Failure Probability Assessment of an API 5L X52 Gas Pipeline with a Wall-thinned Section

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Pressurized gas pipelines are subject to harmful effects from both the surrounding environment and the materials passing through them. Reliable assessment procedures, including fracture mechanics analyses, are required to maintain their integrity. Currently, integrity assessments are performed using conventional deterministic approaches, even though there are many uncertainties to hinder rational evaluations. Therefore, in this study, a probabilistic approach was considered for gas pipeline evaluations. The objectives were to estimate the failure probability of a corroded pipeline in the gas and oil industries and to propose limited operating conditions for different types of loadings. To achieve these objectives, a probabilistic assessment program was developed using a reliability index and simulation techniques, and applied to evaluate the failure probabilities of a corroded API-5L-X52 gas pipeline subjected to internal pressures, bending moments, and combined loadings. The results demonstrated the potential of the probabilistic integrity assessment program.

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NOMENCLATURE

d = defect depth (mm)
D_i = inner diameter (mm)
D_o = outer diameter (mm)
E = Young's modulus (MPa)
l_1 = defect length (mm)
l_2 = defect half-length (= $l_1/2$) (mm)
$M_{crit.}$ = critical bending moment (kN-m)
M_L = plastic moment limit (kN-m)
M_{app} = bending moment (kN-m)
M_{ref} = reference normalizing bending moment (kN-m)
M^C_{ref} = reference normalizing combined moment (kN-m)
$M_{\beta=3.0}$ = bending moment limit (kN-m)
$p_{crit.}$ = critical internal pressure (MPa)
p_f = failure pressure (MPa)
P_f = failure probability
p_L = plastic pressure limit (MPa)
p_{app} = internal pressure (MPa)
p_{ref} = reference normalizing internal pressure (MPa)
$p_{\beta=3.0}$ = internal pressure limit (MPa)
R_m = mean radius (mm)
R_o = outer radius (mm)
t = thickness (mm)
β = reliability index
κ = main curvature of limit state
σ_{local} = equivalent stress along minimum ligament (MPa)
σ_u = ultimate tensile strength (MPa)
σ_y = yield strength (MPa)
θ = total defect half-angle (rad)

1. Introduction

Gas transmission and process pipelines must be able to resist external loads as well as internal pressures. However, they are typically operated in deteriorative environments that cause corrosion, pitting, and erosion. Therefore, complicated assessment procedures are required to maintain their integrity, including fracture mechanic analyses. Currently, integrity assessments are conducted using conventional deterministic approaches even though there are many uncertainties that hinder a rational evaluation of the structural components. These uncertainties are related to the loading history, material properties, and failure mechanisms, and are taken into account with engineering safety factors. Thus, the assessment results are generally too conservative because all the relevant uncertainties are accumulated into a unique safety factor.[1-3]

In this respect, probabilistic fracture mechanics (PFM) may be a useful alternative. The PFM approach is an appropriate methodology for providing reasonable evaluations in risk-based decision making for major structural components since it can deal with various uncertainties quantitatively. To date, several PFM methodologies have been developed to provide integrity assessment tools and resolve industrial issues. For example, a computer program based on the SINTAP procedure was developed to calculate failure probabilities when a defect size is obtained from nondestructive testing (NDT) or nondestructive examinations (NDE).[4] A computer code called PSQUIRT was developed to evaluate probabilistic leak rates in nuclear reactor piping for leak-before-break applications.[5] Reliability assessments for pressurized pipelines containing active corrosion defects have also been performed using several failure pressure models.[6] However, all of the previous stochastic approaches have focused only on the internal pressure loading. There are few efficient methodologies applicable to gas pipelines that reflect practical bending moments or combined loading cases consisting of both internal pressures and bending moments.

The purpose of this paper is to estimate the failure probabilities of typical corroded pipelines used for the oil and gas industries and to propose operating limit conditions for different types of loading. To achieve these objectives, a probabilistic assessment program incorporating a first-order reliability method (FORM), second-order reliability method (SORM), and Monte Carlo simulation (MCS) technique was developed and used to estimate the failure probabilities of a gas pipeline subjected to internal pressures, bending moments, and combined loadings. The effects of several limit state functions (LSFs) were also investigated and the extent of the contribution from each variable on the failure probability was examined through sensitivity analyses. Finally, operating limit conditions of a corroded API-5L-X52 gas pipeline were suggested by adopting a reference reliability index.

2. Development of a Probabilistic Assessment Program

2.1 Fundamentals of the Probabilistic Approach

PFM can be used to determine the failure probabilities (P_f) of components by considering the scatter of applied loads, structural geometries, and material properties. The failure behavior of a component is described by the limit state function $g(x)$, which depends on basic random variables $x = (x_1, x_2, \ldots x_n)$ that denote several parameters. By definition, $g(x) < 0$ implies a failure condition, no failure occurs for $g(x) > 0$, and $g(x) = 0$ defines the limit state. The failure probability is obtained by integrating the probability density function (PDF) of the respective basic variables x_i over the region of $g(x) < 0$. [7]

In general, the failure probability is estimated from either a reliability index technique, such as a FORM or SORM, or from a simulation technique, such as a MCS. In a FORM, the LSF is linearized at the design point so that an approximate failure probability can be determined from

$$P_f = \Phi(-\beta) = 1 - \Phi(\beta), \tag{1}$$

where Φ is the cumulative standard normal distribution function and β is the reliability index that represents the minimum distance between the origin of the space of the basic variables and the design point on the failure surface. In a SORM, the failure surface is approximated by a quadratic hyper-surface associated with the curvature of the nonlinear limit state around the minimum distance point. A simple closed-form solution for the probability computation using a second-order approximation is

$$P_f \approx \Phi(-\beta)\prod_{i=1}^{n-1}(1+\beta\kappa_i)^{-1/2}, \tag{2}$$

where κ_i is the ith main curvature of the limit state and the value of $\prod_{i=1}^{n-1}(1+\beta\kappa_i)^{-1/2}$ is a specific SORM term known as the multiplication factor, even though the definitions of Φ and β are the same as those used in FORM. The MCS method can also be used to estimate the failure probability. It generates sets of random variables according to the given probabilistic distributions of the basic variables and inserts them into the LSF. Therefore, the failure probability can be determined from

$$P_f = \lim_{N_{target}\to\infty}\left[\frac{N_{failure}}{N_{target}}\right] \approx \frac{N_{failure}}{N_{target}}, \tag{3}$$

where $N_{failure}$ is the number of simulation cycles when the failure occurred and N_{target} is the total number of simulation cycles.

2.2 Probabilistic Assessment Program

A wall-thinned gas pipeline integrity assessment program based on PFM was developed using Microsoft Visual C++ Version 6. Fig. 1 shows a flowchart of the developed program, which incorporates reliability index and MCS techniques. The failure probability of wall-thinned gas pipelines can be assessed using this program. In particular, input variable transformation, iteration, and numerical analysis functions are incorporated in the FORM and SORM modules while random number generation, probability distribution generation, and evaluation functions are included in the MCS module. In general, a set of input data, including material properties such as yield and ultimate tensile strengths, defect and pipe geometries associated with the corresponding PDF type, as well as means (μ) and standard deviations (σ) are required for integrity evaluations under deterministic loading conditions. The corresponding coefficient of variation (CV), which is a function of μ and σ, is then determined automatically. An appropriate replication number and a sufficient number of simulations are also required to control the MCS and obtain reliable results. The calculated failure probability is returned with a reliability index in the FORM module and a multiplication factor in the SORM module.

3. Probabilistic Assessment of Gas Pipelines

3.1 Determination of the PDF

The PDFs related to the defect shape, pipeline geometry, and material properties must be determined prior to performing a probabilistic assessment of corroded gas pipelines. Due to a lack of actual field data, most of the PDF μ values as well as the CV of the probabilistic variables were obtained from reference sources[6]. Table 1 indicates the principal probabilistic variables as well as the deterministic variables for the applied loading conditions. Thirty years of operation were assumed when determining the appropriate defect shape mean values, reflecting a defect depth, length, and angle

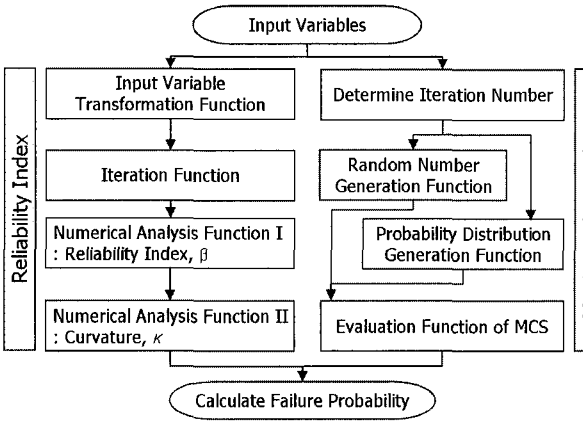


Fig. 1 Flowchart of the wall-thinned gas pipeline assessment program

Table 1 Input variables of the API-5L-X52 gas pipeline used for the PFM analysis

Variable	μ	CV
Defect depth, d (mm)	3.0*	0.1
Defect length, l_2 (mm)	150*	0.1
Defect angle, θ/π	0.055*	0.1
Outer diameter, D_o (mm)	914.4	0.02
Thickness, t (mm)	20.6	0.02
Yield strength, σ_y (MPa)	358.0	0.07
Ultimate tensile strength, σ_u (MPa)	455.0	0.07
Internal pressure, p_{app} (MPa)	15.6–27.3	-
Bending moment, M_{app} (kN-m)	3500–6500	-

*Defect geometry considering 30 years of operation growth rate of 0.1 mm/year, 5.0 mm/year, and 0.018 rad/year, respectively.

3.2 Limit State Functions

A series of probabilistic integrity assessments for a corroded gas pipeline were conducted using the three evaluation methods under different loading conditions, which included internal pressures, bending moments, and combined loadings. The corresponding LSFs were constructed based on the deterministic failure criteria proposed by the modified B31G,[8] the PCORRC,[9] and Kim *et al.*[10,11]

The LSF obtained using the modified B31G (MB31G) equation, which was essentially derived from a plastic limit analysis for cylinders with parabolic shaped cracks under pure internal pressure conditions, can be expressed as

$$g(x_i) = p_f - p_{app}, \tag{4}$$

where p_f is the failure pressure and p_{app} is the internal pressure. For the common situation, the failure pressure is

$$p_f = (\sigma_y + 68.95\,\mathrm{MPa})\frac{2t}{D_i}\left(\frac{1-0.85\frac{d}{t}}{1-0.85\frac{d}{t}M^{-1}}\right), \tag{5}$$

where

$$M = \left(1+0.6275\frac{l_1^2}{D_i t} - 0.003375\frac{l_1^4}{D_i^2 t^2}\right)^{0.5} \quad for\ \frac{l_1^2}{D_i t} \le 50 \tag{6}$$

$$M = 0.032\frac{l_1^2}{D_i t} + 3.3 \quad for\ \frac{l_1^2}{D_i t} > 50\,. \tag{7}$$

In the above equations, σ_y is the yield strength, D_i is the inner diameter, t is the wall thickness, d is the defect depth, and l_1 is the defect length. The LSF obtained using the PCORRC equation, which resulted from curve fitting finite element analysis-based plastic limit solutions for axial flaws with constant depth, takes the same form as Eq. (4). The failure pressure can be represented as

$$p_f = \sigma_u \frac{2t}{D_i}\left(1-\frac{d}{t}M\right), \tag{8}$$

where

$$M = 1-\exp\left(-0.157\frac{l_1}{\sqrt{R_o(t-d)}}\right). \tag{9}$$

The LSF based on Kim's estimation equations, which were derived from a local stress at the deepest point in a semi-elliptically wall-thinned area, is

$$g(x_i) = \sigma_u - \sigma_{local}, \tag{10}$$

where σ_{local} is the equivalent stress at the deepest point of a wall-thinned pipeline subjected to internal pressures, bending moments, and combined loadings.[10,11]

In the case of pure internal pressure, hoop and axial stresses are the only components of the principal stresses due to the thin-wall approximation. Thus, the equivalent stress in the minimum ligament is

$$\sigma_{local}^{P} = \frac{p_{app}}{(p_{ref}/\sigma_y)}, \tag{11}$$

where

$$\frac{p_{ref}}{\sigma_y} = \frac{t}{R_m}\frac{1}{\sqrt{A^2-AB+B^2}} \tag{12}$$

$$A = \frac{1}{(1-\frac{d}{t}+\frac{d}{t}\frac{1}{\phi})};\ B = \frac{R_i}{2R_m};\ \phi = \sqrt{\frac{1.61l_2^2}{R_i d}}\,. \tag{13}$$

In the case of pure bending moments, the limit-load solution derived from the equilibrium stress fields is

$$\sigma_{local}^{M} = \frac{M_{app}}{(M_{ref}/\sigma_y)}, \tag{14}$$

where

$$M_{ref} = \frac{M_L}{1.333};\ M_L = 4\sigma_y R_m^2 t[\cos(\frac{\pi}{8}\frac{d}{t}\theta) - \frac{d}{t}\frac{f(\theta)}{2\theta}] \tag{15}$$

$$f(\theta) = 0.785\theta^2 - 0.0981\theta^4 + 0.004090\theta^6 - 0.000085\theta^8\,. \tag{16}$$

For a combined loading, the equivalent stress (σ_{local}^{p+M}) in the minimum ligament can be expressed from Eqs. (11) and (14):

$$\sigma_{local}^{p+M} = \sqrt{\left(\frac{p_{app}R_i}{2t} - \frac{M_{app}}{M_{ref}^C/\sigma_y}\right)^2 - \left(\frac{p_{app}R_i}{2t} - \frac{M_{app}}{M_{ref}^C/\sigma_y}\right)\left(\frac{p_{app}}{p_L/\sigma_y}\right) + \left(\frac{p_{app}}{p_L/\sigma_y}\right)^2}, \tag{17}$$

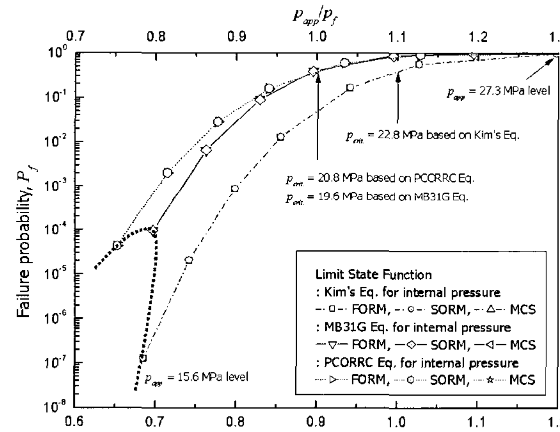
where

$$p_L = \frac{\sigma_y t}{R_m}\left(1-\frac{d}{t}+\frac{d}{t}\frac{1}{\varphi}\right);\ \varphi = \sqrt{1+\frac{1.61l_2^2}{R_i t}} \tag{18}$$

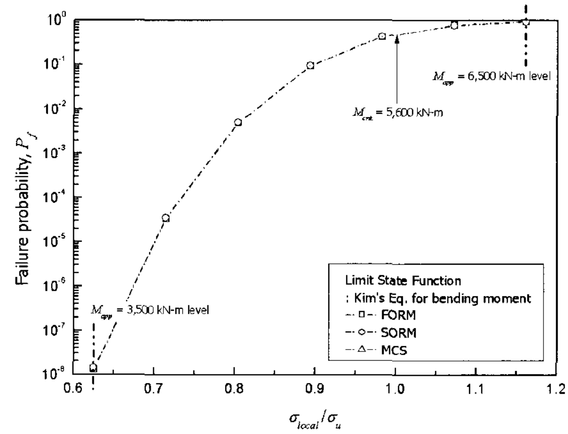
$$M_{ref}^C = 3R_m^2 t\sigma_y\left[\cos\left(\frac{\pi d\theta}{8t} + \frac{\pi p_{app}R_m}{4t\sigma_y}\right) - \frac{d}{t}\frac{f(\theta)}{2\theta}\right]. \tag{19}$$

3.3 Failure Probability Assessment

The failure probabilities of representative corroded gas pipelines were calculated using the aforementioned LSFs and relevant probabilistic variables. Fig. 2 shows the assessment results for a gas pipeline with d = 3.0 mm subjected to internal pressures (p_{app}) and bending moments (M_{app}). For generalization purposes, the failure probabilities were represented in terms of the governing



(a) Internal pressure



(b) Bending moment

Fig. 2 Failure probability results for a gas pipeline with d = 3.0 mm parameters normalized by the corresponding failure criteria, such as σ_{local}/σ_u or p_{app}/p_f.

For the cases with pure internal pressures shown in Fig. 2(a) (15.6 MPa $\leq p_{app} \leq$ 27.3 MPa), the failure probabilities were estimated using LSFs based on the MB31G and PCORRC standards, as well as Kim's equations. Kim's equations gave a critical pressure ($p_{crit.}$) of 22.8 MPa when the estimated local equivalent stress (σ_{local}) in the minimum ligament of the pipeline equaled the ultimate tensile strength (σ_u). The critical pressures calculated from the MB31G and PCORRC equations were 9 and 14% lower than that obtained from Kim's equations. In addition, the failure probability was less than 10^{-8}, regardless of the LSF, for a typical pipeline operating pressure of 7.8 MPa. [6]

For the cases with pure bending moments shown in Fig. 2(b) (3500 kN-m $\leq M_{app,\ X52} \leq$ 6500 kN-m), the critical bending moment ($M_{crit.}$) was defined as the estimated equivalent stress of the pipeline equal to the ultimate tensile strength obtained from Kim's equations. By adopting this criterion, the critical bending moment and corresponding failure probability of a corroded gas pipeline with d = 3.0 mm were 5600 kN-m and 0.5, respectively.

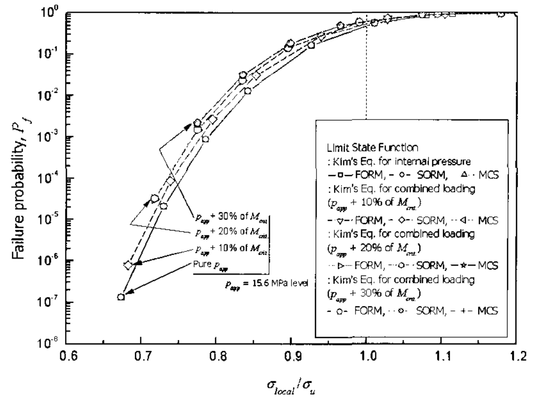
Fig. 3 compares the failure probability assessment results obtained under combined loading conditions with those obtained under pure loading conditions for a gas pipeline with d = 3.0 mm. The corresponding failure probabilities increased significantly from 0 to 30% of the predefined critical values calculated by Kim's equations while the extent values predicted by the three assessment methods remained almost the same. The failure probabilities at the 15.6 MPa level increased by 491 to 6747% for each 10% increment of $M_{crit.}$ due to the applied bending moment, which ranged from 0 to 30% of the critical value of the original pure internal pressure condition, as shown in Fig. 3(a). The failure probabilities at the 3500 kN-m level increased by 455 to 5657% for each 10% increment of p_{crit} due to the internal pressure, which ranged from 0 to 30% of the critical value of the original pure bending condition, as shown in Fig. 3(b). Therefore, the effect of the bending moment was not negligible in the corroded gas pipeline under combined loading conditions.

3.4 Sensitivity Analysis

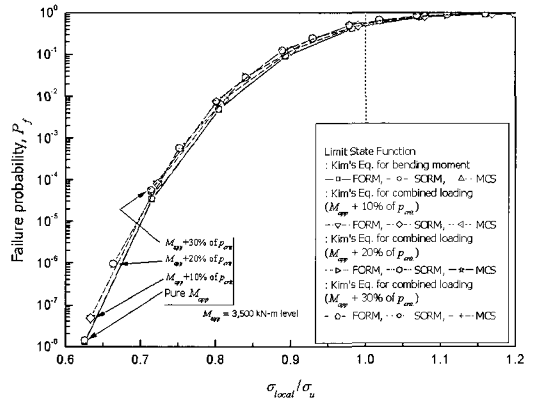
Sensitivity analyses are necessary to confirm the reliability of the assessment results and to measure the uncertainties of probabilistic input variables that may affect the failure probability. As a representative loading condition, sensitivity analyses for a gas pipeline subjected to pure internal pressures were performed using Kim's estimation equation as the LSF. The five essential probabilistic variables indicated in Eqs. (10)–(13) were selected and the effects of the CV (= σ/μ) variation were analyzed for each variable.

Fig. 4 depicts the effects of the CVs on the failure probabilities for a corroded gas pipeline with –25, 25, and 50% variation from its original values. Since the estimated failure probabilities are dependent on the level of the internal pressure, the following description focuses on a gas pipeline at the 15.6 MPa level as a representative pressure for comparison purposes. In Figs. 4(a) and (b), even though the CVs of the defect depth and half-length varied from 0.075 to 0.15, the failure probabilities rarely changed. Fig. 4(c) shows the effect of different outer diameter distributions for which the failure probabilities increased by 50 to 70% with each variation of the CV. The effect of the different thickness distributions on the failure probability is shown in Fig. 4(d). By changing the CVs from 0.015 to 0.03, the failure probabilities increased by 70 to 110% for each variation. Finally, the effect of different ultimate tensile strength distributions is shown in Fig. 4(e). The failure probabilities increased by approximately 1,400% for each variation as the CV changed from 0.0525 to 0.105. Thus, the distribution of key variables such as the ultimate tensile strength and defect depth played an important role in the probabilistic integrity assessments, even when the same mean values were considered.

3.5 Operating Limit Conditions of a Gas Pipeline

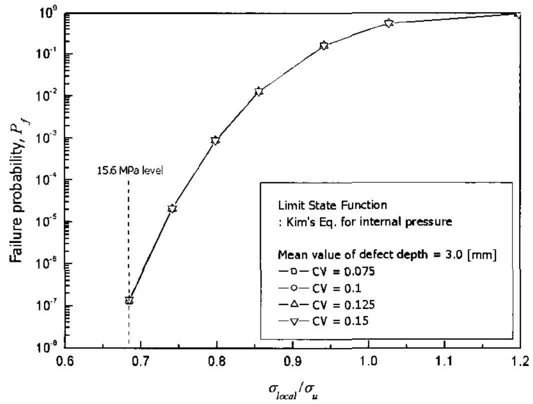


(a) Internal pressure vs. combined loading

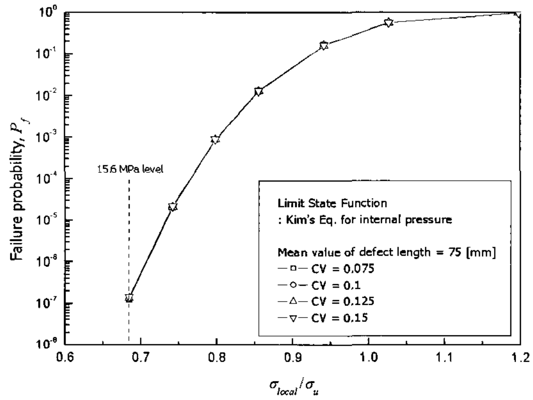


(b) Bending moment vs. combined loading

Fig. 3 Comparison of failure probability evaluation results for a gas pipeline with d = 3.0 mm

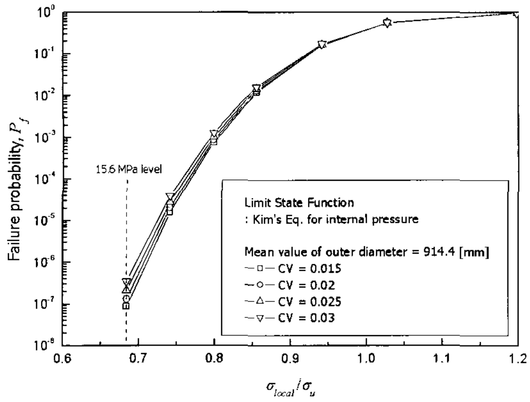


(a) Defect depth

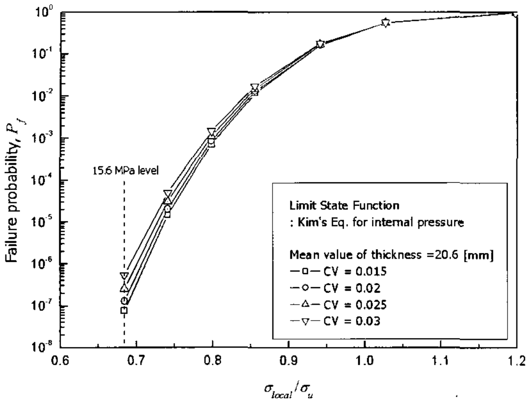


(b) Defect length

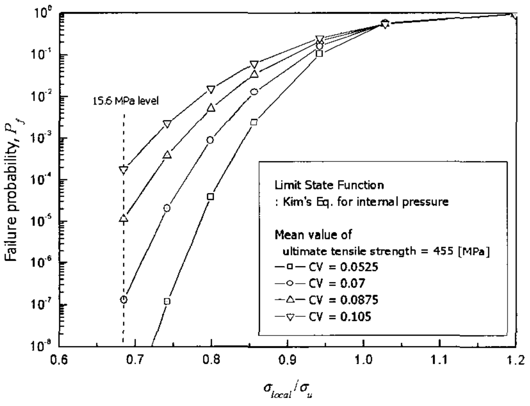
Fig. 4 Effect of probabilistic variables on the failure probability of a gas pipeline subjected to internal pressure



(c) Outer diameter



(d) Thickness

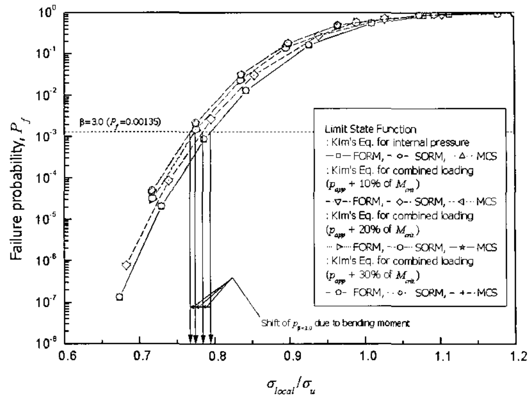


(e) Ultimate tensile strength

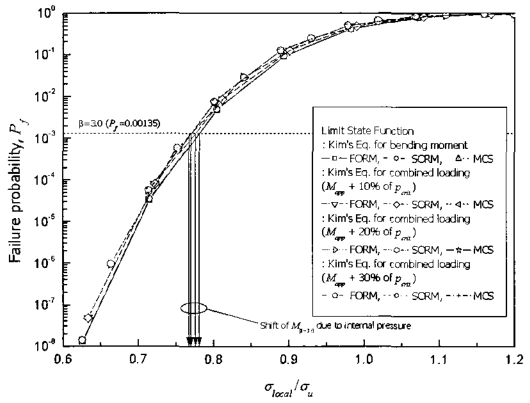
Fig. 4 (continued)

Faber *et al.*[12] proposed several target values that can be used as a reliability index to describe the limit state of major industrial facilities. Based on this previous research, we chose a factor of three (β = 3.0, P_f = 0.00135) as the reference reliability index of a corroded gas pipeline for which the cost to measure defects was high but the consequences of failure were low. The index was used to determine the operating limit conditions of the pipeline under three different loading conditions using Kim's equations.

Fig. 5(a) gives the operating limit conditions for a gas pipeline with d = 3.0 mm subjected to pure internal pressures and combined loadings. The internal pressure limit ($p_{\beta=3.0}$) of the pipeline shifted from 18.4 to 15.4 MPa as additional bending moment was applied; the limit of the internal pressure decreased by 2 to 9% for every 10% increment of M_{crit} from 0 to 30% of the critical moment due to combined loading effect. Fig. 5(b) shows the proposed operating limits for conditions subjected to pure bending moments and combined loadings at the reference reliability index level. For pure bending, the bending moment limit ($M_{\beta=3.0}$) was 4400 kN-m, corresponding to 78% of the critical moment. The bending moment limit decreased by 2 to 7% for each 10% increment of $p_{crit.}$ between 0

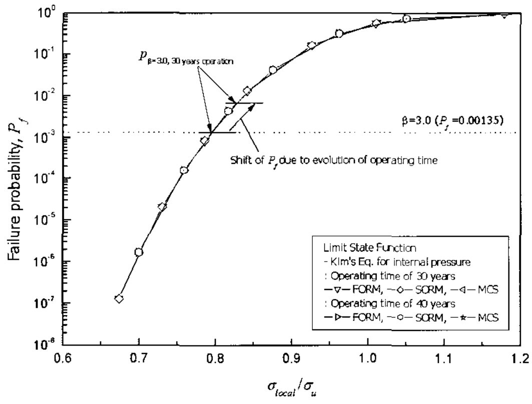


(a) Internal pressure vs. combined loading

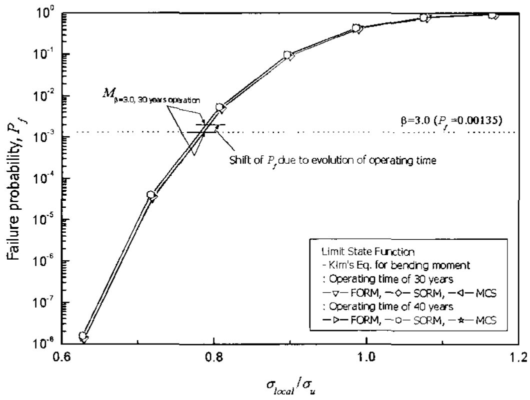


(b) Bending moment vs. combined loading

Fig. 5 Operating limit conditions of combined loadings for a gas pipeline with d = 3.0 mm



(a) Internal pressure



(b) Bending moment

Fig. 6 Comparison of failure probability results for a gas pipeline subjected to internal pressures for different operating times and 30% of the critical pressure.

Therefore, operating limit conditions of 79% of the critical pressure for pure internal pressure loading, 77% of the critical moment for pure bending moment loading, and 65% of the critical pressure or moment for a combined loading are recommended for the gas pipeline. Even though experimental or practical verification is required, it is anticipated that the integrity of a corroded API-5L-X52 gas pipeline will be ensured by applying these operating limit conditions.

However, it is expected that the defects in a gas pipeline will grow with time and that these will influence the failure probability. Fig. 6 compares the operating limit conditions for a gas pipeline subjected to pure internal pressures and bending moments after 30 and 40 years of operation time. The defect shapes were determined from their depth, length, and angle growth rates, and the failure probabilities were estimated using Kim's equations at the reference reliability index. As depicted in Fig. 6(a), by increasing the operating time from 30 to 40 years, the failure probability increased from 1.35×10^{-3} to 6.8×10^{-3} at $p_{\beta=3.0,\ 30\ years\ operation}$. The internal pressure limit decreased from 18.4 to 17.7 MPa, which is still considerably greater than typical operating pressures of 7.8 MPa. As shown in Fig. 6(b), the failure probability increased from 1.35×10^{-3} to 2.03×10^{-3} at $M_{\beta=3.0,\ 30\ years\ operation}$ and the bending moment limit decreased from 4400 to 4350 kN-m, corresponding to 78% of the critical moment. Therefore, even though the level of the operating limit conditions decreased with time, it is anticipated that this concept can be successfully used for realistic applications by incorporating suitable experimental or practical verifications.

4. Conclusions

A probabilistic structural integrity assessment program was developed and applied to estimate the failure probabilities of a corroded API-5L-X52 gas pipeline. The following conclusions were drawn from this study.

(1) The effect of different LSFs on the failure probability was examined for a corroded gas pipeline subjected to pure internal pressures. Among these LSFs, Kim's equations provided the lowest estimates of failure probabilities.

(2) A combined loading condition should be considered to practically evaluate the integrity of a corroded gas pipeline instead of simple pure internal pressure conditions currently described by the MB31G and PCORRC standards since the effect of the bending moments was not negligible in these tests.

(3) The sensitivity analyses demonstrated that the most important variable affecting the failure probability of a corroded gas pipeline was the ultimate tensile strength. This trend was different from that reported for wall-thinned nuclear reactor pipes in which the defect depth and ultimate tensile strength were the major probabilistic variables.

(4) Operating limit conditions of a corroded gas pipeline subjected to internal pressures, bending moments, and combined loadings were proposed after adopting a reference reliability index of 3.0. These results showed promise for practical applications.

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