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## 소프트 판정을 이용한 자력복구 적응 판정제한 채널등화 기법

## (Soft Decision Approaches for Blind Decision Feedback Equalizer Adaptation)

정 원 주\*

(Wonzoo Chung)

## 요 약

이 논문에서는 조정가능한 소프트 판정기를 이용하여 자력복구 판정제한 채널등화기의 적응 모드를 포착모드와 추적 모드 사이에서 최적화 하는 기법들을 제안 한다. 제안된 기법들은 주어진 SNR에 따라 소프트 판정기를 최적화 하여 DFE를 위한 제한 신호를 생성하고 그에 따라 자력복구 IIR 필터 적응모드와 DD-LMS 적응모드를 결합한 적응방식을 적용한다. 제안된 기법들은 포착모드와 추적모드 사이의 최적화된 스위칭을 성취할뿐아니라 DFE 에러 propagation을 최소화 하는데도 기여한다.

## Abstract

In this paper, we propose blind adaptation strategies for decision feedback equalizer (DFE) optimizing the operation mode between acquisition and tracking modes based on adjustable soft decision devices. The proposed schemes select an optimal soft decision device to generate feedback samples for the DFE at a given noise to signal ratio, and apply corresponding adaptation rules which combine a blind infinite impulse response (IIR) filtering adaptation and the decision-directed least mean squared (DD-LMS) DFE adaptation. These adaptation approaches attempt to achieve not only smooth transition between acquisition and tracking of DFE but also mitigation of error propagation.

**Keywords** : blind equalization, decision feedback equalizer, soft decision, blind initialization, joint adaptation

## I. Introduction

The most difficult obstacle in digital data broadcasting over terrestrial wireless channels such as in ATSC DTV is perhaps the severe and long-delay-spread multipath with time-varying nature. A decision feedback equalizer (DFE) has been recognized as a strong candidate to combat such difficult channels with a relatively low computational cost<sup>[1]</sup>.

Unfortunately, the recursive structure of DFE introduces an undesirable effect named error propagation. A single false decision often triggers a chain of false decisions and propagates decision errors especially for low signal to noise ratio (SNR)

<sup>[1]</sup>. Furthermore, in order to initialize the feedback filter of DFE, DFEs require perfect knowledge on channel. Incorrect channel estimation causes irrecoverable error propagation.

Recently, there have been two major attempts to improve the weakness of DFE, mitigation of error propagation and adaptive blind DFE initialization. To mitigate error propagation, soft decision devices are proposed to use soft decisions, instead of hard decisions, to the DFE, to minimize decision error<sup>[2],[3]</sup>.

To relieve the dependency on the reference signals, several authors have proposed blind adaptive approaches for initialization of DFEs<sup>[4],[5],[6]</sup>. Basically, a transversal filter and a recursive filter are updated by a blind adaptive IIR algorithm such as IIR Constant Modulus Algorithm (CMA)<sup>[5]</sup>, Minimum Output Energy (MOE)<sup>[4]</sup>. In practice, IIR-CMA is a

\* 정희원, 명지대학교  
(Myong Ji University)

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widely used initialization method due to its computational efficiency. Once the IIR-CMA filter opens constellation eye, the coefficients of the recursive filter are used for initialization of the DFE filter, and then DFE is updated by the DD-LMS algorithm to track time variation of the channel. The DD-LMS DFE achieves a minimum mean squared error (MMSE) performance, but lacks acquisition ability. On the contrary, the IIR equalizer has acquisition ability, but exhibits poor MSE performance<sup>[7]</sup>. Therefore, in the presence of rapid time varying channels, it is desired an adaptation mechanism optimizing the trade-off between acquisition ability and MMSE performance.

In this paper, we propose adaptive adaptation methods blending IIR equalization and DD-LMS by generalizing the decision device. We view the IIR equalizer and DD-LMS DFE equalizer as two corner cases of a general filtering structure with a soft decision device. We consider two piece-wise soft decision device families, the linear combining decision device [8] and the Run and Go decision device [9], which combines the blind IIR adaptation and the DD-LMS adaptation. We set the blending ratio, which is adjusted by decision device setting, with respect to SNR by optimizing the soft decision device as considered in error propagation mitigation.

In section II we briefly introduce a DFE system model with soft decision device and existing blind adaptation methods. Adaptation rules based on Linear combining decision device and Run and Go decision device are discussed in Section III. Optimization of the soft decision devices is studied in Section IV. Section V presents simulation results, and Section VI concludes.

## II. Soft Decision DFE and DFE Blind Adaptation

### 1. Soft decision DFE

Consider a DFE system model as described in Figure 1. A sequence of sources  $\{s_k\}$  is transmitted through a multipath channel  $\mathbf{c}$  and additive white

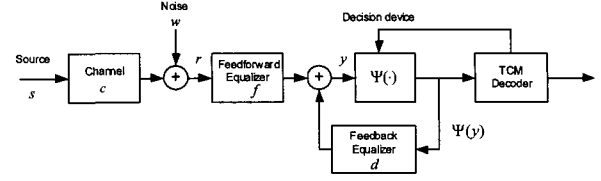


그림 1. DFE 시스템의 Block Diagram

Fig. 1. Block diagram of the DFE system.

Gaussian noise (AWGN)  $\{W_k\}$  of variance  $\sigma_w^2$ . The received signal,  $\{r_i\}$ , is processed by a linear equalizer  $\mathbf{f}$  and the resulting signal is subtracted with the feedback filter  $\mathbf{d}$  output to generate the equalizer output  $\{y_k\}$ .

Ignoring the system delay issue, the equalizer output  $y(k)$  can be written as

$$y_k = \alpha s_k + z_k, \quad (1)$$

where  $z_k$  comprises the residual inter-symbol interference and colored noise terms [10] and  $\alpha$  denote the bias term from the MMSE DFE [11]. In most practical DFE implementations, the bias is removed by a multiplier as considered in [12]. Hence, without loss of generality, we assume  $\alpha = 1$  for decision device derivation. The equalizer output  $y_k$  is sent to a decision device  $\Psi$  and the decision  $\Psi(y_k)$  are fed back to the feedback filter  $\mathbf{d}$ . In a DFE with soft decision device, the decision device  $\Psi$  is optimized to minimize the equalizer output MSE  $\sigma^2$ .

$$\Psi_{opt} = \arg \min_{\Psi} E \| y_k(\Psi) - s_k \|^2. \quad (2)$$

Assuming reasonable statical properties for  $\{Z_k\}$  as done in [3,13], including that  $\{Z_k\}$  is a zero mean Gaussian, the optimal device is given by  $\Psi_{opt} = E\{s_k | y_k\}$ . Specifically, for BPSK signals the optimal decision device becomes

$$\Psi_{opt}(y_k) = \tanh\left(\frac{y_k}{\sigma^2}\right). \quad (3)$$

### 2. Blind Adaptation of DFE

In practice, adaptive methods are commonly used to obtain the MMSE equalizer coefficients. Currently,

in most equalization system a blind IIR algorithm is used for acquisition state and once reliable decisions are available a DD-LMS algorithm is applied to find MMSE coefficients. A general adaptation rule can be written in the following forms

$$\mathbf{f}_n(k+1) = \mathbf{f}_n(k) + \mu r_k^* e(k) \quad (4)$$

$$\mathbf{d}_n(k+1) = \mathbf{d}_n(k) - \mu \Psi^*(y_k) e(k), \quad (5)$$

where  $f_n(k)$  and  $d_n(k)$  denote the  $n$ -th element of  $\mathbf{f}$  and  $\mathbf{d}$  at the  $k$ -th iteration, respectively,  $e(k)$  denotes an error term at  $k$ , and  $(\cdot)^*$  denotes conjugator.

Several blind adaptation approaches generating the error term have been proposed<sup>[4], [5]</sup>. Among them, a method widely used for the acquisition stage (closed eye situation) in practice as proposed in [5] uses constant modulus algorithm (CMA), a popular blind adaptive algorithm for linear equalizers [14], to minimize dispersion of the IIR-filter output,  $J_{CMA} = E(|y_k|^2 - \gamma)^2$ , where  $\gamma$  is known as dispersion constant, by using  $e(k) = y_k(|y_k|^2 - \gamma)$  with the trivial feedback sample generator  $\Psi(y_k) = y_k$ .

Once reasonable estimation of  $\mathbf{f}$  and  $\mathbf{d}$  are obtained, DD-LMS algorithm is applied to track possible time variation of channel, which uses the error term  $e(k) = y(k) - \hat{y}(k)$  with the hard decision feedback samples, i.e.  $\Psi(y_k) = \hat{y}_k$ . DD-LMS achieves MMSE performance compared in the absence of decision errors, but lacks acquisition ability. Therefore, it is desirable to rely on IIR algorithms as little as possible, while maintaining (re)acquisition ability.

In the following section we develop adaptation methods to combine IIR algorithms and DD-LMS algorithms based on adaptive soft decision devices that control the transition between IIR and DD-LMS algorithms.

### III. Proposed Blind DFE Adaptation Schemes

The goal of this section is to provide a strategy to generate the error term  $e$  and the feedback sample generator, or decision device,  $\Psi$  which minimizes the output MSE and has capability of smooth transition from IIR to DD-LMS. The proposed strategies will update the adaptation rule and, at the same time, the feedback sample by changing the decision device as illustrated in Figure 2.

In this paper, without loss of much generality, we focus on PAM cases. For since it can be easily generalized for other constellations (such as QAM). For the QAM case, once can use a blind carrier phase recovery loop such as [?] along with the following adaptation algorithms to remove phase rotation, which cannot be resolved by a CMA-type blind equalizer.

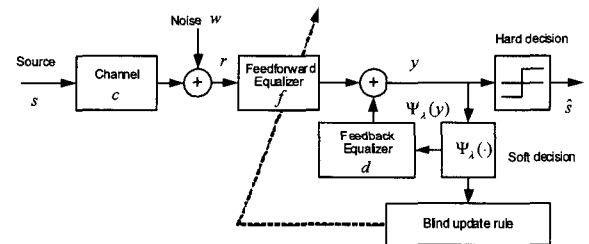


그림 2. 제안된 소프트판정을 이용한 자력복구 DFE 시스템

Fig. 2. Propose adaptive blind adaptation for DFE with Soft Decision Device.

#### 1. Linear Combining Scheme

Define a family of decision devices  $L_\lambda$  for  $\lambda \in [0, 1]$  as

$$L_\lambda(y) = \lambda y + (1 - \lambda) \hat{y} \quad \text{for } 0 < \lambda < 1. \quad (6)$$

$L_\lambda$  linearly combines hard decision and raw output as done in [8] (see Figure 3-a). Using this decision device, the feedback sample  $L_\lambda(y)$  is applied to the feedback filter  $\mathbf{d}$  and the error term is generated as the following

$$e(k) = \lambda y_k (|y_k|^2 - \gamma) + (1 - \lambda)(y_k - \hat{y}_k). \quad (7)$$

These update equations can be viewed as stochastic updates obeying an amalgamated cost function of DD-LMS and IIR-CMA. As  $\lambda$  varies 0 to 1, the update rule varies from DD-LMS to IIR-CMA and the decision device transforms from the slicer to identity function.

## 2. Run and Go scheme

For  $2M$ -level PAM signals, let  $\Gamma$  be the radius of the constellations, i.e.

$$s_k \in \{-(2M-1)\Gamma, \dots, -\Gamma, \Gamma, \dots, (2M-1)\Gamma\}. \quad (8)$$

Recall that for  $2M$ -PAM the hard decision  $\hat{y}$  is given by

$$\hat{y} = \arg \min_{(2k-1)\Gamma} |y - (2k-1)\Gamma| \quad (9)$$

Similarly, we define the nearest boundary value of  $y$ , denoted  $y$ , as the following

$$y = \arg \min_{2k\Gamma} |y - 2k\Gamma| \quad (10)$$

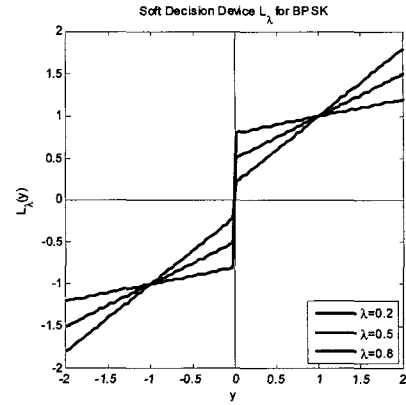
Using these notations, we define a family of soft decision device  $T_\lambda$  for  $\lambda \in [0, 1]$

$$T_\lambda = \begin{cases} \frac{y-y}{\lambda} + y & \text{if } |y-y| < \lambda\Gamma \\ \hat{y} & \text{else} \end{cases} \quad (11)$$

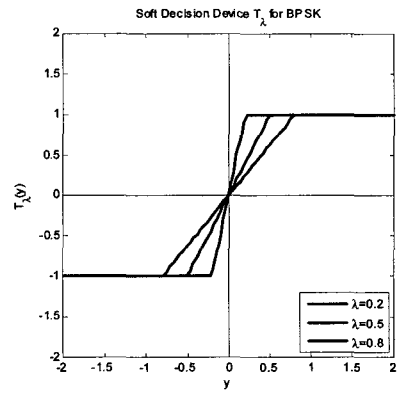
Correspondingly, we update equalizer coefficients with the following error term

$$e(k) = \begin{cases} y_k(|y_k|^2 - \gamma) & \text{if } |y-y| < \lambda\Gamma \\ y_k - \hat{y}_k & \text{else} \end{cases} \quad (12)$$

As  $\lambda$  varies from 0 to 1, the above update rule changes from DD-LMS to IIR-CMA algorithm and the decision device transforms from slicer to identity function (see Figure 3-b)). In stead of stopping adaptation for the unreliable region  $|y-y| < \lambda\Gamma$  as in Stop and Go algorithm<sup>[16]</sup>, the proposed algorithm applies a blind adaptation (Hence, Run and Go"). Although simulation examples never fail to converge, assuring stability and convergence issues of this algorithm is an extremely difficult task and beyond the scope of this paper.



a) Linear Combining



b) Run and go Combining

그림 3. lambda 값 변화에 따른 판정기의 변화  
Fig. 3. Decision Devices for different.

## IV. Adaptation Optimization

The proposed adaptation rules and decision devices are parameterized by  $\lambda \in [0, 1]$  and  $\lambda$  determines the "blending ratio" between IIR and DD-LMS adaptation. In this section, we optimize  $\lambda$ , i.e. determining adaptation rule between IIR and DD-LMS adaptation, such that the resulting decision device is optimal in the MSE sense. W.

Recall that the DFE output is written as

$$y_k = s_k + z_k. \quad (13)$$

We investigate the optimal  $\lambda$  for the given  $\sigma^2 (= E(|z_k|^2))$  under the following assumption

- i)  $\{s_k\}$  is an i.i.d. sequence.
- ii)  $z_k$  is a zero mean Gaussian random process

with variance  $\sigma^2$ .

iii)  $\{S_k\}$  and  $\{Z_k\}$  are uncorrelated.

### 1. Optimal Linear Combining Adaptation

The optimal  $L_\lambda$  for given  $\sigma^2$  is obtained by  $\lambda_\sigma$  satisfying

$$\lambda_\sigma = \arg \min_{\lambda} E(L_\lambda(y_k) - s_k)^2 \quad (14)$$

Using  $L_\lambda = \lambda y_k + (1-\lambda)\hat{y}_k$ , we have a quadratic equation of  $\lambda$  by expanding  $E(L_\lambda(y_k) - s_k)^2$ . Hence the optimal  $\lambda$  is given as

$$\lambda_\sigma = \frac{E(\hat{y}_k - s_k)^2 - E\{\hat{y}_k z_k\}}{E(\hat{y}_k - s_k)^2 - 2E\{\hat{y}_k z_k\} + \sigma^2} \quad (15)$$

By calculating  $E(\hat{y}_k - s_k)^2$  and  $E\{\hat{y}_k z_k\}$ , one can obtain the optimal  $\lambda$  for a given  $\sigma^2$ . In practice, the  $\lambda_\sigma$  can be computed based on numerical simulation and used as a table saved in a memory. In BPSK case a closed form expression of  $\lambda_\sigma$  can be obtained as the following

$$E(\hat{y}_k - s_k)^2 = 2^P \{\hat{y}_k \neq s_k\} = 4Q\left(\frac{1}{\sigma}\right) \quad (16)$$

$$E\{\hat{y}_k z_k\} = \frac{2}{\sqrt{2\pi\sigma}} \int_1^\infty x e^{-\frac{x^2}{2\sigma^2}} = \sigma \sqrt{\frac{2}{\pi}} e^{-\frac{1}{2\sigma^2}}, \quad (17)$$

where  $Q(\cdot)$  is the Q-function. Consequently, we have the optimal combining weight for linear combining decision device

$$\lambda_\sigma = \frac{4Q\left(\frac{1}{\sigma}\right) - \sigma \sqrt{\frac{2}{\pi}} e^{-\frac{1}{2\sigma^2}}}{4Q\left(\frac{1}{\sigma}\right) - 2\sigma \sqrt{\frac{2}{\pi}} e^{-\frac{1}{2\sigma^2}} + \sigma^2} \quad (18)$$

### 2. Optimal Run and Go Adaptation

The optimal Run and Go device for given  $\sigma^2$  is obtained by

$$\lambda_\sigma = \arg \min_{\lambda} E \|T_\lambda(y_k) - s_k\|^2. \quad (19)$$

The expectation value expands as

$$\begin{aligned} & E(T_\lambda(y_k) - s_k)^2 \\ &= E_{|y-y|<\lambda\Gamma} \left\{ \left( \frac{y_k - y_k}{\lambda} + y_k - s_k \right)^2 \right\} \\ &+ E_{|y-y|>\lambda\Gamma} \left\{ (\hat{y}_k - s_k)^2 \right\} \end{aligned} \quad (20)$$

Unlike the linear combining case, obtaining the closed form optimal  $\lambda$  from (20) is quite difficult, although numerical simulation method can be used for  $\lambda_\sigma$ .

To obtain a closed form expression for BPSK signals, we simplify the above equation using the first order Taylor approximation of Gaussian p.d.f.

$$\frac{e^{-\frac{x^2}{2\sigma^2}}}{\sqrt{2\pi\sigma}} \approx \begin{cases} \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2\sigma^2}} x & (0 < x \leq \sigma^2) \\ 0 & (x > \sigma^2) \end{cases} \quad (21)$$

Assuming  $\sigma^2 \leq 1$ , we now have

$$E(T_\lambda(y_k) - s_k)^2 \approx \frac{4\sigma}{\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}} \quad (22)$$

for  $\lambda \geq \sigma^2$  and for  $(\lambda \leq \sigma^2)$

$$\begin{aligned} & E(T_\lambda(y_k) - s_k)^2 \approx \\ & \frac{e^{-\frac{1}{2\sigma^2}}}{\sqrt{2\pi}} \left[ \frac{4}{3\sigma^3} \lambda^2 + \frac{8}{3\sigma} \left(1 + \frac{1}{\sigma^2}\right) \lambda + \frac{1}{2\sigma^3} (\lambda - \sigma^2)^2 \right] \end{aligned} \quad (23)$$

The approximation is a quadratic function of  $\lambda$  for  $0 < \lambda \leq \sigma^2$  and has the minimum at  $\lambda = \sigma^2$ .

Therefore, the optimal  $\tilde{\lambda}_\sigma$  minimizing the approximated MSE is given by

$$\lambda_\sigma = \sigma^2 \quad (24)$$

This approximation turns out to be the piece-wise linear approximation of the optimal decision device  $\tanh(y_k/\sigma^2)$  as proposed in [13]. For higher order-PAM and QAM,  $\lambda_\sigma$  can be approximated

using the piece-wise linear approximation of the soft decision device as in [13].

### V. Simulation

In this section we present simulation results of our proposed soft decision device with DFE adaptation. In practice  $\sigma^2$  can be obtained from training sequence or approximately estimated by computing  $E(y_k - \hat{y}_k)^2$  based on the block-by-block calculation or leakage integration. The first simulation result, Figure 4, shows acquisition ability of the proposed algorithms in comparison with IIR-CMA. BPSK signals are transmitted through a severe multipath channel  $c_1 = [0.3, 1, 0, 0.2, 0, 0.7, 0, -0.5]$  under 30dB SNR. The DFE is equipped with a 10 tap feedforward filter and a 10 tap feedback filter, initialized with conventional single spike method<sup>[14]</sup>, all zeros but  $f_s = 1$ . Both proposed algorithms successfully equalize the received signal with satisfactory residual MSE for  $\mu = 0.001$ , while IIR-CMA alone barely successes to open eyes for the same step size. The second example shows tracking ability of the proposed algorithms in the presence of sudden change of the channel. The channel in the first simulation,  $c_1$ , changes abruptly to  $c_2 = [1, 0, 0, 0, 0, 0, 0, 0.3]$  at the 2,500th baud sample. As shown in Figure 5 the both proposed

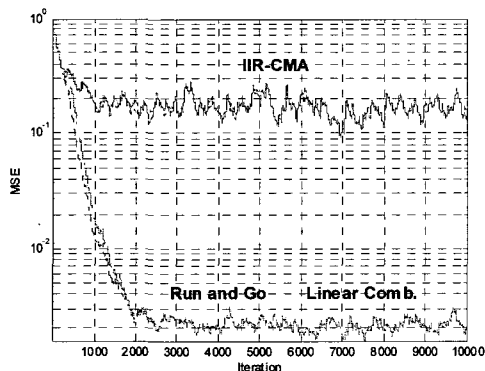
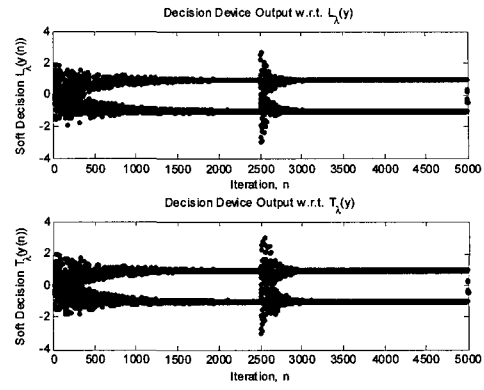
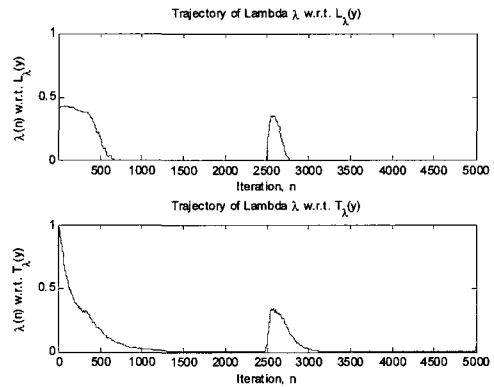


그림 4. 제안된 DFE들의 초기화 신호 포착 성능  
Fig. 4. Acquisition ability of proposed DFEs.



a) Soft Decision Device Outputs



b) Trajectories of  $\lambda$

그림 5. 제안된 DFE들의 신호 재포착 성능  
Fig. 5. Re-acquisition ability of proposed DFE adaptation methods

algorithms successfully track the transition. Figure 5-a) show the equalized symbols as the DFE runs. Clearly, one can observe a burst of symbols during the transition of channels from  $c_1$  to  $c_2$ . Figure 5-b) shows the trajectories of  $\lambda$  for the both proposed algorithms. As the estimated MSE of DFE output increases in the transition period, the  $\lambda$  value of each algorithm increase to re-acquire data. Once the algorithms transit from DD-LMS to mixed ones, the MSE of DFE output decreases and finally the algorithm returned to DD-LMS.

### VI. Conclusion

In this paper, we have proposed blind DFE adaptation methods based on adjustable soft decision

devices. The proposed algorithms, Linear Combining method and Run and Go method, blend blind IIR-CMA adaptation for the acquisition stage and DD-LMS adaptation for the tracking stage. We proposed to optimize the combined adaptation rule by optimizing the soft decision device for given SNR. We presented closed form expression of such optimization for BPSK signal. Simulation results show that the proposed algorithms successfully deal with dynamic change of channels by smoothly changing adaptation modes. Rigorous proof on the stability and convergence of the proposed algorithms are subjects of future study.

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저 자 소 개

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정 원 주(정회원)

received the B.A. degree in mathematics from Korea University, Seoul, Korea and the M.S. and Ph.D. degrees in Electrical Engineering from Cornell University, Ithaca, NY.

He has worked as the Senior System Architect at Dotcast, Inc, Seattle, WA. He is currently Associate Professor at Myong Ji University. His main research area includes digital signal processing for digital communication systems.