나카가미 페이딩 채널에서 M-ary FSK 변조된 리드솔로몬 부호화의 성능분석

강희조*

Performance Analysis of Reed-Solomon Coded M-ary FSK Modulation in Nakagami Fading Channels

Heau-jo Kang*

요 약

본 논문에서는 낮은 전력과 낮은 데이터 율의 M-ary FSK 시스템의 응용하고 성능 개선에 관해서 분석한다. 제안 시스템에서는 최대비 합성 다이버시티, 부호화와 비 부호화 일 때 가우시안 잡음, $m=2$, $m=3$, 레일리, 반가우신안 페이딩 채널에서 리드솔로몬 부호화를 이용하여 최적의 변조 값을 구하였다. 시뮬레이션 분석 결과로부터 슬로우 페이딩 모델에 응용이 적당함과 응용시스템에 유용함을 확인하였다.

ABSTRACT

In this paper we analyze the performance improvement of the M-ary FSK systems for low power and low data rate applications. This contribution presents a unified analysis of its MRC diversity, uncoded and coded performance in AWGN, m=2, m=3, Rayleigh and one sided Gaussian fading channels using optimum noncoherent demodulation with Reed-Solomon(RS) codes. The results of this paper should be useful as benchmarks of obtainable performance and as a reference for validating the results of simulation studies when slow fading models are applicable.

키워드

MRC diversity, Nakagami fading, Reed-Solomon, one sided Gaussian fading($m=0.5$)

Ⅰ. Introduction

M-ary frequency shift keying modulation (M-ary FSK) is a power efficient modulation method that is currently being considered for low power and low data rate applications. Its power efficiency improves as M (the number of orthogonal frequencies used) increases, but unfortunately, so does its complexity. In principle, M-ary FSK can be detected coherently to obtain optimum performance, but in practice this is mot feasible for the following reasons: (a) A precise frequency and carrier phase reference must be obtained at the receiver for each of the transmitted orthogonal carriers. For large M, this would make a coherent receiver very complex and difficult to implement. (b) As shown in section Ⅱ, as M increases, the performance of ideal noncoherent demodulation approaches that of ideal coherent demodulation

* 목원대학교 컴퓨터공학부

접수일자 : 2006. 4. 4

with a significant advantage in receiver complexity. A noncoherent demodu lation for M-ary FSK can be designed using FFT based demodulator resulting in cost efficient receivers[1]-[2]. Due to these two factors, the results presented in this paper consider only optimum noncoherent demodu lation.

This paper applies MRC diversity and RS code M-ary FSK in a slow nonselective Nakagami fading channel with additive white Gaussian noise.

The structure of the paper is as follows: In section Ⅱ describes the system model under investigation. The utility of these channels in modeling propagation conditions ranging from space communications to land maritime aeronautical mobile satellite communications to terrestrial mobile communications, respectively, is well known. The results of section Ⅲ MRC diversity performance analysis of the performance of uncoded M-ary FSK modulation with optimum noncoherent demodulation for Nakagami-m slow fading channels is presented. Section Ⅳ to derive the coded performance using RS codes. These codes are considered because of their powerful error correction capability and the straightforward manner in which the RS/M-ary FSK coding modulation scheme can be implemented. In section Ⅳ, optimum codes are derived as function of the modulation's signal set size, M and the channel model using a commonly specified bit error probability criterion.

Ⅱ. System Model

The block diagram of the system under investigation is shown in figure 1. The input bit stream with a bit rate of f_b b/s is segmented in to $q-bit$ symbols, and the resulting symbol sequence with a symbol rate of $f_s = f_b/q$ symbols/s is fed to the RS code encoder. The RS (N,K) code defined over the galois field GF(2^q) is used for error correction, where N is the code length and K is the information length of the code. The coded symbol, to be transmitted with a transmission symbol rate of $f_s \times N/K$, is assigned to one of the M $(=2^q)$-ary FSK signals, where orthogonality of the M frequencies is assumed. For each symbol, fading is assumed to be nonfrequency selective. Noncoherent detection is used for the reception of the M-ary FSK signal. The detector output symbol sequence and samples of the envelope detector output for each received symbol are then fed to the RS decoder[3]-[4]. Unfortunately, constructing a soft decision decoder for an RS code is impractical except for some restricted classes of codes.

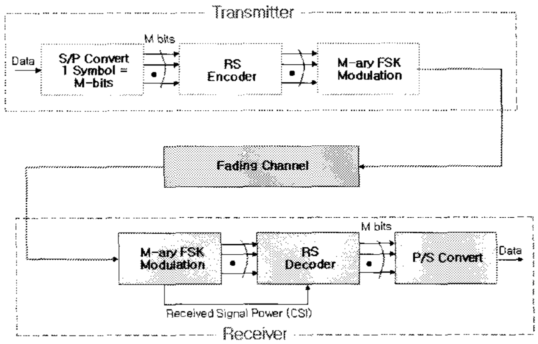


그림 1. 시스템 모델
Fig. 1. System model

Ⅲ. Uncoded Performance

For orthogonal M-ary FSK on a Nakagami-m channel, the received signal in the interval $(0, T)$ has the form

$$r(t) = R\sqrt{2E_s/T}\cos(w_i t + \theta) + n(t) \tag{1}$$

where E_s is the average received symbol energy (under a normalization described below), $n(t)$ is AWGN with one sided power spectral density N_o, θ is a uniformly distributed random variable $(0,2\pi)$, w_i is one of M orthogonally spaced frequencies, and R is a Nakagami-m random variable whose p.d.f. is given by [5]

$$p(R) = \frac{2m^m R^{2m-1}}{\Gamma(m)\Omega^m} e^{-(\frac{m}{\Omega}R^2)}, \quad R \ge 0 \tag{2}$$

where

$$\Omega = \overline{R^2}$$

and

$$m = \frac{\Omega^2}{var(R^2)} \geq \frac{1}{2}$$

are the scale and shape parameters of the distribution. For $m = 1$ we have a Rayleigh fading channel, and as $m \to \infty$ we have a channel that becomes nonfading (as the p.d.f. tends to an impulse function), At the other extreme, for $m = 1/2$ we have a one-sided Gaussian fading distribution.

We restrict out discussion to slow nonselective fading. With no loss of generality, we adopt the convenient normalization $\Omega = 1$ so that the received energy in the fading channel, $R^2 E_s$, has an average value $\overline{R^2} E_s = \Omega E_s = E_s$. With $\Omega = 1$, the p.d.f. in (2) is reduced to a one parameter distribution so that all of the results can be expressed in terms of the single parameter m. In Fig. 1 we see several members of this one parameter family of Nakagami-m distributions.

Following Barrow [6]. we find the symbol error probability for Nakagami-m fading by averaging the nonfading symbol error probability over the underlying fading random variable. Barrow considered the binary case only, but in the following we generalize to M-FSK[7]. Here the symbol error probability is given by

$$P_s = \int_0^\infty \sum_{i=1}^{M-1} \binom{M-1}{i} \frac{(-1)^{i+1}}{i+1} e^{\frac{-i}{i+1}\frac{E_s R^2}{N_0}} p(R) dR \tag{3}$$

where $p(R)$ is given by (2) with $\Omega = 1$. Evaluating (3) and converting form symbols to bits using

$$P_b = \frac{M}{w(M-1)} P_s \text{ and } E_b = \frac{E_s}{\log_2 M}$$

we find the M-ary FSK bit error probability on a Nakagami-m channel to be

$$P_s = \sum_{i=1}^{M-1} \binom{M-1}{i} \frac{(-1)^{i+1}}{i+1} \bullet \left[\frac{m}{m + \frac{i}{i+1}\frac{E_s}{N_o}} \right]^m \tag{4}$$

In (4) for $m \to \infty$ and $m = 1$, we get the familiar nonfading and Rayleigh fading special cases.

Assuming a maximal ratio combining approach with L identical branches, the diversity effect is examined as follows. The output signal to noise power ratio after maximal ratio combining is equal to the sum of the signal to noise power ratio of the various combining branches. It can be shown that pdf of the resulted signal to noise power ratio is [8]-[9].

$$p(\gamma) = \frac{\gamma^{mL-1}}{\Gamma(mL)} \left(\frac{m}{\Gamma'}\right)^{mL} \exp\left(-\frac{m\gamma}{\Gamma'}\right) \tag{5}$$

The average error probability of M-ary FSK signal over the Nakagami-m fading channel with MRC diversity reception is

$$P_s = \int_0^\infty P_S \, p(\gamma) d\gamma \tag{6}$$

In figure 2, we show the corresponding comparison of the SER performance of the M-ary FSK system, when communicating over Nakagami-m one-sided Gaussian ($m = 0.5$) fading models. The curves in the figure were plotted against the average SNR per bit of E_b/N_0 for the parameters $M = 8$, $M = 16$, $M = 32$, $L = 2$ and $L = 3$. From the results of figure 2 we observe that for a given SNR per bit value, the MRC diversity branch number $L = 3$ achieves a lower SER, than the MRC diversity branch number $L = 2$ regardless of $M = 8$, $M = 16$ and $M = 32$.

Figure 2 shows the average symbol error probability P_s as a function of E_s/N_o with the diversity order L as a parameter. It is found from this figure that for a large values of E_s/N_o, P_s reduces inversely proportional to $\left(\frac{E_s}{N_o}\right)^L$ for any value of M.

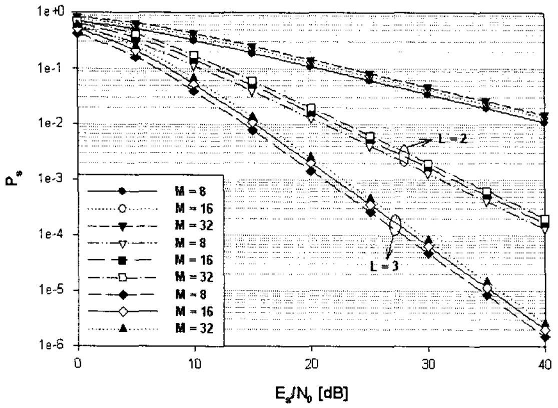


그림 2. MFSK 변조방식의 심볼 에러율
Fig. 2. Average symbol error rate

Ⅳ. Coded performance

In this section, the performance of RS coded M-ary FSK modulation is presented. The choice of RS coding for this application is particularly appropriate because of the straight forward manner in which the RS encoded codewords can be mapped to the M-ary FSK signal set. Figure 7 shows a block diagram of the M-ary FSK transmitter with RS encoding. Application of this transmission scheme to a fading channel will generally require that data be interleaved after encoding in order to randomize bit errors due to long error bursts caused by long fades, thus improving decoder performance [10]. In obtaining coded performance results for fading channels, infinitely long interleaving has been assumed.

An(N, K) RS nonbinary code takes K symbols and maps them into N symbols. Each RS symbol can be represented by k bits. The parameters N and K, the code's minimum distance, $d_{\min}$, and the number of symbol errors (t) the code can correct are given by [1]

$$
\begin{aligned}
&N = 2^k - 1, \text{symbols} \\
&K = 1, 3, \ldots, N-2, \text{symbols} \\
&d_{\min} = N - K + 1, \text{symbols} \\
&t = (N-K)/2, \text{symbols}
\end{aligned} \tag{7}
$$

The first equation in (7) is an existence condition on RS codes and explains why the parameter k sets the codeword length (N) as well as the signal set size (i.e.,$M = 2^K$). In Figure 3, each output from the RS (N, K) encoder contains Nk bits. For an (N, K) RS code, there are a total of $q = M = 2^K$ code symbols. Therefore, each code symbol out of the encoder can be mapped to one of the M frequencies in the M-ary FSK signal set. Also note that if T is taken as the time required to transmit each code symbol or one of the M frequencies, the transmission rate is given by $R_b = k/T$bits/sec and the information rate is given by $R_I = rR_b$bits/sec, where $r = K/N$ is the code rate. Finally, note that with this scheme, the decoder output consists of Kk decoded bits every NT seconds.

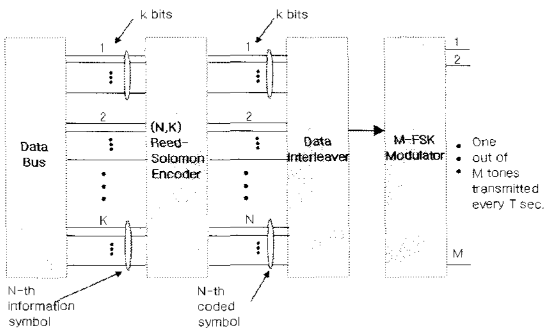


그림 3. RS부호를 채용한 M-ary FSK 블록 다이어그램(N, K)
Fig. 3. M-ary FSK with (N, K) reed-solomon encoder block diagram

The performance of this coding modulation scheme can be computed analytically in a straightforward manner when the receiver uses hard decisions on the received symbols. Note that this is directly compatible with noncoherent demodulation of the M-ary FSK signal set. Although several decoding alternatives are available, it is assumed that when the received codeword contains less than t symbol errors, the decoder corrects these errors and when the number of symbol errors exceeds t, the decoder makes no attempt to correct the errors.

$$
P_b = \frac{M}{2(M-1)} \left[\frac{1}{N} \sum_{i=t+1}^{N} i C_i^N P_s^i (1-P_s)^{N-i} \right] \tag{8}
$$

where P_s is the MRC diversity symbol error probability given by (7) with the signal to noise ratio (γ_s)appropriately modified to account for the code rate.

In order to find optimum RS codes as a function of M and of the channel model, (8) has been used to determine the E_b/N_o required to achieve $P_b = 10^{-6}$. Figure 4 shows the required E_b/N_o for 32-FSK modulation for AWGN, Nakagami-m ($m = 3, m = 5,$) Rayleigh ($m = 1$) and one-sided Gaussian($m = 0.5$) fading channels as a function of the number of information symbols, K, for asch RS code. Note how for the AWGN channel the optimum codes have relatively high rate whereas for fading channels the optimum code rate decrease as fading becomes more severe. Similar results have been given in [10] in studying the performance of RS $(31, K)$codes with BPSK modulation in fading channels.

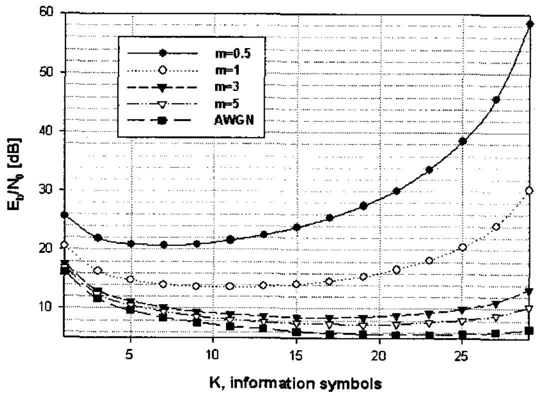


그림 4. 나카가미 페이딩 채널에서 경판정 복호를 이용한 32-FSK 변조방식의 RS 부호율에 따른 성능
Fig. 4. E_b/N_o required for $P_b = 10^{-6}$ for 32-FSK modulation with $(31, K)$ RS coding with hard decision decoding for Nakagami-m fading channels

Following a similar procedure for different modulation signal set size, optimum codes can be determined. This becomes more evident as M increases and may provide some flexibility in the choice of RS code to be used for a particular application.

Figures 5 show the performance of the optimum codes and MRC diversity for Nakagami fading with one sided Gaussian respectively. In examining the coded results given in these figures, it is worth noting that whereas the performance of uncoded M-ary FSK does not improve significantly as M increases, particularly as fading becomes more severe, the same is not true of the coded performance when optimum codes are considered. In fact, as fading becomes more severe, the benefits of coding and diversity become more significant as M and L increases.

The spectral efficiency of the above transmission scheme is measured by the information rate to bandwidth ratio. For RS encoded M-ary FSK, the information rate is $R_i = Kk/NT$ and the occupied bandwidth, assuming minimum frequency spacing between orthogonal carries, is $W = M/T$. The theoreti cal spectral efficiency is then given by $n = R_I/W = Kk/NM = rk/M$(bps/Hz). For example, 32-FSK encoded with RS $(31, 15)$ yields a theoretical spectral efficiency of 0.08 bps/Hz .In practice, the actual spectral efficiency of this transmission scheme would be significantly less due to the need to increase the frequency spacing between tones to allow some margin for frequency uncertainties, phase noise, etc.

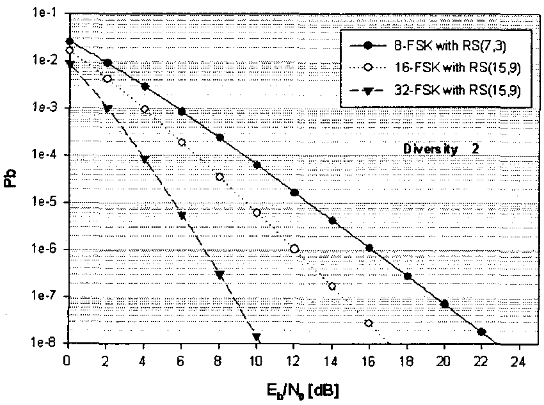


그림 5. 반 가우시안 분포 페이딩 환경에서 MRC 다이버시티와 최적 RS 부호를 갖는 MFSK 변조방식의 오율 성능
Fig. 5. Performance of MRC diversity and optimum RS codes with MFSK modulation in Nakagami-m fading with one sided Gaussian($m = 0.5$)

V. Conclusions

A unified analysis of the uncoded and coded performance of M-ary FSK with MRC diversity, RS encoding and ideal noncoherent demodulation has been presented for AWGN, Nakagami-m ($m=2$), ($m=3$), Rayleigh($m=1$) fading and one sided Gaussian($m=0.5$) fading channels. The analytic method developed for finding the uncoded symbol error probability circumvents the difficulties encountered in evaluating performance for large M when using the alternating series formulation. Optimum RS codes have been derived as a function of M and of the channel model for a given bit error probability criterion assuming sufficiently long interleaving (i.e., theoretically infinite). In practice, the use of limited interleaving will introduce additional degradation which, when found through simulations, could be quantified by referring to the infinite interleaving case treated here.

The results given in this paper should be useful in validating the results of simulation studies and can serve as benchmarks of obtainable performance in slow fading situations.

REFERENCES

[1] J. Proakis, Digital Communications, 2nd Ed., New York:McGraw-Hill, 1989.

[2] R. Khalona, "Optimum Reed-Solomon codes for M-ary FSK modulation with hard decoding in Rician-fading channels", IEEE Trans. Commun., vol. 44, pp. 409-412, Apr. 1996.

[3] W. C. Jakes, Microwaves Mobile Communi cations, New York:Wiley, pp. 45-60, 1974.

[4] T. Matsumoto and A. Higashi, "Performance analysis of RS-code M-ary FSK for frequency-hopping spread spectrum mobile radios", IEEE Trans. on Veh. Technol., vol. VT-41, no. 3, pp. 266-270, Aug. 1992.\

[5] M. Nakagami, "The m-distribution-A general formula of intensity distribution of fading", in Statistical Methods in Radio Wave Propagation, W. C. Hoffmain, Ed. New York: Pergamon, 1960.

[6] B. B. Barrow, "Error probabilities for data transmission over fading radio paths", SHAPE Air Defence Technical Center, The Hague, Tech. Memo TM-26, 1962.

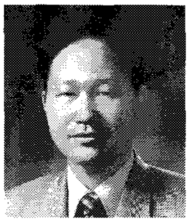
[7] P. Crepeau, "Unicoded and coded perfor mance of M-FSK and DPSK in Nakagami fading channels", IEEE Trans. on Commu., vol. COM-40, pp. 487-493, March 1992.

[8] A. H. Wojnar, "Unknown bounds on perfor mance in Nakagami channels", IEEE Trans. Commun., vol. COM-34, pp. 22-24, Jan. 1967.

[9] Y. Miyagaki, N. Morinaga, and T. Name kaws, "Error probability characteristics for CPSK signal through m-distributed fading channel", IEEE Trans. Commun., vol. COM-26, pp. 88-100, Jan. 1978.

[10] E. Lutz, "Code and interleaver design for data transmission over fading channels", Proceedings of IEEE CLOBECOM, pp. 381-386, Atlanta, November 1984.

저자소개



강 희 조(Heau-JO Kang)

received th Ph. D. degree in Aviation Electronic Engineering from Hankuk Aviation University in 1994.

In 1990 he joined the faculty of Mokwon University where he is currently a professor in the Department of Computer Engineering.

His research interests include multimedia communication systems, mobile communication systems, ubiquitous, mobile computing, smart home networking, post PC, UWB,EMI/EMC, intelligent transport system(ITS), millimeter- wave communication, 4G, and radio frequency identification(RFID) system. He has published more than 350 papers in journals and conferences and has filed more than 10 industrial property. He is a Member of IEEE, KICS, KEES, KIMICS, KMS, KONI, KIEEME, KSII, and DCS.