# An Adaptive Tracking Controller for Vibration Reduction of Flexible Manipulator

## Yoon-Gyeoung Sung<sup>1,#</sup> and Kyu-Tae Lee<sup>1</sup>

1 Dept. of Mechanical Engineering, Chosun University, Gwangju, South Korea # E-mail: sungyg@chosun.ac.kr, TEL: +82-62-230-7181, FAX: +82-62-230-7055

KEYWORDS: Sliding-mode control, Input shaping technique, Least square method, Tracking control, Flexible manipulator

An adaptive tracking controller is presented for the vibration reduction of flexible manipulator employed in hazardous area by combining input shaping technique with sliding-mode control. The combined approach appears to be robust in the presence of severe disturbance and unknown parameter which will be estimated by least-square method in real time. In a maneuver strategy, it is found that a hybrid trajectory with a combination of low-frequency mode and rigid-body mode results in better performance and is more efficient than the traditional rigid-body trajectory alone which many researchers have employed. The feasibility of the adaptive tracking control approach is demonstrated by applying it to the simplified model of robot system. For the applications of the proposed technique to realistic systems, several requirements are discussed such as control stability and large system order resulted from finite element modeling.

Manuscript received: July 29, 2005 / Accepted: January 12, 2006

## 1. Introduction

Numerous researches have been carried out for controlling flexible manipulator in space, industry, and radioactive facilities. There have been many approaches in control methodology such as  $H^{\infty}$  control, adaptive control, optimal control, and so on. <sup>1,8</sup> However, they are mainly focused on a maneuver based on rigid-body-like operation without allowing flexibility on the system. Since the 1980s, control systems have become more sophisticated due to considerations of flexibility in addition to rigid-body dynamics. Of course, it is natural for flexible manipulator to adopt the rigid-body maneuver concept of satellite systems in 1970s. As a result, many of the flexible systems have suffered from vibration during operations even if the maneuver is intended to be a rigid-body operation. This is a motivation to present an alternative approach to reduce the vibration of flexible manipulator to be applied to the high radiation areas.

The objective of reducing the overall vibration and fuel consumption is accomplished while allowing vibration in the system during the operation, especially in the application of a point-to-point maneuver. In other words, the system vibration is utilized in the operation rather than attempting to completely eliminate the vibration. As an example of this operation, a simplified model is utilized for the evaluations with respect to nonlinear damping, uncertain parameters, and external disturbances. In order to develop a robust controller, Sung and Wander<sup>6</sup> evaluated the input shaping technique of Singer and Seering<sup>5</sup> with a linear system model. The performance was remarkable in the rotating maneuver. However, real systems will be exposed to non-ideal effects such unmodeled dynamics, parameter uncertainties, time-varying parameters, high dimensionality, etc. It is not expected that the input shaping technique alone can handle the

non-ideal effects. Therefore, an adaptive tracking control approach is presented by combining the input shaping technique with sliding-mode control to reduce the residual vibration. By taking this approach we attempt to reduce the residual vibration and fuel consumption at the same time.

The organization of this article is as follows. In the next section, the adaptive tracking controller is developed with a simplified model by including parameter estimation. Then, the input shaping technique is briefly discussed as a feedforward control so that the system will experience a natural vibration during a maneuver. By changing the parameters with a nonlinear damping element<sup>7</sup> in the model, several cases of numerical simulation are conducted with respect to nonlinear damping, unknown parameters and sinusoidal disturbance. Following this are numerical results for comparisons of non-ideal effects and the conclusion is presented.

### 2. Flexible Manipulator Modeling

In hazardous area, a flexible manipulator<sup>2</sup> on mobile robot is utilized to retrieve radioactive small-scale waste material and to reach an object in distance as in Fig. 1. In order to develop advanced robotic technologies accurately to perform given tasks, the system is simplified as a two-mass-damped-spring model for the application of input shaping technique. In the ideal case,<sup>6</sup> the input shaping technique shows excellent performance. However, the performance must be tested in a non-ideal case. A simple generic model is shown in Fig. 2 and is used to illustrate the control concept and methodologies.

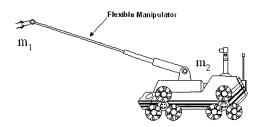


Fig. 1 Mobile robot with flexible manipulator

The equations of motion of Fig. 2 are

$$m_1\ddot{x}_1 + D(\dot{\hat{x}}, \hat{x})\dot{\hat{x}} + k\hat{x} = u + w$$
 (1)

$$m_2\ddot{x}_2 + D(\dot{\hat{x}}, \hat{x})\dot{\hat{x}} + k\hat{x} = 0$$
 (2)

where  $\hat{x} = x_1 - x_2$ .  $x_1$  and  $x_2$  are the position of mass,  $m_1$  and mass  $m_2$  respectively. u and w are control command and external disturbance. This system can also be represented in state space form as

$$\dot{x} = Ax + Bu + w \tag{3}$$

where

$$x = [x_1, x_2, x_3, x_4]^T$$

Numerous experimental results, such as those in the Spacecraft Control Laboratory Experiment (SCOLE),<sup>7</sup> indicate that proportional damping assumptions are not sufficient and that there is a great need for understanding the damping mechanism that may be inherently nonlinear. Various nonlinear models, such as linear dampers with clearance, coulomb friction dampers, velocity-nth power damping, and so forth, have been investigated. The following damping model <sup>7</sup> is representative of a variety of nonlinear damping mechanisms formed as

$$D(\dot{\hat{x}}, \hat{x}) = c \left| \dot{\hat{x}} \right|^a \left| \hat{x} \right|^b \tag{4}$$

where C is a constant, and a, b > 0.

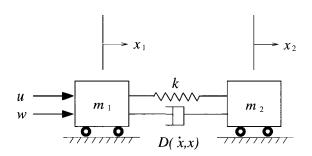


Fig. 2 Simplified model with disturbance

#### 3. Sliding-Mode Tracking Controller

Consider the coupled single-command system in Fig. 2.

$$\ddot{x}_{1} = \frac{1}{m} \left( -D(\dot{\hat{x}}, \hat{x})\dot{\hat{x}} - k\hat{x} + u + w \right) \tag{5}$$

$$\ddot{x}_2 = \frac{1}{m_2} (-D(\dot{\hat{x}}, \hat{x})\dot{\hat{x}} - k\hat{x})$$
 (6)

where the true nonlinear damping term  $D(\hat{x}, \hat{x})\hat{x}$  is estimated as  $\hat{c} \hat{x}$  where  $\hat{c}$  is a constant with

$$\left| \hat{c}\dot{\hat{x}} - D\left(\dot{\hat{x}}, \hat{x}\right)\dot{\hat{x}} \right| \le \left| \dot{\hat{x}} \right| = F \tag{7}$$

We define the sliding plane  $s_1(x,t)$ , namely

$$s_1(x,t) = 0$$
 with  $s_1 = \dot{\widetilde{x}}_1 + \lambda \widetilde{x}_1$  (8)

where  $\widetilde{x}_1 = x_1 - x_{d1}$  is tracking error. To satisfy a sliding condition, we define a tracking control law u as

$$u = k\hat{x} + \hat{c}\dot{\hat{x}} + m_1(\ddot{x}_{d1} - \lambda x) - K\operatorname{sgn}(s)$$
(9)

where  $\operatorname{sgn}(s) = -1$  for s < 0 and  $\operatorname{sgn}(s) = +1$  for s > 0 and  $K = \eta + F$  and a strictly positive constant  $\eta$ . Of course, we might have a chattering problem so that the concept of boundary layer with saturation function<sup>5</sup> could be employed. Several techniques are found in Mostafa and Oz <sup>3</sup> for more advanced chattering reduction.

#### 4. Input Shaping Technique

Singer and Seering developed the technique by employing the impulse response of a linear second-order damped system. More detailed descriptions can be found in reference. Here, the result of a two-impulse sequence is presented in order to shape a command. By selecting  $t_1=0$  for the time of the first impulse, and  $A_1=1$  for the amplitude, the simple solution can be obtained

$$A_2 = e^{-\zeta \omega \Delta T}, \quad \Delta T = \frac{\pi}{\sqrt{1 - \zeta^2}} \tag{10}$$

The previous result is an example of a one vibrational mode. As in Fig. 3, the combined response by two impulse is shown with no residual vibration. However, the impulse sequence can also be used to deal with higher modes (see the reference<sup>5</sup> for more detailed descriptions). If a two-impulse sequence is designed for each of the two modes of a closed-loop system, respectively, they can be convolved to form a sequence which moves this two-mode system without residual vibration. In order to generate arbitrary commands that make the same motion without vibration, any arbitrary desired commands to the system can be obtained by convolution with the impulse sequences.

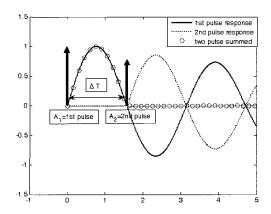


Fig. 3 Two impulse response

#### 5. Parameter Estimation

For the preliminary investigation of the combined control approach (input shaping technique and sliding-mode control), a parameter estimation simulation is performed to see how well the

approach works. Furthermore, it is necessary to evaluate the sensitivity of the combined approach with respect to a time-varying parameter. Parameter estimation based on least-square of errors with exponential forgetting<sup>5</sup> is employed to deal with time-varying parameters in a real time process. If exponential forgetting of data is incorporated into least-square estimation, one minimizes

$$J = \int_0^t e^{-\int_s^t \mu(r) dr} |y(s) - W(s)\hat{a}(t)|^2 ds$$
 (11)

where vector y(s) contains the outputs of the system, the vector  $\hat{a}$  is unknown parameters, W(s) is a signal matrix, and  $\mu(t) \ge 0$  is a time-varying forgetting factor. The update law is

$$\dot{\hat{a}} = -PW^T e_1 \tag{12}$$

where  $e_1 = \hat{y} - y$  and  $\hat{y} = \hat{a}W^T$  is the predicted output at time t. For simplicity, the nonlinear damping term is assumed to be known in constant stiffness estimation so that the effect of unknown stiffness can be evaluated with respect to the flexible maneuver trajectory.

For the simplified model with unknown scalar stiffness  $\hat{a} = k$ , each element can be written as

$$y = m_1 \ddot{x}_1 - m_2 \ddot{x}_2 - u + 2c |\dot{x}_1 - \dot{x}_2|^a |x_1 - x_2|^b (\dot{x}_1 - \dot{x}_2)$$
(13)

and

$$W^T = -2(x_1 - x_2)$$

the scalar gain update

$$\dot{P} = -\mu P - P W^T W P \tag{14}$$

In this implementation, the forgetting factor is given as

$$\mu = \mu_0 \left( 1 - \frac{|P|}{k_0} \right) \tag{15}$$

where  $\mu_0$  and  $k_0$  are positive constants.

# 6. Numerical Simulation

In the simulation,  $u(t) = 2\sin^2 \Omega t$  is an command chosen to be like the simulation of the point-to-point maneuver where  $\Omega = 1 \, rad \, / \, sec$ . The parameter values for nonlinear damping of Eq. (4) are a=2, b=1 and c=0.01. It is taken that  $m_1=m_2=1$  and k=1 with appropriate units and time is in units of seconds.

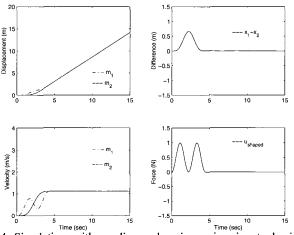


Fig. 4 Simulation with nonlinear damping using input shaping technique

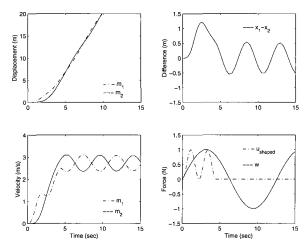


Fig. 5 Simulation with nonlinear damping and disturbance using input shaping technique

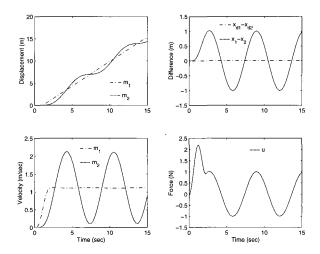


Fig. 6 Simulation with nonlinear damping using input shaping technique and sliding-mode control

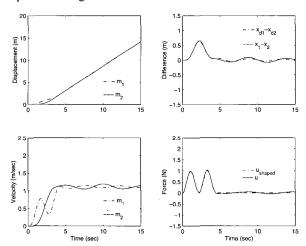


Fig. 7 Simulation with nonlinear damping and disturbance using input shaping technique and sliding-mode control

The control force u(t) acts on mass  $m_1$ . An external disturbance<sup>1</sup>,  $w(t) = \sin(0.5t)$  is acting on mass  $m_1$ . In the simulations, phase uncertainty is not considered. Two impulses are used for the input shaping technique because of the demands of real-time application.

First, the input shaping technique is tested with respect to nonlinear damping and external disturbance. The two-impulse sequence is used under the consideration of adaptive control implementation. In Fig. 4, the results show that the input shaping technique cannot treat the nonlinear damping effect perfectly, unlike the linear problem. There is some residual vibration after the end of

the operation. In Fig. 5, the sinusoidal disturbance is included. It shows that the input shaping technique can not handle the disturbance whatsoever with large oscillation in both bodies. In the previous two simulations, the performance of the input shaping technique is investigated with respect to non-ideal effects. It implies that input shaping technique alone cannot be used for non-ideal problems since it has the nature of an open-loop control.

Second, in order to treat non-ideal effects such as unmodeled dynamics, system uncertainties, and others, it is typical to employ a feedback control technique. Hence, sliding-mode control is used to increase robustness and performance. Sliding-mode control is incorporated with the input shaping technique. The approach is different from popular methods in the sense of tracking the states of the linear flexible mode as a strategy using minimum energy with respect to the desired command. Results from simulation which combines the input shaping command of Fig. 5 with the sliding-mode control of Eq. (9) is shown in Fig. 6. It is shown that mass  $m_1$  tracks the desired path, but mass  $m_2$  is vibrating slightly. In this simulation, the sliding-mode controller is on all the time after the end of motion so that both bodies are vibrating lightly after 20 seconds. We could stop the controller in the case of internal disturbances in order to not generate an adverse effect. However, it is promising that mass  $m_1$ is controlled. In Fig. 7, the external disturbance is included to see how the approach works. Some control exists for both bodies even with the sinusoidal disturbance present, unlike the input shaping technique alone in Fig. 5. After ending the move, the controller is properly counteracting the disturbance. In Fig. 8, a 20% parameter error in stiffness is provided without the sinusoidal disturbance in order to see the effect of the modeling error. It shows that even though mass  $m_1$  closely tracks the desired path, mass  $m_2$  oscillates severely. It seems that the sliding-mode control is rejecting the interactions between the two bodies after 5 seconds. The simulation indicates that the precise system model is required.

Third, a parameter estimator is implemented in order to cope with modeling error. All state information is assumed to be available in this preliminary simulation. During on-line operation, a parameter in the input shaping technique and sliding-mode control is updated. The approach is different from other methods in the aspect of adaptive incorporation of input shaping technique and sliding-mode control. With a 50% error in the initial stiffness estimate, three cases are compared with regard to rigid, flexible and combination trajectories. In the last simulation, all non-ideal effects are considered with the proposed control algorithm. In Fig. 9, a rigid-body trajectory based on the kinematics of two bodies is used. The results are more

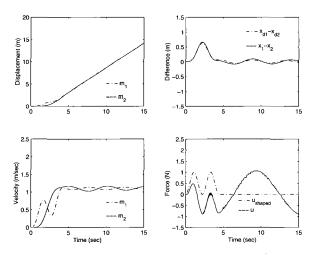


Fig. 8 Simulation with nonlinear damping and 20% modeling error in stiffness using input shaping technique and sliding-mode control

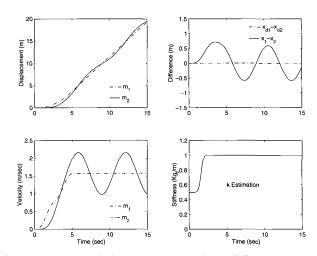


Fig. 9 Rigid-body Path: input shaping technique, sliding-mode control and estimation with nonlinear damping and 50% error in stiffness

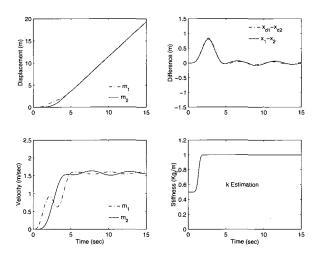


Fig. 10 Flexible Path: input shaping technique, sliding-mode control, and estimation with nonlinear damping and 50% error in stiffness

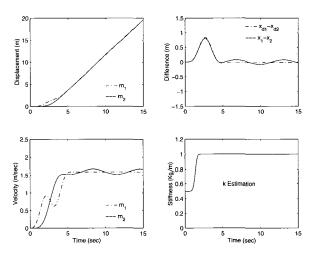


Fig. 11 Combination Path: input shaping technique, sliding-mode control, and estimation with nonlinear damping and 50% error in stiffness

desirable than those of Fig. 7. Mass  $m_1$  tracks the desired path. On the other hand, mass  $m_2$  has large overshoot and residual vibration. It seems that a rigid-body path is not appropriate in the control of the flexible manipulator.

A flexible-body trajectory based on the frequency of the system is implemented as shown in Fig. 10. It shows that the results are much better than those of the rigid-body path with small overshoot and

small residual vibration. However, the reference model has an oscillation in the plot of relative distance after 5 sec since the model is continuously updated, unlike the fixed reference model of the previous cases. In Fig. 11, a hybrid trajectory separated into a flexible-body path and a rigid-body path is used to eliminate the residual oscillation of Fig. 10. By using the path synthesis, no residual vibration on mass  $m_1$  is achieved. However, some oscillation of mass  $m_2$  exsits because of the non-collocation of actuator and sensor

In the parameter estimation of the adaptive cases, the convergence of the estimator must be reasonably fast so that the second location of impulse sequence can be properly decided. As long as the time-varying parameter is asymptotically approaching the true value, combined method appears to be realized. Among the three trajectories, the hybrid path could be selected if a stabilizer is provided to mass  $m_2$ . Through the evaluations, the adaptive tracking approach appears to be feasible to cope with disturbances, modeling error, and nonlinear effect.

#### 7. Conclusions

The vibration reduction approach of flexible manipulator on mobile robot in hazardous area is proposed by devising an adaptive tracking control method. First of all, the input shaping technique is tested under realistic situations. As expected, the input shaping technique is very sensitive with respect to plant variation such as parameter uncertainty. Moreover, the method may not be successfully used in the presence of disturbances.

As an approach for efficient operation, an adaptive tracking controller is presented for the control of flexible manipulator. The feasibility is confirmed by the combined implementation of the input shaping technique and sliding-mode control as long as a system is properly modeled. The combined approach appears to be robust in the presence of severe disturbances. In a maneuver strategy, it is found that a hybrid trajectory which is the combination of a low-frequency mode and a rigid-body mode results in better performance than the rigid-body trajectory alone.

As indicated in the implementation for a one-dimensional flexible manipulator on the mobile platform, the combined technique can reduce the vibration of the flexible manipulator as well as achieve the attitude control of a rigid-body at the same time. However, a stabilizing ability is required to completely eliminate the residual vibration in applications of the tracking control concept to realistic multi-dimensional flexible manipulator. In a realistic system, the control system also faces issues of enormous large system order and multi-input multi-output (MIMO) problem unlike the one-dimensional system. In addition, the combined technique is expected to provide an efficient approach to reduce vibration of flexible structure for point-to-point problems with non-ideal effects.

#### **ACKNOWLEDGEMENT**

This study was supported(in part) by research funds from Chosun University, 2004.

#### REFERENCES

- 1. Wei, B., Liu, Q. and Byun, K. W., "Robust  $H^{\infty}$  Control Synthesis Method and Its Application to Benchmark Problems," Journal of Guidance, Control and Dynamics, Vol. 15, No. 5, pp. 1140-1148, 1992.
- Carteret, B. A., "Light Duty Utility Arm System Development Plans and Applications," American Nuclear Society Seattle, Washington, WHC-SA-2669-FP, 1996.

- Mostafa, O. and Oz, H., "Chatter Elimination in Variable Structure Control Maneuvering of Flexible Spacecraft," Journal of the Astronautical Sciences, Vol. 37, No. 4, pp. 529-550, 1989.
- Singer, N. C. and Seering, W. P., "Preshaping Input Commands to Reduce System Vibration," Journal of Dynamic Systems, Measurement and Control, Vol. 112, pp. 76-82, 1990.
- Slotine, J. J. E. and Li, W., "Applied Nonlinear Control," Prentice-Hall Inc., New Jersey, pp. 358-382, 1991.
- Sung, Y. G. and Wander, J. P., "Applications of Vibration Reduction of Flexible Space Structure Using Input Shaping Technique," Proceedings of Southeastern IEEE Conference, Athens, OH, pp. 333-336, 1993.
- Taylor, L., "4th Annual SCOLE Workshop," NASA TM-101503, 1987
- Kapila, V., Tzes, A. and Yan, Q., "Closed-loop Input Shaping for Flexible Structures Using Time-Delay Control," Journal of Dynamic Systems, Measurement and Control, Vol. 122, pp. 454-460, 2000.