# MHD Hartmann flow of a Dusty Fluid with Exponential Decaying Pressure Gradient

**HAZEM A. ATTIA***
*Department of Mathematics, College of Science, Al-Qasseem University, P.O. Box 237, Buraidah 81999,*
*On leave from : Department of Engineering Mathematics and physics, Fac. of Engineering, El-Fayoum University, El-Fayoum, Egypt*

In the present study, the unsteady Hartmann flow with heat transfer of a viscous incompressible electrically conducting fluid under the influence of an exponentially decreasing pressure gradient is studied. The parallel plates are assumed to be porous and subjected to a uniform suction from above and injection from below while the fluid is acted upon by an external uniform magnetic field applied perpendicular to the plates. The equations of motion are solved analytically to yield the velocity distributions for both the fluid and dust particles. The energy equations for both the fluid and dust particles including the viscous and Joule dissipation terms, are solved numerically using finite differences to get the temperature distributions.

## 1. Introduction

The importance and application of solid/fluid flows and heat transfer in petroleum transport, wastewater treatment, combustion, power plant piping, corrosive particles in engine oil flow, and many others are well known in the literature (Lohrabi, 1980 ; Chamkha, 2000 ; Saffman, 1962 ; Gupta and Gupta, 1976 ; Prasad and Ramacharyulu, 1979 ; Dixit 1980 ; Ghosh and Mitra, 1984). Particularly, the flow and heat transfer of electrically conducting fluids in channels and circular pipes under the effect of a transverse magnetic field occurs in magnetohydrodynamic (MHD) generators, pumps, accelerators, and flow meters and has possible applications in nuclear reactors, filtration, geothermal systems, and others. The possible presence of solid particles such as ash or soot in combustion MHD generators and plasma MHD accelerators and their effect on the performance of such devices led to studies of particulate suspensions in conducting fluids in the presence of magnetic fields. For example, in an MHD generator, coal mixed with seed is fed into a combustor. The coal and seed mixture is burned in oxygen and the combustion gas expands through a nozzle before it enters the generator section. The gas mixture flowing through the MHD channel consists of a condensable vapor (slag) and a non-condensable gas mixed with seeded coal combustion products. Both the slag and the non-condensable gas are electrically conducting (Lohrabi, 1980; Chamkha, 2000). The presence of the slag and the seeded particles significantly influences the flow and heat transfer characteristics in the MHD channel. Ignoring the effect of the slag, and considering the MHD generator start-up condition, the problem reduces to unsteady two-phase flow in an MHD channel (Singh, 1976 ; Mitra and Bhattacharyya, 1981 ; Borkakotia and Bharali, 1983 ; Megahed et al., 1988 ; Aboul-Hassan et al., 1991).

In the present work, the transient Hartmann flow with heat transfer of an electrically conduct-

* **E-mail** : ah1113@yahoo.com
**TEL** : +00966-6-3826928; **FAX** : +00966-6-3800911
Department of Mathematics, College of Science, Al-Qasseem University, P.O. Box 237, Buraidah 81999, On leave from : Department of Engineering Mathematics and physics, Fac. of Engineering, El-Fayoum University, El-Fayoum, Egypt. (Manuscript **Received** April 4, 2005; **Revised** May 29, 2006)

ing, viscous, incompressible, dusty fluid is studied. The fluid is acted upon by an exponentially decaying with time pressure gradient. The fluid is assumed to be incompressible and electrically conducting and the particle phase is assumed to incompressible, electrically non-conducting dusty and pressureless. The fluid is flowing between two infinite electrically insulating porous plates maintained at two constant but different temperatures while the particle phase is assumed to be electrically non-conducting. The fluid is subjected to a uniform suction from above and a uniform injection from below and mass conservation is assumed. An external uniform magnetic field is applied perpendicular to the plates while no electric field is applied and the induced magnetic field is neglected by assuming a very small magnetic Reynolds number. This configuration is a good approximation of some practical situations such as heat exchangers, flow meters, and pipes that connect system components. The equations of motion are solved analytically using the method of Laplace Transform to obtain the velocity distributions for both the fluid and dust particles as functions of space and time. The energy equations including the viscous and Joule dissipation terms are solved numerically using the finite difference approximations to obtain the temperature distributions for both the fluid and dust particles. The effect of the magnetic field, the Hall current, the ion slip, and the suction velocity on both the velocity and temperature fields are reported.

## 2. Description of the Problem

The dusty fluid is assumed to be flowing between two infinite horizontal porous plates located at the $y=\pm h$ planes. A uniform pressure gradient, which is taken to be exponentially decaying with time, is applied in the $x$-direction. The plates are subjected to a uniform suction from above and a uniform injection from below. Thus the $y$-component of the velocity of the fluid is constant and denoted by $v_o$. The dust particles are assumed to be electrically non-conducting spherical in shape and uniformly distributed throughout the fluid and to be big enough, so that they are not pumped out through the porous plates and have no $y$-component of velocity. The two plates are assumed to be electrically non-conducting and kept at two constant temperatures $T_1$ for the lower plate and $T_2$ for the upper plate with $T_2 > T_1$. A uniform magnetic field $B_o$ is applied in the positive $y$-direction. This is the only magnetic field in the problem as the induced magnetic field is neglected by assuming a very small magnetic Reynolds number (Megahed et al., 1988). The fluid motion starts from rest at $t=0$, and the no-slip condition at the plates implies that the fluid and dust particles velocities have neither a $z$ nor an $x$-component at $y=\pm h$. The initial temperatures of the fluid and dust particles are assumed to be equal to $T_1$. It is required to obtain the time varying velocity and temperature distributions for both fluid and dust particles. Due to the inclusion of the Hall current term, a $z$-component of the velocities of the fluid and of dust particles is expected to arise. Since the plates are infinite in the $x$ and $z$-directions, the physical quantities do not change in these directions that is $\partial/\partial x=\partial/\partial z=0$ and the problem is essentially one-dimensional. The governing equations for this study are based on the conservation laws of mass, linear momentum and energy of both phases. In this work, it is assumed that both phases are treated as two interacting continua. The interaction between the phases is restricted to the interphase drag force which is modeled by Stokes linear drag theory and the interphase heat transfer (Lohrabi, 1980; Chamkha, 2000).

## 3. The Velocity Distribution

The flow of fluid is governed by the momentum equation

$$\rho\frac{Dv}{Dt}=-\nabla P+\mu\nabla^2 v+JxB_o-K_N(v-v_p) \quad (1)$$

where $\rho$ is the density of clean fluid, $\mu$ is the viscosity of clean fluid, $v$ is the velocity of the fluid, $\mathbf{v}=u(y,\ t)\mathbf{i}+v_o\mathbf{j}$, $v_p$ is the velocity of dust particles, $\mathbf{v}_p=u_p(y,\ t)\mathbf{i}$, $\mathbf{J}$ is the current density, $N$ is the number of dust particles per unit volume,

$K$ is the Stokes constant $=6\pi\mu a$, and "$a$" is the average radius of dust particles.

The first three terms in the right-hand side of Eq. (1) are, respectively, the pressure gradient, viscosity, and Lorentz force terms. The last term represents the force due to the relative motion between fluid and dust particles. It is assumed that the Reynolds number of relative velocity is small. In such a case the force between dust and fluid is proportional to the relative velocity (Saffman, 1962). The current density **J** from the generalized Ohm's law is given by (Crammer and Pai, 1973; Sutton and Sherman, 1965)

$$J=\sigma[E+vxB_o] \quad (2)$$

where $\sigma$ is the electric conductivity of the fluid (Crammer and Pai, 1973; Sutton and Sherman, 1965). Solving Eq. (2) for **J** and substituting the result in Eq. (1), the two components of Eq. (1) read

$$\rho\frac{\partial u}{\partial t}+\rho v_o\frac{\partial u}{\partial y} =-\frac{dP}{dx}+\mu\frac{\partial^2 u}{\partial y^2}-\sigma B_o^2 u-KN(u-u_p) \quad (4)$$

The motion of the dust particles is governed by Newton's second law applied in the $x$ and $z$-directions

$$m_p\frac{\partial u_p}{\partial t}=KN(u-u_p) \quad (5)$$

where $m_p$ is the average mass of dust particles. It is assumed that the pressure gradient is applied at $t=0$ and the fluid starts its motion from rest. Thus,

$$t\leq 0: u=u_p=0$$

For $t>0$, the no-slip condition at the plates implies that

$$t>0,\ y=\pm h: u=u_p=0$$

The problem is simplified by writing the equations in the non-dimensional form. The characteristic length is taken to be $h$, and the characteristic time is $\rho h^2/\mu$ while the characteristic velocity is $\mu/h\rho$. We define the following non-dimensional quantities

$$(\hat{x},\hat{y},\hat{z})=(x,y,z)/h,\ \hat{t}=t\mu/\rho h^2,$$
$$\hat{P}=P\rho h^2/\mu^2,\ \hat{u}=\mu\rho h/\mu,\ \hat{u}_p=u_p\rho h/\mu$$

$S=\rho h v_o/\mu$ is the suction parameter,
$G=m_p\mu/\rho h^2 K$ is the particle mass parameter,
$H_a^2=\sigma B_o^2 h^2/\mu$ is the Hartmann number squared,
$R=KNh^2/\mu$ is the particle concentration parameter.

In terms of the above non-dimensional quantities the velocity equations read

$$\frac{\partial u}{\partial t}+S\frac{\partial u}{\partial y} =-\frac{dP}{dx}+\frac{\partial^2 u}{\partial y^2}-H_a^2 u-R(u-u_p) \quad (6)$$

$$G\frac{\partial u_p}{\partial t}=u-u_p \quad (7)$$

with the initial and boundary conditions,

$$t\leq 0: u=u_p=0,\ t>0: y=\pm 1,\ u=u_p=0 \quad (8)$$

Equations (6)-(8) may be solved using the method of Laplace Transform (LT) [19] to obtain $V$ and $V_p$ as functions of $y$ and $t$. The real part of $V$ or $V_p$ represents the $x$-component of the velocity while the imaginary part represents the $z$-component. Taking LT with respect to the time of Eqs. (12) and (13) we have

$$\frac{d^2\bar{u}}{dy^2}-S\frac{d\bar{u}}{dy}-A\bar{u}-s\bar{u}-R(\bar{u}-\bar{u}_p)=-F(s) \quad (9)$$

$$Gs\bar{u}_p=\bar{u}-\bar{u}_p \quad (10)$$

where $\bar{u}=\bar{u}(y,s)$ and $\bar{u}_p=\bar{u}_p(y,s)$ are respectively, the LT of $u(y,t)$ and $u_p(y,t)$, $A=H_a^2$, and $-F(s)$ is the LT of the pressure gradient. $\bar{u}$ and $\bar{u}_p$ must satisfy the boundary conditions, $\bar{u}=\bar{u}_p=0$ at $y=\pm 1$. All the bars will be dropped for convenience. Eliminating $u_p$ gives

$$\frac{d^2u}{dy^2}=S\frac{du}{dy}-K_1 u=-F(s) \quad (11)$$

where $K_1\equiv K_1(s)=A=s+R(1-1/(1+Gs))$.

The solution of the above equation gives

$$u(y,s)=\frac{F(s)}{K_1}\left(1+\exp(Sy/2)\left[\frac{\sinh(S/2)\sinh(qy)}{\sinh(q)}-\frac{\cosh(S/2)\cosh(qy)}{\cosh(q)}\right]\right)$$

and from Eq. (10) we obtain

$$u_p(y,\, s) = \frac{F(s)}{K_2}\left(1+\exp(Sy/2)\left[\frac{\sinh(S/2)\sinh(qy)}{\sinh(q)} - \frac{\cosh(S/2)\cosh(qy)}{\cosh(q)}\right]\right)$$

where $q^2 = (S+4K_1)/4$ and $K_2 = K_1(1+Gs)$.

Using the complex inversion formula and the residue and convolution theorems (Spiegel, 1986), the inverse transforms of $u(y, s)$ and $u_p(y, s)$ are given as

$$\begin{aligned} u(y,\, t) = C\sum_{n=1}^{\infty}\Big( & \frac{PI_1}{PN_1+a}\exp(PN_1xt) - \exp(-at)) \\ & + \frac{PI_2}{PN_2+a}(\exp(PN_2xt) - \exp(-at)) \\ & + \frac{PI_3}{PN_3+a}(\exp(PN_3xt) - \exp(-at)) \\ & + \frac{PI_4}{PN_4+a}(\exp(PN_4xt) - \exp(-at))\Big) \end{aligned} \tag{12}$$

$$\begin{aligned} u_p(y,\, t) = C\sum_{n=1}^{\infty}\Big( & \frac{PIP_1}{PN_1+a}\exp(PN_1xt) - \exp(-at)) \\ & + \frac{PIP_2}{PN_2+a}(\exp(PN_2xt) - \exp(-at)) \\ & + \frac{PIP_3}{PN_3+a}(\exp(PN_3xt) - \exp(-at)) \\ & + \frac{PIP_4}{PN_4+a}(\exp(PN_4xt) - \exp(-at))\Big) \end{aligned} \tag{13}$$

where

$$-\frac{dP}{dx} = C\exp(-at)$$

$$PN_1 = \frac{NN_1xG - B + \sqrt{(NN_1xG-B)^2 - 4xGx(A-NN_1)}}{2G}$$

$$PN_2 = \frac{NN_1xG - B - \sqrt{(NN_1xG-B)^2 - 4xGx(A-NN_1)}}{2G}$$

$$PN_3 = \frac{NN_2xG - B + \sqrt{(NN_2xG-B)^2 - 4xGx(A-NN_2)}}{2G}$$

$$PN_4 = \frac{NN_2xG - B - \sqrt{(NN_2xG-B)^2 - 4xGx(A-NN_2)}}{2G}$$

$$PI_1 = \frac{NN_3}{KPN_1xDPN_1}$$

$$PI_2 = \frac{NN_3}{KPN_2xDPN_2}$$

$$PI_3 = \frac{NN_4}{KPN_3xDPN_3}$$

$$PI_4 = \frac{NN_4}{KPN_4xDPN_4}$$

$$PIP_1 = \frac{PI_1}{(1+GxPN_1)}$$

$$PIP_2 = \frac{PI_2}{(1+GxPN_2)}$$

$$PIP_3 = \frac{PI_3}{(1+GxPN_3)}$$

$$PIP_4 = \frac{PI_4}{(1+GxPN_4)}$$

$$B = 1 + G(A+R)$$

$$NN_1 = -\pi^2(n-1)^2 - S^2/4$$

$$NN_2 = -\pi^2(n-0.5)^2 - S^2/4$$

$$NN_3 = 2\pi(-1)^n(n-1)\exp(Sy/2)\sinh(S/2)\sin(\pi(n-1)y)$$

$$NN_3 = 2\pi(-1)^n(n-1)\exp(Sy/2)\sinh(S/2)\sin(\pi(n-1)y)$$

$$NN_4 = 2\pi(-1)^{n+1}(n-0.5)\exp(Sy/2)\cosh(S/2)\cos(\pi(n-0.5)y)$$

$$KPN_1 = \frac{GxPN_1^2 + BxPN_1 + A}{1+GxPN_1}$$

$$KPN_2 = \frac{GxPN_2^2 + BxPN_2 + A}{1+GxPN_2}$$

$$KPN_3 = \frac{GxPN_3^2 + BxPN_3 + A}{1+GxPN_3}$$

$$KPN_4 = \frac{GxPN_4^2 + BxPN_4 + A}{1+GxPN_4}$$

$$DPN_1 = \frac{(1+GxPN_1)(2GxPN_1+B) - G(GxPN_1^2+BxPN_1+A)}{(1+GxPN_1)^2}$$

$$DPN_2 = \frac{(1+GxPN_2)(2GxPN_2+B) - G(GxPN_2^2+BxPN_2+A)}{(1+GxPN_1)^2}$$

$$DPN_3 = \frac{(1+GxPN_3)(2GxPN_3+B) - G(GxPN_3^2+BxPN_3+A)}{(1+GxPN_3)^2}$$

$$DPN_4 = \frac{(1+GxPN_4)(2GxPN_4+B) - G(GxPN_4^2+BxPN_4+A)}{(1+GxPN_4)^2}$$

The summation parameter $n$ and the coordinate $y$ are included in the above quantities.

## 4. The Temperature Distribution

Heat transfer takes place from the upper hot plate to the lower cold plate by conduction through the fluid. Since the hot plate is above, there is no natural convection, however, there is a forced convection due to the suction and injection. In addition to the heat transfer, there is a heat generation due to both the Joule and viscous dissipations. The dust particles gain heat from the fluid by conduction through their spherical surface. Since, the problem deals with a two-phase flow, two energy equations are required (Crammer and Pai, 1973; Schlichting, 1968). The energy equations describing the temperature distributions for both the fluid and dust particles read

$$\rho c\frac{\partial T}{\partial t}+\rho c v_o\frac{\partial T}{\partial y} =k\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2+\sigma B_0^2u^2+\frac{\rho_p c_s}{\gamma_T}(T_p-T) \tag{14}$$

$$\frac{\partial T_p}{\partial t}=-\frac{1}{\gamma_T}(T_p-T) \tag{15}$$

where $T$ is the temperature of the fluid, $T_p$ is the temperature of the particles, $c$ is the specific heat capacity of the fluid at constant volume, $k$ is the thermal conductivity of the fluid, $\rho_p$ is the mass of dust particles per unit volume of the fluid, $\gamma_T$ is the temperature relaxation time, and $c_s$ is the specific heat capacity of the particles.

The last three terms on the right-hand side of Eq. (14) represent the viscous dissipation, the Joule dissipation $(j^2/\sigma)$, and the heat conduction between the fluid and dust particles respectively. The temperature relaxation time depends, in general, on the geometry, and since the dust particles are assumed to be spherical in shape, the last term in Eq. (14) is equal to $4\pi aNk(T_p-T)$. Hence

$$\gamma_T=\frac{3\,\mathrm{Pr}\,\gamma_p c_s}{2c}$$

where $\gamma_p$ is the velocity relaxation time $=2\rho_s a^2/9\mu$, Pr is the Prandtl number $=\mu c/k$, and $\rho_{ss}$ is the material density of dust particles $=3\rho_p/4\pi a^3 N$.

$T$ and $T_p$ must satisfy the initial and boundary conditions

$$t\leq 0:\ T=T_p=0$$

$$t>0,\ y=-h:\ T=T_p=T_1$$

$$t>0,\ y=h:\ T=T_p=T_2$$

To put these equations in the non-dimensional form, we use the same non-dimensional quantities defined in the velocity distribution section in addition to the following non-dimensional quantities

$$\hat{T}=\frac{T-T_1}{T_2-T_1},\ \hat{T}_p=\frac{T_p-T_1}{T_2-T_1}$$

$E_c=\mu^2/h^2c\rho^2(T_2-T_1)$ is the Eckert number, $L_o=\rho h^2/\mu\gamma_T$ is the temperature relaxation time parameter.

In terms of the above non-dimensional variables and parameters, Eqs. (14) and (15) become

$$\frac{\partial T}{\partial t}+S\frac{\partial T}{\partial y}=\frac{1}{\mathrm{Pr}}\frac{\partial^2 T}{\partial y^2}+E_c\left(\frac{\partial u}{\partial y}\right)^2 +H_a^2E_cu^2+\frac{2R}{3\,\mathrm{Pr}}(T_p-T) \tag{16}$$

$$\frac{\partial T_p}{\partial t}=-L_o(T_p-T) \tag{17}$$

$T$ and $T_p$ must satisfy the initial and boundary conditions

$$t\leq 0:\ T=T_p=0 \tag{18a}$$

$$t>0,\ y=-1:\ T=T_p=0 \tag{18b}$$

$$t>0,\ y=1:\ T=T_p=1 \tag{18c}$$

If the values of the velocity components are substituted in the right-hand side of the energy Eq. (16), the resulting energy equations are cumbersome and too difficult to solve analytically. Therefore we resort to numerical techniques and solve the equations using the finite difference approximation (Ames, 1977). We choose the Crank-Nicolson implicit method. The finite difference equations are written at the midpoints of the computational cell and the different terms are replaced with their second order central difference approximations in the $y$-direction. The diffusion terms are replaced with the average of the central differences at two successive time levels. The Joule and the viscous dissipation terms are evaluated using the velocity components and their derivatives in the $y$-direction, which are obtained from the exact solution.

## 5. Results and Discussion

Figure 1 presents, respectively, the profiles of the velocity components and temperature of the fluid $u$ and $T$ and particles $u_p$ and $T_p$ for various values of time $t$. The figures are plotted for $H_a=1$ and $S=1$. As shown in Figs. 1(a) and 1(b) the profiles of $u$ and $u_p$ are asymmetric about the plane $y=0$ because of the suction. It is observed that the velocity component and temperature of the fluid phase reach the steady state faster than that of the particle phase. This is because the fluid velocity is the source for the dust particles' velocity. It is shown that the velocity components and temperatures of the fluid and dust particles do not reach the steady state monotonically due to the effect of the pressure gradient.

Figure 2 shows the time evolution of the velocity components and temperature at the centre of the channel $(y=0)$, respectively, for the fluid and particle phases for various values of the Hartmann number $H_a$ and $S=0$. Figures 2(a) and 2(b) indicate that increasing $H_a$ decreases $u$ and $u_p$ for all $t$, as a result of increasing the damping force on $u$ which decreases $u$ and consequently decreases up. Figures 2(c) and 2(d) indicate that the variation of $T$ and $T_p$ with $H_a$ depends on time. It is clear that for small $t$, increasing $H_a$ increases $T$ and $T_p$ due to increasing the Joule dissipation. But, for large $t$, increasing $H_a$ decreases $T$ as a result of decreasing the velocities $u$ and up and consequently decreases the viscous and Joule dissipations.

Figure 3 presents the time evolution of the velocity components and temperature at the centre of the channel $(y=0)$, respectively, for the fluid and particle phases for various values of the suction parameter $S$ and $H_a=0$. It is clear from Figs. 3(a) and 3(b) that increasing the suction parameter decreases both $u$ and $u_p$ due to the convection of the fluid from regions in the lower half to the centre which has higher fluid speed. Figures 3(c) and 3(d) show that increasing S decreases the temperature at the centre of the channel. This is due to the influence of convection in pumping the fluid from the cold lower half towards the centre of the channel. It is observed from Figs. 2 and 3 that the suction has a more pronounced effect on the steady state time of the

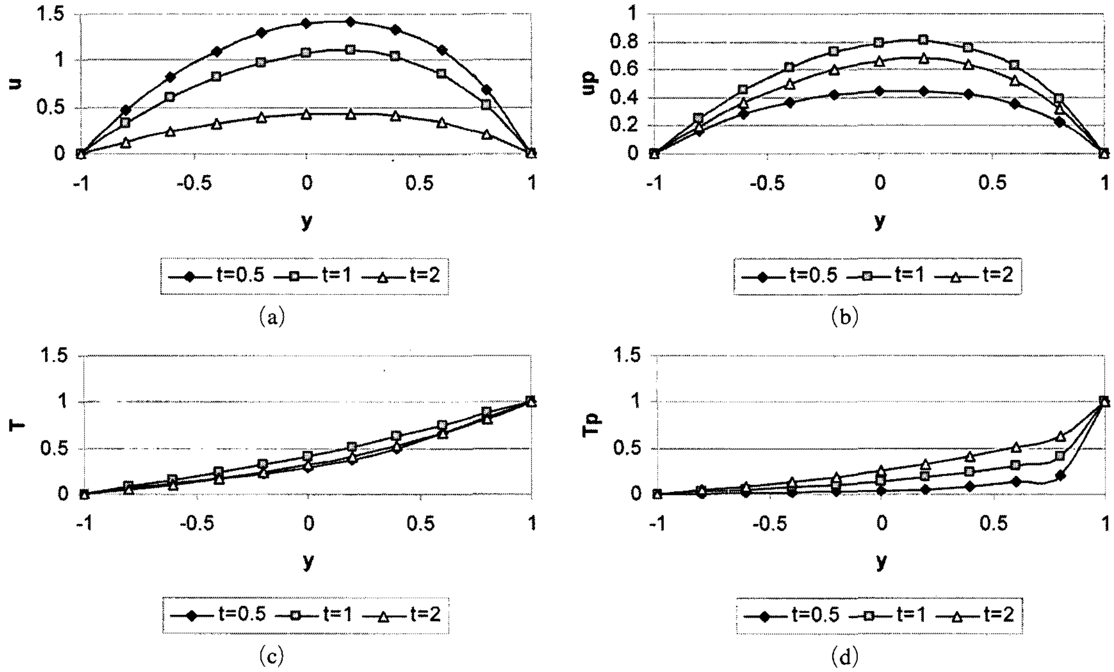


**Fig. 1** Time variation of the profiles of: (a) $u$; (b) $u_p$; (c) $T$ and (d) $T_p$.($H_a=1$ and $S=1$)

velocity and temperature of the particles than that of the magnetic field.

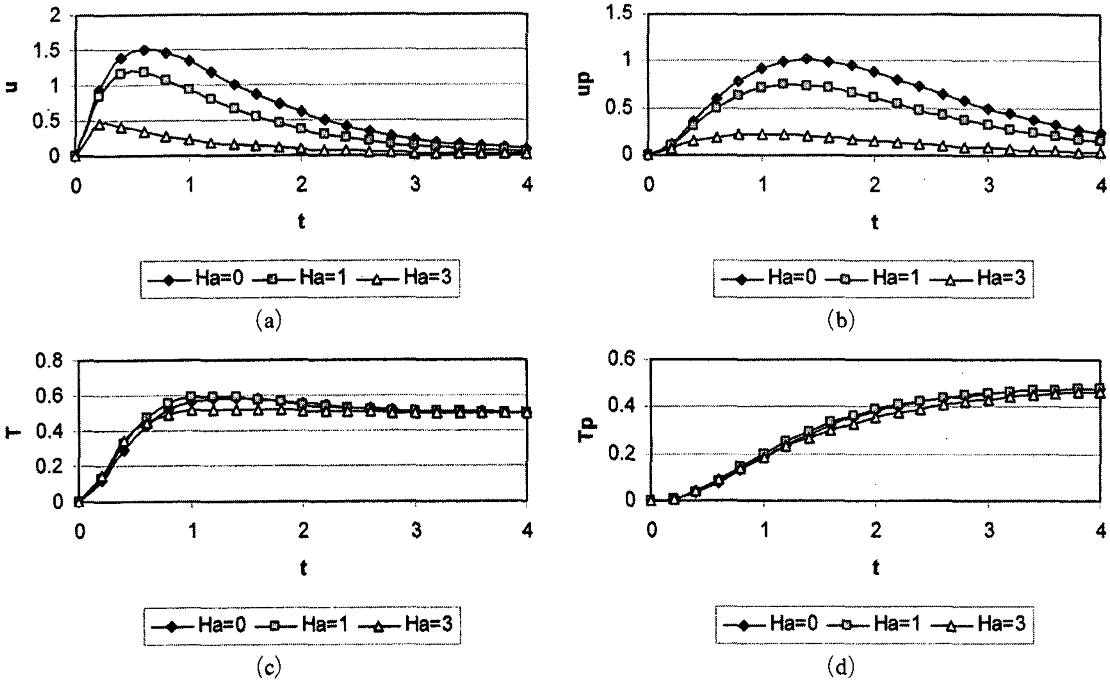


**Fig. 2** Effect of Ha on the time variation of: (a) $u$ at $y=0$; (b) $u_p$ at $y=0$; (c) $T$ at $y=0$ and (d) $T_p$ at $y=0$. ($S=0$)

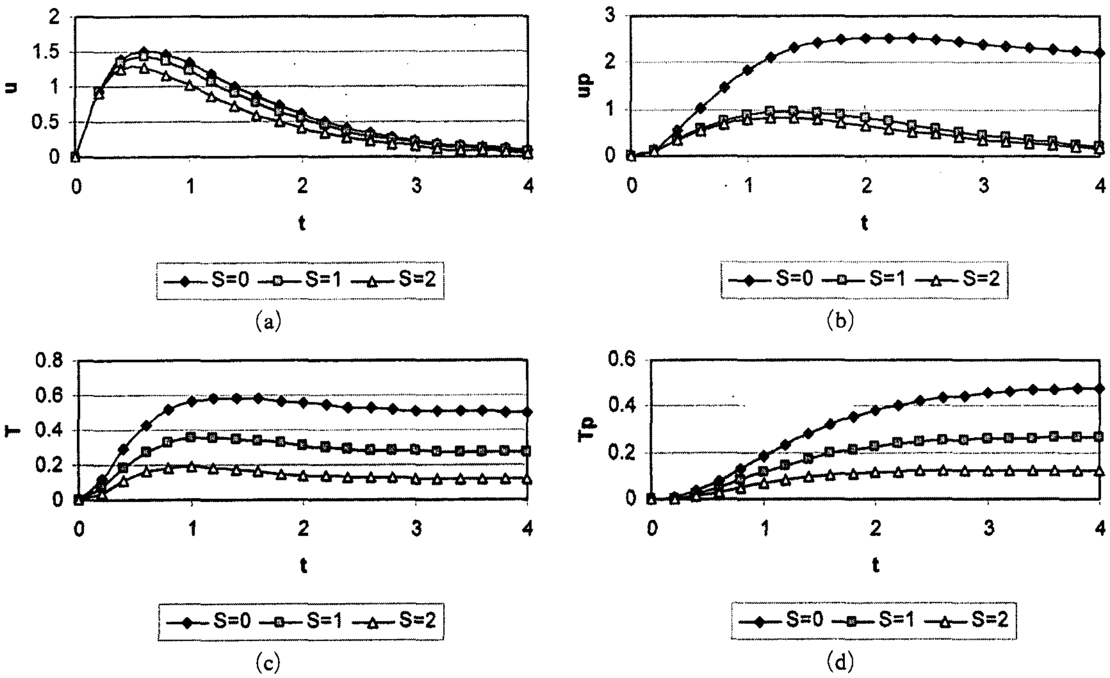


**Fig. 3** Effect of S on the time variation of: (a) $u$ at $y=0$; (b) $u_p$ at $y=0$; (c) $T$ at $y=0$ and (d) $T_p$ at $y=0$. ($H_a=0$)

## 6. Conclusions

The unsteady flow with heat transfer of a dusty conducting fluid under the influence of an applied uniform magnetic field has been studied in the presence of uniform suction and injection and an exponential decaying pressure gradient. An analytical solution for the equations of motion has been obtained while the energy equation has been solved numerically using finite differences. The effect of the magnetic field, and the suction and injection velocity on the velocity and temperature distributions for both the fluid and particle phases has been investigated. It is of interest to see that the effect of the magnetic field on the temperatures of the fluid and particles depends on time. Also, it is observed that the suction velocity has a more apparent effect than the magnetic field on the steady state time of the velocity and temperature of the dust particles.

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