

A Method of Evaluating the Spatial Difference between Two Numerical Surfaces

Jung-Eun Lee* and Yukio Sadahiro**

두 개의 수치 평면에 대한 공간적 차이의 측정 방법

이정은* · 사다히로 유키오**

Abstract : Surface data generally represent continuous distribution of geographical or social phenomena of a region in urban analysis. Instances include distribution of temperature, population of region, and various distributions related to human activities. When spatial data are given in the form of surface, surface comparison is required as a way of comprehending the surface change or the relationship between two surfaces. As for previous approaches of surface comparison, there are visualization, quantitative methods and qualitative method. All those approaches, however, show the difference between two surfaces in a limited way. Especially, they are not able to distinguish spatial difference between two surfaces. To overcome such problem, this paper proposes a method of comparing two surfaces in terms of their spatial structure. Main concept of the method comes from earth moving problem and the method is named minimum surface transformation, here. When a surface is transformed into another, total surface volume moved in the process of transformation should be the minimum. Both quantitative and spatial differences between two surfaces are evaluated by total surface volume moved and the distribution of moved surface volume of each cell respectively. The method is applied to hypothetical and actual data. From the former, it is understood that the method explains how two surfaces are quantitatively and spatially different. The result of the latter shows that moved total surface volume decreases as time goes by which fits the actual situation that population change rate gets smaller. Concerning the other measure of surface difference, the distribution of X_{ij} describes detailed flow of surface volume than that of simply subtracting surface volume by indicating to what direction the population change occurs.

Key Words : surface comparison, spatial difference, earth moving, minimum surface transformation

요약 : 도시분석에서 평면 데이터는 지리학적 혹은 사회적 현상의 연속적인 분포를 표현하는데 사용된다. 예를들면, 어떤 지역의 온도분포, 인구밀도 등의 다양한 인문활동 및 자연현상이 평면 데이터로 표현될 수 있다. 앞서 언급된 현상들이 평면 데이터 형식으로 주어졌을 때, 두 평면간의 비교를 통해 평면간의 상호관계 혹은 평면간의 차이를 이해할 수 있게된다. 평면간의 비교 방법에 의한 기존 연구들은 시각화기법, 정량적 방법 및 정성적 방법이 존재한다. 하지만, 이들 방법은 평면간의 차이를 국한된 측면에서 구분한다는 제한점이 있다. 특히, 평면간의 공간적 특성에 의한 차이점은 명확하게 구분하지 못한다. 이와같은 문제점을 해결하기위해, 본 연구에서는 평면의 공간적 특성에 입각한 차이점을 구분해내는 비교방법을 제안하고자 한다. 평면 비교 방법론의 기본 개념은 토목공학에서 주로 사용되는 토량계산 문제에 근거하고 있으며, 본 연구에서는 이를 변형하여, 최소 평면 변환으로 부르기로 한다. 최소 평면 변환의 목적은 서로 다른 두 평면이 있을 때, 한 평면을 다른 평면으로 변환시키고, 그 과정에서 발생하는 평면값의 총 이동량을 최소화함에 있다. 평면의 변환과정에서 발생하는 평면값의 이동의 총합 및 레스터 형식으로 표현된 평면의 각 셀에서 발생한 이동 평면값을 통해, 평면간의 정량적 및 공간적 차이를 구분할 수가 있다. 제안된 방법은 가상 데이터 및 실제데이터에 적용되었다.

* JSPS Research Fellow, Department of Urban Engineering, University of Tokyo, nalalee@ua.t.u-tokyo.ac.jp

** Associate Professor, Department of Urban Engineering, University of Tokyo, sada@ua.t.u-tokyo.ac.jp

가상 데이터를 적용한 결과, 제안된 방법으로 두 평면 데이터 간의 정량적 차이 및 공간적 차이가 구분됨을 확인할 수 있었다. 또한, 실제 데이터를 적용한 결과에서 볼 때, 평면 변환과정에서 얻어진 평면값의 총 이동량이 시간대별로 점차 작아짐을 알 수가 있었다. 이는 실제 분석대상 지역의 인구밀도 변화율이 점차 줄어드는 사실을 반영하는 결과로, 제안된 방법을 통해 얻어진 결과가 실제 상황을 제대로 기술하고 있음이 파악되었다. 또한, 제안된 방법론에서 제시된 평면 데이터간의 공간적 차이를 구분하기 위한 기준인 평면변환 과정에서 발생한 평면값의 이동 분포를 통하여, 단순히 두 평면값을 각 셀 단위에서 뺄셈을 한 경우의 분포보다 두 평면 데이터간의 평면값 이동의 흐름이 보다 자세히 기술되어, 인구이동의 흐름이 대상지역에서 어느 방향으로 발생되었는지를 알 수 있다.

주요어 : 평면 비교, 공간적 차이, 토량 계산 문제, 최소 평면 변환

1. Introduction

From the geographical point of view, surface data is treated as one of spatial objects depicting not only various human activities that occur in the region but environmental phenomena as well. For instance, distribution of temperature or population of region, CO₂ emission and relationship between market area potential and the population of the area can be described as surface data. Such examples are often represented in continuous format.

However surface data are represented in 2D format being divided by a lattice which gives us simplified approach for surface data analysis. When such surface data are visualized on GIS, it is easier to understand how the data is spatially distributed and other related factors affect such distribution. It is also possible to compare several surface data to see how the distributions are different or how it has been changed by time, in addition.

Thus, surface comparison is an approach for further understanding of information represented by surface data. Surface comparison can be performed from various aspects depending on the focus of comparison. Even though each approach has different point of view, the main concept of surface comparison lies on distinguishing similarity or difference between two

surfaces. As for approaches for surface comparison, there are qualitative, quantitative and visualization approaches.

Qualitative characteristics of surface data are considering topological structures of surface (Okabe and Masuda 1984). Concerning such feature, the concept of surface network is used which indicates the connection among pit, col and peak of surface (Pfaltz 1976). This concept is modified and developed by Flower and Little (1979) and Wolf (1984). Since this approach is effective to show structural feature of surface data, it is able to compare surfaces on topological aspect.

However, it is not always effective where two surfaces have similar structure, since same surface network cannot distinguish basic transformation of surface that keeps the same spatial configuration.

In figure 1, three different surfaces and their surface networks are indicated. Even though the surfaces are not the same in their configuration, surface network is the same for all cases. This shows the limit of surface network described above that it is not able to distinguish and measure transformation cost and spatial difference among surfaces.

Concerning quantitative approaches of surface comparison, there are studies of using measures of Pearson's correlation coefficient, Kappa index

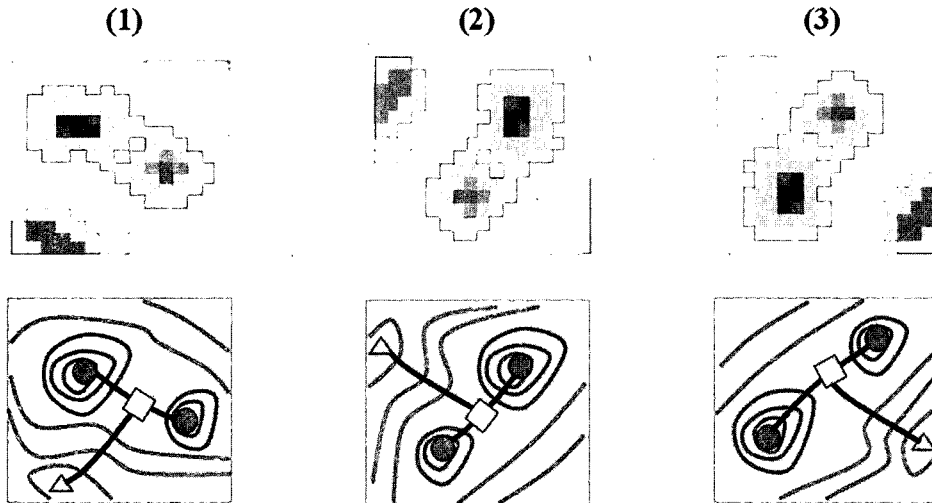


Figure 1. Examples of surface network of three surfaces with different spatial configuration

of agreement (Congalton and Mead 1983; Rosenfield and Fitzpatrick-Lins 1986; Pontius et al. 2003; Trosset 2005) and Kullback-Leibler information number (Okabe and Sadahiro 1994). Pearson’s correlation coefficient is used to calculate the similarity between two samples that are linearly related. Kappa index of agreement is a measure that is mainly used in the field of remote sensing to compare two images to find whether their differences are due to chance or real disagreement or agreement. As for Kullback-Leibler information number, it assesses the similarity between two surfaces where they are given in probability density function form. Quantitative methods only distinguish the differences of surfaces at local level and show weakness distinguishing difference where spatial configuration is different. As described in table 1, quantitative methods cannot distinguish the difference of surface transformation for instance.

Visualization is a way of comparing two surfaces as they are displayed. It is the simplest way to compare surfaces if the difference is

Table 1 Result of surface comparison

	(1) · (2)	(1) · (3)
Pearson’s Correlation Coefficient	-0.1250	-0.1250
Kappa Index of Agreement	-0.0015	-0.0015
Kullback-Leibler Information Number	N/A	N/A
Surface Transformation (Rotation)	1	2

* (1), (2), (3) represents the surface described in figure 1.

easily distinguished. As the technology of GIS is developed more and more, distinguishing surface difference by visualization becomes easier. However, this approach has a limit to figure out spatial difference between two surfaces unless the difference is obviously represented.

This becomes critical where the volume of data becomes larger. It is only possible to analyze surface difference at simple level. In figure 2, there are two different surface data showing the distribution of population in a region. Except circled area, it is practically hard to distinguish

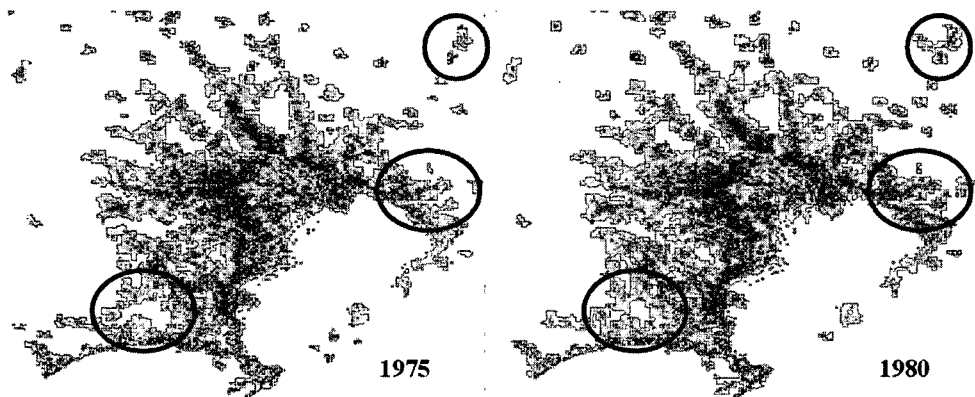


Figure 2. Comparing two different surface data by visualization

the difference between them.

If we consider surface comparison from a different aspect, it is possible to classify by the type of surface data, categorical and numerical surface data. For example, categorical map is used for surface comparison for evaluating quantification error versus location error in the study of Pontius (2000), Pontius and Schneider (2001) and Pontius et al (2003). Similar to this, Hagen (2003) and Hagen et al. (2005) also treat categorical map as surface data and differences are distinguished by fuzzy set approach. However, few methods are studied for numerical surface data comparison.

Therefore, in this study, we are trying to evaluate the surface difference in the context of their difference in spatial configuration by proposing a measure which can distinguish both quantitative and spatial difference. Since a type of surface data treated in this study is numerical, this study has a meaning in analyzing spatial difference of numerical surface especially.

In the next chapter, the concept of methodology and the procedure of it showing how it is applied in this study are explained. Then, method application to hypothetical data and

actual data is discussed with evaluated results respectively in chapter 3. At last, in chapter 4, this paper is concluded with summarized contents of this study.

2. Methodology

In the previous chapter, it is understood that existing methods have limits for distinguishing surface difference. For visualization approach, detailed difference cannot be easily found. Difference in transformation cost between two surfaces cannot be distinguished by using previous quantitative methods. Considering qualitative approach, difference in transformation cost and spatial configuration of surfaces are not distinguished. To overcome these limits, therefore, the goal of this research is to measure and compare the difference of two surface data in terms of their spatial configuration with effective method. Here, it is expected that comparing two surfaces in the context of not only quantitative difference but spatial differences as well is possible with proposed measure.

1) Preconditions for surface data before comparison

Before surface data are compared by the method of surface transformation, they have to be arranged for calculation. First precondition is that two surface data should have the same total volume to be comparable. For this reason, standardization process is required if two surface data have different total surface volume. Secondly, two different surfaces should be divided by the same lattice. By satisfying these two preconditions, two surface data are now ready to be compared.

2) Methodology

Basic concept of the methodology comes from the problem of earth moving which is mostly used in the field of civil engineering (Murai and Obayashi 1973; Bernold and Halpin 1984; Easa 1988; Wada et al. 1991; Nakagawa et al. 1994). This concept is a problem of transporting amount of soil from one location to another with minimum cost of hauling distance and amount of soil. In this problem, important components are hauling distance and amount of soil to be transported. Of course, there are other factors to be considered when this concept is applied to real construction field. From such aspect, in the previous studies of earth moving, Wada et al. (1991) tried earth moving simulation considering actual factors of types of earth layer and changing rate of earth that may occur during the work. Nakagawa et al. (1994) focus on the methodology of solving earth moving problem by using fuzzy theory since there exist many complicated factors for earth moving in the real world situation. Due to such reason, it is

important to calculate the amount of earth, which would be cut and filled during earth moving, very close to the real value according to the shape of ground (Easa 1988; Davis et al. 1981).

Even though there are many factors to consider in earth moving due to the verified configuration of earth, only the basic concept of earth moving is adopted here. The method that is proposed in this study is named minimum surface transformation. Definition of the method is that minimized volume of the surface should be moved when a surface is stretched or compressed to fit the other in the process of transformation by cell unit of given surface data. However, there is an assumption that hauling distance is set to be constant so that the movement of surface volume is limited to adjacent cells of each cell only.

Since the sum of moved surface volume indicates the difference in spatial distance between two surface data, by using this measure, it would be possible to classify surface data and understand the spatial factors explicitly that affect the difference of spatial data, which plays an important role in regional analysis if the method is further applied. Furthermore, with those concepts mentioned above, detailed analysis to which direction the surface volume moves would be possible. This will give us a new understanding of spatial difference in surface comparison.

The sum of moved surface volume calculated in the process of surface transformation would be classified according to its value scale for comparison; small, large and similar values. These values correspond to the degree of similarity or difference between two surfaces. Smaller the value is, closer the distance between two surfaces.

If it is supposed that there are two surfaces

named S_k and S_m in discrete form, then, minimum surface transformation from S_k into S_m by a cell unit is performed as described in figure 3. For instance, transformation is performed from cell s_{ik} into s_{im} , then, transformed surface volume, $trans_s_{ik}$ would be calculated by adding all surface volume to the value of s_{ik} which moves into it, and subtracting all surface volume from the value of s_{ik} that moves out from it. Then the value of $trans_s_{ik}$ would be the same as that of s_{im} .

$$S_k = \{s_{1k}, s_{2k}, \dots, s_{nk}\}, S_m = \{s_{1m}, s_{2m}, \dots, s_{nm}\}$$

$$trans_s_{ik} = s_{ik} - \sum_{j \in ADJ(i)} X_{ij_out} + \sum_{j \in ADJ(i)} X_{ji_in}$$

$$= s_{im} \quad (1)$$

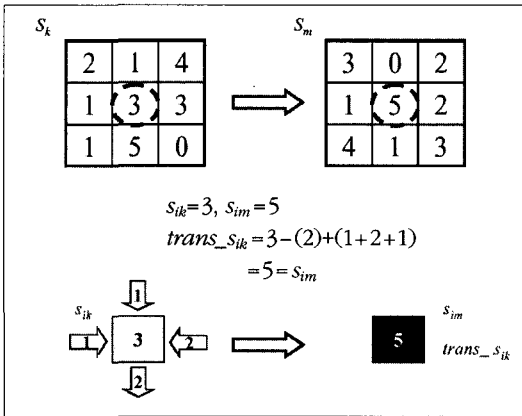


Figure 3. A process of minimum surface transformation

The object function and constraints of the method are defined as following.

$$\min \sum X_{ij} \quad (2)$$

$$trans_s_{ik} = s_{ik} - \sum_{j \in ADJ(i)} X_{ij} + \sum_{j \in ADJ(i)} X_{ji} \quad (3)$$

$$s.t. X_{ij} \geq 0, trans_s_{ik} = s_{im}$$

X_{ij} : surface volume moving from cell i to cell j
 $j \in ADJ(i)$: adjacency set where cell i and cell j are adjacent

$\sum_{j \in ADJ(i)} X_{ij}$: sum of surface volume moving out from cell i ($\sum X_{ij_out}$)
 $\sum_{j \in ADJ(i)} X_{ji}$: sum of surface volume moving into cell i ($\sum X_{ji_in}$)

From the definition of the model described above, the sum of moved surface volume indicates the quantitative difference between two surfaces. Concerning spatial difference between two surfaces, it is shown by the flow of each surface volume. The model is optimization problem and can be solved by a simple linear programming approach.

3. Method Applications

1) Application to hypothetical data

The method is applied to hypothetical data for the evaluation first. Four types of surfaces are generated with variations in the number and location of peaks which are determined randomly. Details of data are described in figure 4. Each surface consists of 25 cells and the total surface volume is equal to one. For each type of surface, the number of peak is differently given. For surface 1, there is no peak and the surface is flat with the value of each cell as 0.04. Surface 2 has only one peak with the value 1.0 and the rest of cells have no surface volume. In Surface 3, there are two peaks in the surface and each has the surface volume 0.5. Surface 4 is having three peaks of which the value is 0.33.

Surface transformation is performed in six different cases; transformation from surface 1 to 2, 1 to 3, 1 to 4, 2 to 3, 2 to 4, and 3 to 4. The

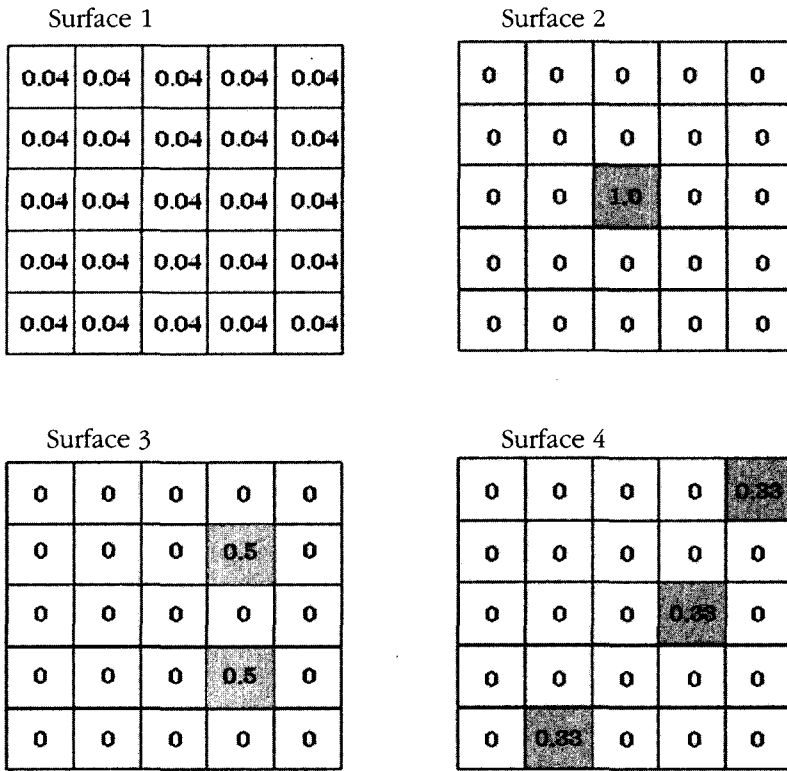


Figure 4. Surfaces with different number of peaks

sum of moved surface volume in the process of surface transformation notated as X_{ij} for each case of transformation is summarized in table 2. The value of $\sum X_{ij}$ is the smallest in the case of transformation from surface 1 into 4. This gives us a finding that surface 1 and 4 is the closest pair among others.

From these results, it is understood that proposed method gives a measure of closeness between two surfaces being compared, meaning that the value gets smaller when original surface is compared to smoother surface where the number of peaks is increased. Among four surfaces given in the example, then, it is possible to say that surface 1 and 4 are the most similar surfaces, in other words.

To see the movement of surface volume in

Table 2 The result of surface transformation

Surface Transformation Case	From 1 to 2	From 1 to 3	From 1 to 4
$\sum X_{ij}$	2.40	2.00	1.80
Surface Transformation Case	From 2 to 3	From 2 to 4	From 3 to 4
$\sum X_{ij}$	2.00	2.67	1.99

terms of spatial structure for surface transformation, the value of X_{ij} for each cell is visualized as shown in figure 5 and figure 6. Surfaces described in figure 5 indicate the cases where surface 1, which is flat, is transformed into surface 2, 3 and 4 respectively. In figure 6, it shows the cases where surface 2 and 3, which have number of peaks, are transformed into

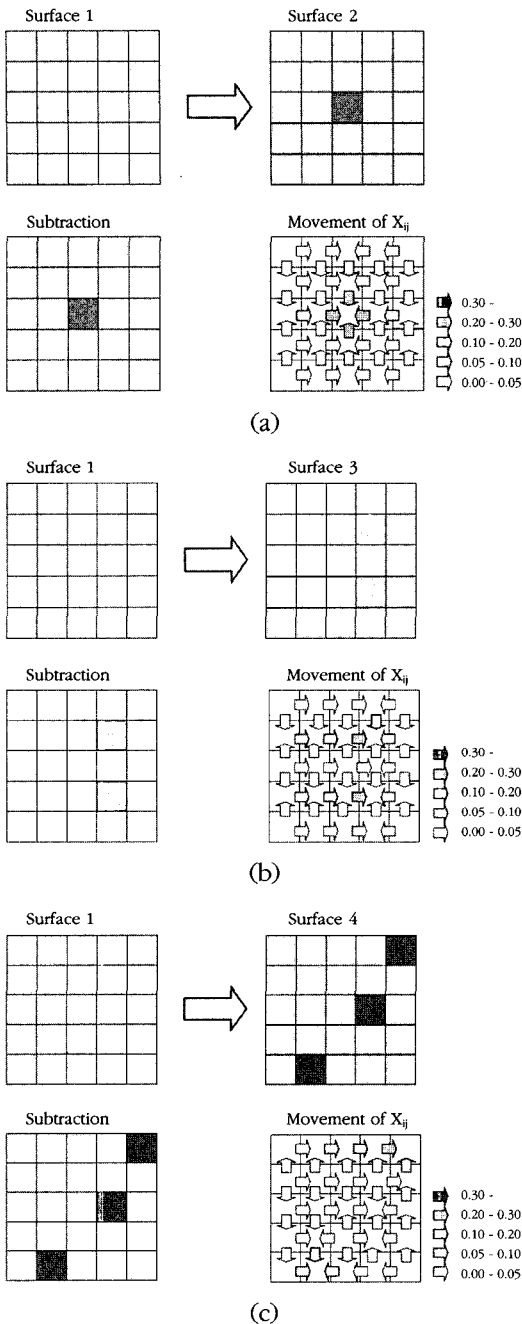


Figure 5. Visualization of X_{ij} movement for each cell where surface 1 is transformed into surface 2, 3, 4

other surfaces.

From figure 5, the movement of X_{ij} value of cells shows that larger value tends to move

toward cells located near the peak of goal surface into which the original surface is transformed, in general. For other cells located in far distance from the peak of goal surface, the distribution of moved X_{ij} value depends on the shape of the original surface as well. Where surface 1 is the original one, since it has no peak and its shape is flat as shown in figure 5, the distribution of movement of X_{ij} shows larger value near the peak of goal surface. In other words, surface volume is moved toward the peak of goal surface. To see the effectiveness of proposed method, distribution of X_{ij} is compared to that of subtraction of surface volume at each cell. As described in figure 5, it is clear that the distribution of subtraction only shows the difference of each cell while that of X_{ij} describes how two surfaces are spatially different by indicating surface volume movement.

However, where the original surface has number of peaks such as surface 2 and 3, the movement of X_{ij} seems to have the characteristics of both original and goal surface which is different from the case depicted in figure 5. For instance, where surface 2 is transformed into surface 3, surface 3 can be obtained by dividing the surface volume of the peak of surface 2 into two parts. Therefore, movement of X_{ij} is moved from the peak of surface 2 to those of surface 3 as shown in figure 6. This shows that surface volume is moved toward the peak of goal surface and that of original surface is moved out from the peak of original surface for transformation. When the distribution of X_{ij} movement is compared with that of subtraction, different from figure 5, it seems that two distributions look relatively similar. Main reason for such result is that surface volume of cells is zero which is not the peak of surface. This indicates that surface

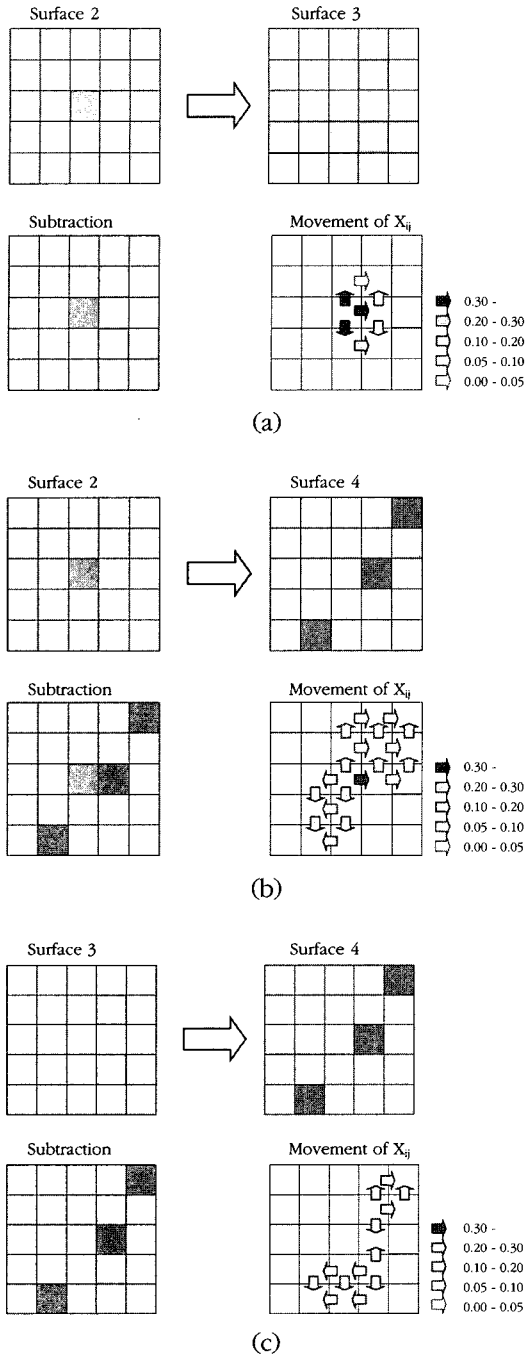


Figure 6. Visualization of X_{ij} movement for cell where surface 2 and 3 are transformed into

volume movement occurs at locations around the peaks of two surfaces in such case.

From the analyzed results described above, it can be found that proposed method shows the spatial difference between two surfaces both in local and global aspects in detail than the distribution obtained by surface volume subtraction. Therefore, proposed method shows that it is effective for distinguishing spatial difference between two surfaces. According to the result described in table 2, the total of moved surface volume between surface 1 - 3 and 2 - 3 are the same of which the value is 2.0. However, these two cases have different spatial difference as shown in figure 5 (b) and 6 (a) respectively.

2) Application to actual data

Proposed method is evaluated with hypothetical data for its effectiveness in previous section. From the result, it is understood that the method distinguishes both quantitative and spatial difference of surfaces from the measures of $\sum X_{ij}$ and movement of X_{ij} . For the quantitative difference, the model calculates the minimum value of $\sum X_{ij}$ which shows that the result obtained is reasonable. In addition, the movement of X_{ij} shows how two surfaces are spatially different in detail even though the quantitative difference is the same. Where the transformation is performed from flat surface to the other that of having a number of peaks, surface volume moves gradually to the direction of the peak of goal surface. When transformation is performed between surfaces having a number of cells, surface volume is spread at the location of original surface and move toward that of goal surfaces. In addition, movement of X_{ij} shows how two surfaces are spatially different by representing the movement of surface volume occurred during surface transformation.

For method application to actual data, the data of population distribution in Saitama and Tochigi prefecture located in north from Tokyo, Japan, from 1975 to 1990 with 5-year interval are used as source.

The source, Census data of Japan, is provided in mesh data format every 5 year with approximately 300 fields including items such as total population, population by age group. The data used in this study is given in 500m mesh format. Three pairs of surface comparison are performed for method evaluation; years of 1975-1980, 1980-1985 and 1985-1990.

The first pair, comparing the population distribution between 1975 and 1980, the first measure indicating the degree of surface difference, the sum of X_{ij} is calculated and the value is 18.84. From the original surfaces, several locations with obvious difference can be found. However, it is not easy to see detailed information on how these two surfaces are different in terms of their spatial configuration.

To analyze the surface difference in detail, the distribution of subtraction from one surface volume to the other and that of X_{ij} movement is calculated respectively and compared. From the figure of subtraction, left hand at the bottom of figure 7, it can be seen that population tends to change from center to boundaries at global scale. Such fact also can be found from the distribution of X_{ij} , right hand at the bottom of figure 7. However, the distribution of X_{ij} shows the surface change in more detailed way. The distribution of subtraction only shows the surface difference at each location in local scale and approximate difference in global scale, while the movement of X_{ij} shows the direction of surface volume movement in continuous and smoother way. The population tends to change from area

of blue colored cells (X_{ij_OUT} value of the legend) to red colored cells (X_{ij_IN} value of the legend) as shown in figure 7.

As for the second pair, the population distribution between 1980 and 1985, the first measure which indicates the degree of surface difference, sum of X_{ij} is calculated with the value of 16.49. The value of X_{ij} is smaller than that of previous pair and this indicates that there is less change in the distribution of population. Therefore, it is not easy to distinguish the obvious difference between two surfaces. However, if we look and compare the distribution of subtraction and that of X_{ij} movement, the difference can be easily found between two surfaces.

As shown in figure 8, the difference between the distribution of subtraction and that of X_{ij} is not clear at global scale. However, at local scale, the distribution of X_{ij} , right hand at the bottom of figure 8, shows more information than that of subtraction from two aspects. The first one is that there is no cell in some locations where the subtraction value is very close to zero. From this fact, it is understood that the movement of X_{ij} shows essential information of surface volume movement. Second, colored cells represent detailed direction of surface volume movement, population changes from blue colored cells

(X_{ij_OUT} value of the legend) to red (X_{ij_IN} value of the legend) colored cells. The movement is shown in relatively more continuous and smoother way.

The third pair, the population distribution between 1985 and 1990, the sum of X_{ij} is calculated with the value of 13.33 which is the smallest among three cases. From this result, it is understood that original surfaces are the pair of the closest in the context of their surface volume. In other words, the change in population is the

least during 1985 and 1990. Even though obvious differences between two surfaces are not able to be detected easily, since two surfaces have the least difference at quantitative level, the difference between the distribution of subtraction and that of X_{ij} movement is quite clear in this case which is shown in the bottom of figure 9. These two

distributions have different patterns in the distribution at global scale as shown in figure 9. The change of distribution of population is clearly appeared in the distribution of X_{ij} movement. The population is aggregated densely into the middle area of the region showing three clear clusters. However, such pattern is not

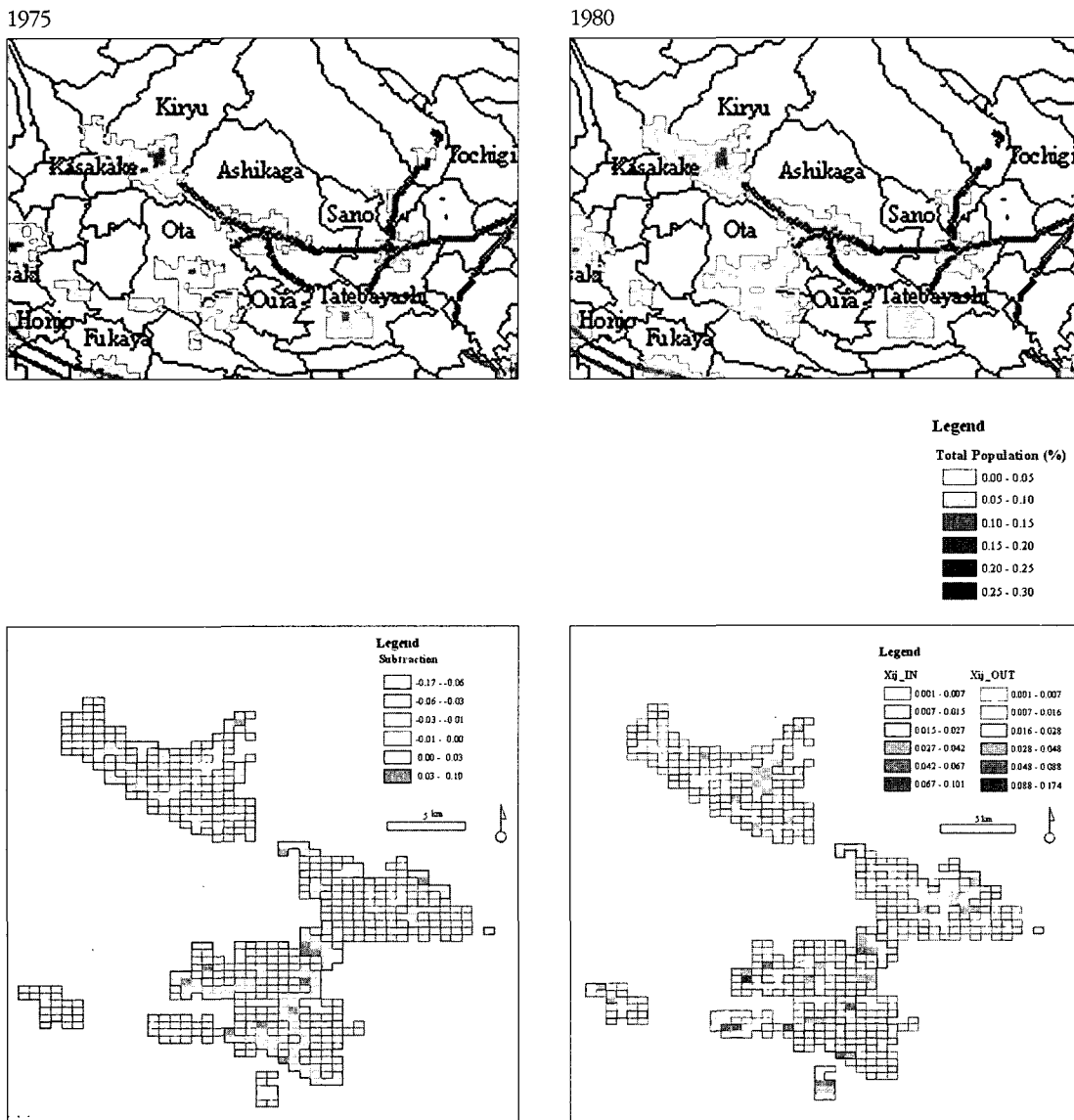


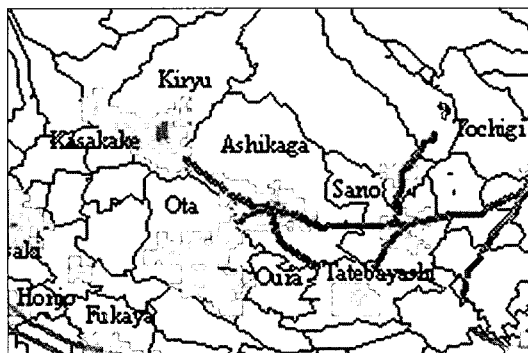
Figure 7. Original surfaces of population distribution of 1975 and 1980 (above) Distribution of subtraction (left bottom) and X_{ij} movements (right bottom)

clearly indicated in the distribution of subtraction.

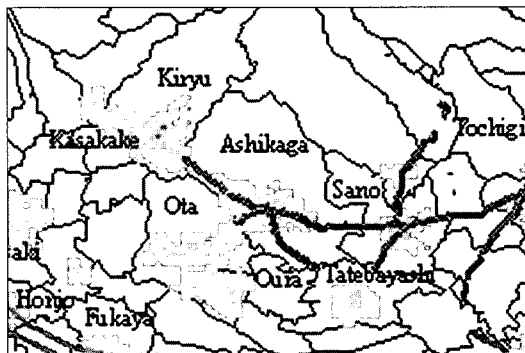
To evaluate the performance of the proposed method, the obtained result by actual data analysis is compared with actual situation that is understood from the data. Concerning the total population of each period, it is known that the change rate of the population decreases as time goes by. From the surface transformation, the

obtained value of $\sum X_{ij}$ which is the optimized one for each period decreases as well showing that the result fits the real situation of population change. The distribution of X_{ij} value describes detailed information of surface volume change. It shows not only the distribution of surface volume subtraction but the flow of surface volume as well.

1980



1985



Legend

Total Population (%)

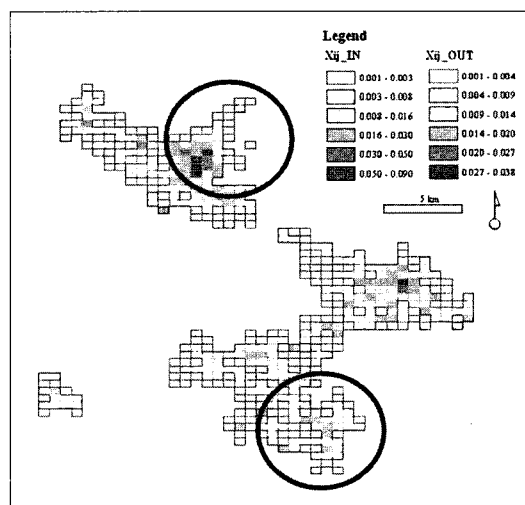
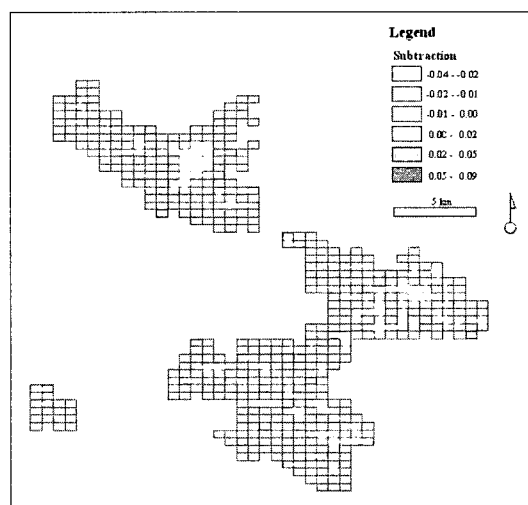
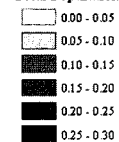


Figure 8. Original surfaces of population distribution 1980 and 1985 (above) Distribution of subtraction (left bottom) and X_{ij} movements (right bottom)

quantitative and spatial difference between two surfaces by measures of sum and movement of surface volume that is occurred during the process of surface transformation. As a method of transformation, the concept is based on the problem of earth moving and it is modified and named minimum surface transformation. The goal of the method in this study is to transform one surface into the other with the minimum movement of surface volume. As a constraint for transformation, which is different from the concept of earth moving, it is assumed that moving range of surface volume of a cell should be limited to only its adjacent cells.

Hypothetical data are used for preliminary evaluation of the method. From the result, it is seen that the method distinguishes quantitative difference between two surfaces by the measure of the sum of X_{ij} which indicates the total moved surface volume in the process of surface transformation. The sum of X_{ij} is smaller where a surface is transformed into smoother surface and the value gets larger in contrast situation. Another finding from the result is the movement of X_{ij} obtained from surface transformation. From this measure, it is understood to which direction surface volume is moved by the distribution of X_{ij} value obtained in the process of transformation. Where the original surface is flat with no peaks, the movement of X_{ij} can be found overall area of surface. And, each X_{ij} is moving toward the peak of goal surface. Where original surface has number of peaks, the movement of X_{ij} seems to move from the peaks of original surface to those of goal surface. These findings support that proposed method effectively shows spatial difference between two surfaces in terms of spatial configuration of surface.

Based on the evaluation of the method with

hypothetical data, method application to actual data of population distribution of Saitama and Tochigi prefecture in Japan is performed. Surface comparison between two surfaces is analyzed by three different time period. From the measure of the sum of X_{ij} , at global scale, the value gets smaller as the time goes by, and this indicates that the change in population rate gets smaller. From the analysis at local scale, the movement of X_{ij} shows the trend of population change by each time period. The period with larger value of sum of X_{ij} , the movement of X_{ij} shows similar distribution as that of subtraction of two surfaces. However, as the sum of X_{ij} gets smaller, the movement of X_{ij} indicates clear pattern with the trend of surface change while such pattern is hard to be understood from the distribution of surface subtraction. From the result, it can be said that proposed method is effective to distinguish spatial difference between two surfaces where their quantitative difference is smaller.

For further application of the method, the relationship analysis not only between two surface data but that of other spatial objects that are related to the surface data will be possible. The relationship analysis between population change and the distribution of convenience stores or railway stations may be an example of further application.

Acknowledgement

This research is financially supported by Research Fellowship of the Japan Society for the Promotion of Science (No.17-11032). Authors thank anonymous referees for their valuable comments.

References

- Bernold, L. and Halpin, D. W., 1984, Microcomputer Cost Optimization of Earth Moving Operation, *Proceedings of the 4th International Symposium on Organization and Management of Construction*, 2, 333-341.
- Congalton, R. G. and Mead, R. A., 1983, A Quantitative Method to Test for Consistency and Correctness in Photointerpretation, *Photogrammetric Engineering and Remote Sensing*, 49(1), 69-74.
- Davis, R., Foote, F., Anderson, J., and Mikhail, E., 1981, *Surveying: Theory and Practice*, McGraw-Hill Book Co., New York, N.Y.
- Easa, S. M., 1988, Estimating Pit Excavation Volume Using Nonlinear Ground Profile, *Journal of Surveying Engineering*, ASCE 114(2), 71-83.
- Flower, R. J., and Little, J. J., 1979, Automatic Extraction of Irregular Network Digital Terrain Models, *Computer Graphics*, 13, 199-207.
- Hagen, A., 2003, Fuzzy Set Approach to Assessing Similarity of Categorical Maps, *International Journal of Geographical Information Science*, 17(3), 235-249.
- Hagen, A., Straatman, B., Uljee, L., 2005, Further Developments of a Fuzzy Set Map Comparison Approach, *International Journal of Geographical Information Science*, 19(7), 769-785.
- Murai, S., Obayashi, S., 1973, Simulation of Earth Moving on Residential Land Development, *Proceedings of 28th Annual Meeting of Japan Society of Civil Engineers*, 5, 267-268.
- Nakagawa, Y., Takata, T. Sada, T. and Hayasi, T., 1994, Simulation of Earth Moving by using EWS Management System. *Proceedings of 49th Annual Meeting of Japan Society of Civil Engineers*, 4, 558-559.
- Okabe, A. and Masuda, S., 1984, Qualitative Analysis of Two-dimensional Urban Population Distributions in Japan, *Geographical Analysis*, 16(4), 301-312.
- Okabe, A. and Sadahiro, Y., 1994, A Statistical Method for Analyzing the Spatial Relationship between the Distribution of Activity Points and the Distribution of Activity Continuously Distributed over a Region, *Geographical Analysis*, 26(2), 152-167.
- Pfaltz, J. L., 1976, Surface Networks, *Geographical Analysis*, 8(1), 77-93.
- Pontius, Jr. R. G., 2000, Quantification error versus location error in comparison of categorical maps. *Photogrammetric Engineering & Remote Sensing*, 66, 1011-1016.
- Pontius, Jr. R. G., and Schneider, L. C., 2001, Land-cover Change Model Validation by an ROC Method for the Ipswich Watershed, Massachusetts, USA. *Agriculture, Ecosystems and Environment*, 85, 239-248.
- Pontius, Jr. R. G., Agrawal, A., Huffaker, D., 2003, Estimating the Uncertainty of Land-cover Extrapolation while constructing a Raster Map from Tabular Data, *Journal of Geographical Systems*, 5, 253-273.
- Rosenfield, G. H. and Fitzpatrick-Lins, K., 1986, A Coefficient of Agreement as a Measure of Thematic Classification Accuracy, *Photogrammetric Engineering and Remote Sensing*, 52(2), 223-227.
- Trosset, M. W., 2005, Visualizing Correlation, *Journal of Computational & Graphical Statistics*, 14(1), 1-19.
- Wada, K., Takahasi, T. and Yamamoto, K., 1991, Application of Fuzzy Theory to Earth Moving Problem in the Field of Construction, *Proceedings of 46th Annual Meeting of Japan Society of Civil Engineers*, 4, 140-141.
- Wolf, G. W., 1984, A Mathematical Model of Cartographic Generalization, *Geo-Processing*, 2, 271-286.
- Correspondence : Jung-Eun Lee, Department of Urban Engineering, University of Tokyo, 7-3-1 Hongo, Bunkyo-Ku, Tokyo 113-8656 (e-mail : nalalee@ua.t.u-tokyo.ac.jp)
- 교신 : 이정은, ☎113-8656 東京都文京区本郷 東京大學都市工學専攻 (이메일 : nalalee@ua.t.u-tokyo.ac.jp)

Received May 16, 2006

Accepted June 18, 2006