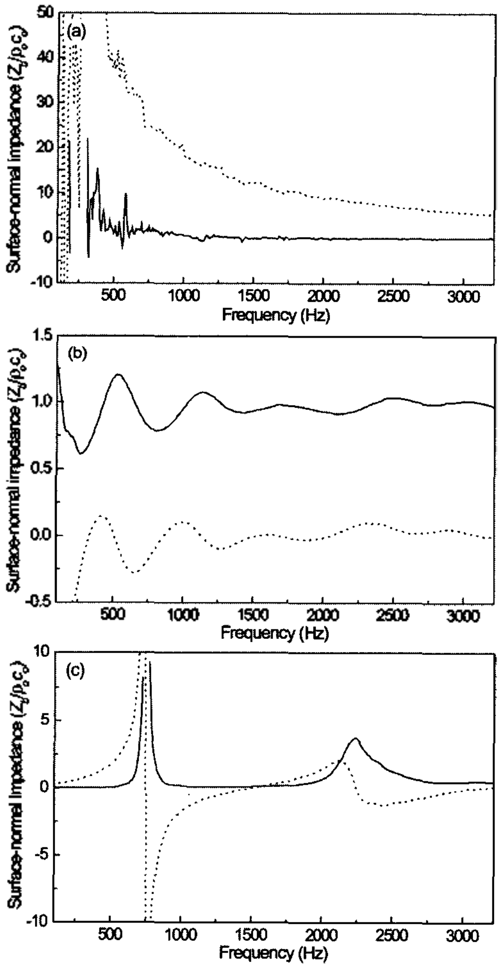

Fig. 3. Measured surface-normal impedances in association with the terminations of (a) rigid end, (b) absorbing end, and (c) open tube: ———, real part; ………, imaginary part.

Normalized characteristic impedances as illustrated in Fig. 4 could be calculated from a pair of measured surface impedance values corresponding to the selected pair of two backing conditions for each sample with a predetermined thickness.

One can see that the obtained results after applying different load sets for a sample agree generally very well, except at a distinctive frequency band in the case of using the open tube. This frequency band is centered at the resonance frequency related to the length of the backing pipe behind the sample as shown in Fig. 3(c). If an appropriate length for an open pipe in the backing were selected, all the results might appear similar. The other aspect one can observe is that the result of using load set No.5 deviates slightly from others in the frequency range of 1.2~2.5 kHz, particularly for 15-mm-thick sample, that is the thinnest one among the specimens used. The main difference between the load set No.5 and the others is whether the rigid end is employed or not. The measured results in Fig. 4 indicate that any pair of different terminations backing the specimen is not satisfactory in obtaining the acoustic parameters, i.e., characteristic impedance and propagation constants, of the sound absorbing porous material; The result contrasts with the fact that, mathematically, two loads are enough for extracting such acoustic parameters.

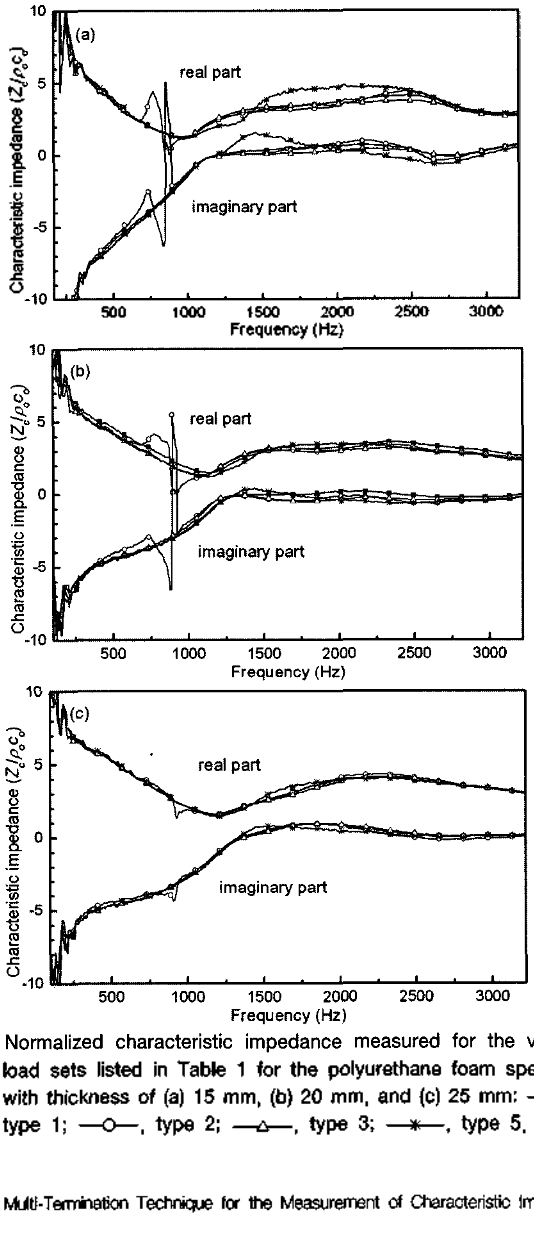

Fig. 4. Normalized characteristic impedance measured for the various load sets listed in Table 1 for the polyurethane foam specimen with thickness of (a) 15 mm, (b) 20 mm, and (c) 25 mm: —■—, type 1; —○—, type 2; —△—, type 3; —✳—, type 5.

IV. Multi-termination method

Figure 5 illustrates the characteristic impedance measured for specimens having different thicknesses, which were obtained by using the load pairs of type Nos. 1 and 3.

For an ideal situation, the results for three identical samples having different thicknesses should be same, but the results suggest that it is not the case, at least for some load combinations. This may be due to the difference in boundary conditions and elasticity of foam materials [16-18]. Moreover, the measured specimens were relatively thin as mentioned before and, thus, there might be a problem arisen from mounting the specimens. It has been known that structural edge constraints have any effect mainly on low frequency range [18] and the

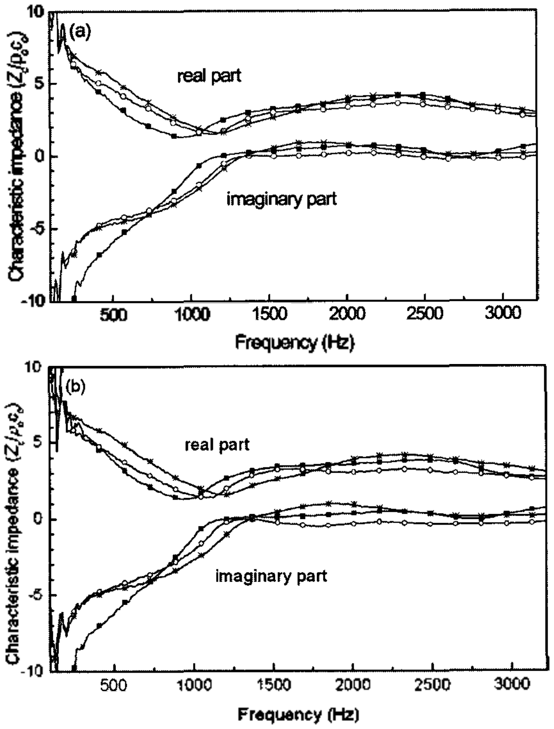

Fig. 5. Normalized characteristic impedance measured for specimens having different thicknesses, L, by using the load pair of (a) type No.1 and (b) type No.3: —■—, L = 15 mm; —○—, L = 20 mm; —✳—, L = 25 mm.

air-gaps around a sample affect high frequency range [16]. In addition to such causes of data deviations, there exists always some random error sources related to the subtle difference in test preparation such as sample mounting method, load exchange process, and so on.

In this study, a multi-termination method was proposed in order to reduce random errors in the measurement for modifying the conventional two-load method. The impedance could be extracted from an over-determined problem by using three or more backing conditions. Using N backing terminations for a specimen, the measured surface-normal impedances are given by Eq. (1) (i = 1 , 2, ⋯ N). Because Eq. (1) is not linear about Zc and γ an over-determination cannot be made directly. In order to linearize Eq.(1), the following changes in variables are required:

$$x \equiv Z_c^2,\quad y \equiv Z_c \cot(\gamma L). \tag{6}$$

Using Eq. (6), Eq. (1) can be rewritten in terms of x and y as

$$x + j(Z_{b,i} - Z_{n,i})y = Z_{b,i}Z_{n,i}.\ (\mathrm{i} = 1,\ 2,\ \cdots\ \mathrm{N}) \tag{7}$$

This can alternatively be expressed in the matrix form as

$$\begin{bmatrix} 1 & j(Z_{b,1} - Z_{n,1}) \\ 1 & j(Z_{b,2} - Z_{n,2}) \\ \vdots & \vdots \\ 1 & j(Z_{b,N} - Z_{n,N}) \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{Bmatrix} Z_{b,1}Z_{n,1} \\ Z_{b,2}Z_{n,2} \\ \vdots \\ Z_{b,N}Z_{n,B} \end{Bmatrix} \text{ or } \mathbf{AX} = \mathbf{B}.$$

(8)

By using the pseudo-inverse of the matrix A, the solution X can be obtained as

$$\mathbf{X} = (\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H\mathbf{B}, \tag{9}$$

where the superscript 'H' denotes the Hermitian of a matrix. The characteristic impedance and the propagation constants can be calculated from Eq. (6) as follows:

$$Z_c = \sqrt{x},\quad \gamma = \cot^{-1}(y/Z_c)/L. \tag{10 a, b}$$

Figure 6 shows the estimated characteristic impedances by using the two cases of multiple terminations: (a) rigid end + sound absorbing termination + closed cavity with 20 mm in depth, and (b) rigid end + closed cavity with 20 mm in depth + closed cavity with 40 mm in depth. Here, the backing conditions were chosen so as to reduce the load exchange process to the minimum, and the rigid termination was included for a reference.

When Fig. 6 is compared with Fig. 5, one can observe that a better agreement has been achieved between the results for 20 and 25-mm-thick samples when multiple terminations were employed. Note that the result for 15-mm-thick specimen still deviates from other results. This implies that a special care is needed in mounting thin samples in an impedance tube. Although only the open-cell PU-foam was tested in this work, it can be said that the same conclusion is available for other general porous materials.

V. Conclusions

The effect of backing conditions on the measurement of basic acoustic parameters of porous absorbing materials in an impedance tube has been investigated experimentally, in which several pairs of generally used end conditions were employed. The usual *two-load technique* yielded the characteristic impedance and propagation constants that similar outputs could be obtained

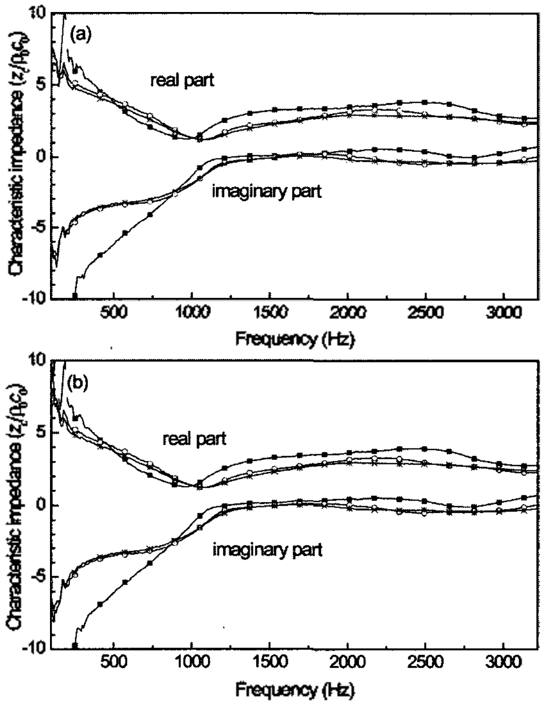

Fig. 6. Normalized characteristic impedance measured by multi-termination method using (a) rigid end + absorbing termination + air-cavity with a depth of 20 mm, and (b) rigid end + air-cavity with a depth of 20 mm + air-cavity with a depth of 40 mm: —■—, L = 15 mm; —○—, L = 20 mm; —∗—, L = 25 mm.

for most pairs of termination conditions except the case of using an open pipe end if its length were not chosen carefully. On the other hand, it was observed that the measured characteristic impedances for a specimen of different thickness, which should be identical, did not agree well with each other. This fact suggests that the repeatability in the impedance measurement is not much affected by the backing condition, but depends strongly, in relative sense, on randomness arisen during the test preparation. In this work, such random errors occurring in relation to the measurement procedures have been investigated and the multi-termination method has been suggested in order to reduce such random errors. By using the three backing conditions instead of two, as a smallest combination example of the method, it was shown that there is a good possibility of improving the agreement among the measured impedance for several samples having different thicknesses, but with an identical material and structure.

Acknowledgment

This work was partly supported by BK21 Project.

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