Adaptive Fault Diagnosis using Syndrome Analysis for Hypercube Network

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ABSTRACT

System-level diagnosis plays an important technique for fault detection in multi-processor systems. Efficient diagnosis is very important for real timesystems as well as multiprocessor systems. Feng[1] proposed two adaptive diagnosis algorithms HADA and IHADA for hypercube system. The diagnosis cost, measured by diagnosis time and the number of test links, depends on the number and locaton of the faults. In this paper, we propose an adaptive diagnosis algorithm using the syndrome analysis. This removes unnecessary overhead generated in HADA and IHADA algorithmsand give a better performance compared to Feng's Method.

Key Words : Hypercube, Fault, Diagnosis, Adaptive, Syndrome

I. Introduction

The high complexity of a multiprocessorsystem frequently generates failure. The downtime and repair cost caused by a failure seriously offset the benefit of multiprogramming. Among the various multicomputer available in the market, hypercubes are very popular structures. A hypercube has the property of regularity and hierarchy to develop a highly efficient fault diagnosis scheme.

The most well-studied model to represent a diagnosable system S is the model introduced by Preparata et al(henceforth referred to as the PMC model)[3], in which the system is represented by n unit level, each unit is tested by several other units of the system.

This model can be represented by a digraph $G=(V,E)$ where V is a set of nodes, $[V]=n$, each node representing a unit and E is the set of edges, each edge $(u(i),u(j))$ is labeled with $a(i,j)$ to represent the result of testing : $a(i,j)= 0$ if $u(i)$ evaluates $u(j)$ to be fault-free and $a(i,j)=1$ if $u(i)$ evaluates $u(j)$faulty. A set of all $a(i,j)$'s is said to be a syndrome of S. S is called t-diagnosable if for every syndrome, the faulty units can be uniquely identified so long as the number of faulty units does not exceed t.[3] If there are more than d faulty units in the system, which is referred as the over-d faulty problem[3], complete diagnosis cannot be guaranted but correct diagnosis should be pursued.

Nakajima has proposed an adaptive diagnosis algorithm in [4] to dynamically assign tests instead of assigning all tests and then decoding the outcomes, proposed earlier. Thus, adaptive diagnosis requires a low number of tests. However, a completely connected system must be assumed. This restricts its applicability.

There are two measures of cost for on-line system level diagnosis : (1) Diagnosis time. (2) number of test links. We assume the diagnosis is performed without shutting down the whole system.

Since hypercube has the property of regularity and hierarchy, it is widely used for parallel processing system. In this study, we use the host ofhypercube system as a syndrome analyzer. Diagnosis is measured by the number of rounds. Within the round, tests are assumed to be performed in parallel using one unit of time. We also assume that overhead for the communication of the test syndrome is negliglible compared to the testing among units. Also unit can

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not be tested and test its neighbors at the same time.

Ⅱ. Previous Results

Feng[1] proposed a new scheme for adaptive system level diagnosis for hypercube systems. In order to obtain formal analysis results, Feng first proposed a conceptual diagnosis algorithm HADA(Hypercube Adaptive Diagnosis Algorithm). Then they present its practical variant IHADA(Improved HADA) which is more efficient.

Hypercube system have its hierarchical construction, i.e, an n-cube consists of two (n-1)-cubes. Moreover, a ring which consists of all nodes in an n n-cube can be found by using gray code[5]. On a ring, a single loop testing can be performed, which is a minimal-edge test assignment[6]. The basic idea of Feng[1]'s approach is take advantage of the hierarchical feature so that the diagnosis process can be performed in a divide-and-conquer fashion. HADA algorithm either assigns tests to subcubes when faulty unit cannot locate(but detected), or terminate otherwise. A status-identified subcube is used to diagnosis its counterpart. At each iteration, tests are assigned in a ring embedded in the subcube rather than in all network links of the subcube.

If a ring, represented by the set R, is tested bidirectionally, the testing graph has a connectivity of 2. By [2], the testing graph is *1-FL-(|R|-2)-FD*, where |R| is the number of units in the ring. Here, *t-FL-u-FD*(t-fault-locating-*u*-fault-detecting) implies that the faulty units can be identified provided that the number of faulty units is $\leq t$, and the existence of more than t but no more than $u$faulty units can be detected. Hence, if there is only one faulty unit in a ring, then it must be the only unit which is tested by both of its neighbors with test outcomes of ls.

Feng[1] use the property of ring and diagnosis hypercube system as follows. First, they embed ring to hypercube using *RGC*(reflected-gray-code). Then they split ring into two directed subcubes which contains at most one faulty unit. If there is subcube which contains at most one faulty unit, the central observer analyzes again the syndrome of the previous testing phase to identify the remaining faulty units.

Ⅲ. System-level diagnosis for hypercube using Adaptive cube Partition

HADA and IHADA algorithms are lower in cost compared to previous algorithms and these present a better algorithm for over-d fault problem which is inherently tackle through a deterministic method. But Feng did not fully consider the hypercube property when split ring into subrings (subcube). HADA and IHADA's *i*th split are performed by d_i(*node number* : $d_n d_{n-1} \cdots d_2 d_1$) in predetermined pattern. But there are n possible way to split n-cube into 2*(n-1)-cubes. Accordingly, if we choose the split method which find the subcube contain minimum number of faulty units, we can reduce the number of splits.

In this paper, we propose an hypercube diagnosis algorithm which split cubes using syndrome analysis. We expect non-faulty units using syndrome analysis which found during the split(n-cube into 2*(n-1)-cubes). We select a split method which generate maximum number of non-faulty units in (n-1)-cube. Then, we show that the proposed algorithm give a better in diagnosis time, number of tests and the number of test links compared to HADA and IHADA algorithm.

We Propose a diagnosis algorithm as given below.

Diagnosis(n)
Input : n-cube
Step1 : Map n-cube into ring using RGC (Reflected Gray Code)
Step2 : Call *Ring_Diagnosis(n, 0)*
If number_of_faulty_node in ring <= 1 than stop
// You can find the location of faulty unit //
else goto Step 3.
Step3 : *d =Find_Dimension_of_Partition(n)* // Split degree(d) is returned
Perform *Subcube_Diagnosis(n, d)* using d which generated by syndrome analysis

Function *Ring_Diagnosis(n, d)*
Input : n-cube, dimension of split(d)
Step1 : If d==0 then perform the *clockwise_testing*
else nodes which have done clockwise_testing perform corresponding testings

while remaining nodes do *clockwise_testing* // Parallel processing //

Step2 : If d==0 then perform the *counterclockwise_testing*
else nodes which have done counterclockwise_testing perform corresponding testings while remaining nodes do *counterclockwise_testing* // Parallel Processing //
If only one node reponses both tests1 in Step1 and Step2 with test outcomes of 1's, then this is faulty.
else *n*-cube is faulty node and needs further tsests to locate faulty units'.
return(diagnosis results)

Function ***Find_Dimension_of_Partition(n)***
Input : Syndrome of *n*-cube
Step1 : Build a set of units FF_{guess} where both the test outcomes generated by clockwise or counterclockwise are fault-free and adjacent nodes in RGC is not tested faulty from clockwise and counterclockwise testings.
Step2 : Assume that *n*-cube is split into two (n-1)-subcubes such as $SC0_i$ and $SC1_i$ by d_i of $d_n\ d_{n-1} \cdots d_2 d_1$. Let $FF0_i = FF_{guess} \cap node(SC0_i)$ and $FF1_i = FF_{guess} \cap node(SC1i)$.
Return(i) where, $k_i = \max_{1 \le i \le n} \{|FF0_i|, |Ff1_i|\}$

Function ***Subcube_Diagnosis(n, d)***
Input : *n*-cube, *d*
Step1 : Split n-cube into two (*n*-1) subcubes using *d*
Step2 : Perform Ring_Diagnosis(*n-1, d*) for each (*n-1*)-subcubes.
Step3 : Perform *d0=Find_Dimension_of_Partition(SC0)* and *d1=Find_Dimenstion_of_Partition(SC1)* using the subcubes generated in Step2.
Step4 : Perform *Subcube_Diagnosis(SC0, d0)* and *Subcube_Diagnosis(SCI, d1)*

Ⅳ. Correctness of Algorithm

We compare the proposed algorithm with the algorithm of HADA. [1] uses testing rounds and test links for performance cost evaluation. Feng[1] assumes that a node can not be tested while it is performing tests.

Therefore, on a ring, a single loop of testing can be performed in two rounds. Based on the assumption, the testing rounds and test links are as follows. Let $TL_n\ (f_1,f_2)$ and $TR_n(f_1,f_2)$ be the number of testing rounds and number of test links required in HADA. Let f_1,f_2 bethe number of faulty units in each subring when n-cube is subdivided into 2*(n-1)-subcubes.

Theorem1. *the number of test links required in HADA is*[1]:

$$TL_n(f_1,f_2) = \begin{cases} 2^n \\ 3\times 2^{n-1} \\ \\ 3\times 2^{n-1}+1 \\ 2^{n-1}+TL_{n-1}(f_1,f_2)+TL_{n-1}(f'_1,f'_2) \end{cases}$$

$f_1 = 0$ *and* $f_2 = 0,1$
$f_1 = 0$ *and* $1 < f_2 \le n;$ *or*
f_1 *and* $1 \le f_2 \le n-1$
f_1 *and* $1 < f_2 \le n-1$
$f_1, f_2 > 1, 1 < f_1 + f_2 \le n,$
$f_1 + f_2 = f_1,$ *and* $f'_1 + f'_2 = f_2$

the required diagnosis time is[1]:

$$TR_n(f_1,f_2) = \begin{cases} 2 \\ 4 \\ 5 \\ 6 \\ 7 \\ 2+\max(TR_{n-1}(f_1,f_2), TR_{n-1}(f'_1,f'_2)) \end{cases}$$

$f_1, f_2 = 0$
$f_1 = 0$ *and* $f_2 = 1$
$f_1 = 0$ *and* $1 < f_2 \le n$
$f_1 = 1$ *and* $f_2 = 1$
$f_1 = 1$ *and* $1 < f_2 \le n-1$
$f_1, f_2 > 1, 1 < f_1 + f_2 \le n,$
$f_1 + f_2 = f_1,$ *and* $f'_1 + f'_2 = f_2$

The proposed algorithm in this paper requires same number of diagnosis time and test links as in [1] if the range of the faulty units in pre-divided subring is as follows.

$f_1, f_2 = 0,$
$f_1 = 0$ and $f_2 = 1,$
$f_1 = 0$ and $1 < f_2 \le n$

On the other hand, it requires less diagnosis time and test link if the range of the faulty units as follows.

$f_1 = 1$ and $f_2 = 1,$
$f_1 = 1$ and $1 < f_2 \le n-1,$
$f_1, f_2 > 1,\ 1 < f_1 + f_2 \le n,\ f_1 + f_2 = f_1$ *and* $f'_1 + f'_2 = f_2$

Lemma 1. *Let map the ring using RGC and the maximum number of continuous faulty units in a ring be MAX(t), Then, if* $MAX(f) \le 4$, $n \ge 3$ *is true, then the set of nodes* FF_{guess}*which is predicted to be fault-free by find_Dimension_of_Partition() sat-*

isfies the following conditions.

$FF_{guess} \subset F$, where FF is a set of fault-free nodes

Proof : *We consider four cases for MAX(f)*

Case 1: $MAX(f)=1$, $n \geq 2$.

The faulty unit which is adjacent with fault-free unit in clockwise as well as counterclockwise is tested with 1. The faulty unit can be removed from FF_{guess} according to the step1 of *Find_Dimension_of_Partition(). Therefore, $FF_{guess} \subset F$.*

Case 2: $MAX(f)=2$, $n \geq 3$.

Twoconsecutive faulty units have one fault-free neighbor respectively. Therefore, faulty unit is tested as 1 by fault-free neighbor at least one of the test directions. The two faulty units can be removed from FF_{guess}.

Therefore, $FF_{guess} \subset F$.

Case 3: $MAX(f)=3$, $n \geq 3$.

Two faulty units out of three consecutive faulty units are adjacent to one fault-free neighbor respectively.

These faulty unit are tested as 1 at least one test direction(Clock or counterclockwise), so the faulty units are removed form FF_{guess}. A faulty unit surrounded by faulty units is guessed as a faulty unit by step1, it is removed from FF_{gues} by step1. So, all three faulty units are removed. Therefore, $FF_{guess} \subset F$.

Case 4: $MAX(f)=4$, $n \geq 3$.

Two faulty units from the four consecutive faulty units are adjacent to one fault-free neighbor respectively.

These units are tested as 1 by at least one of tests and removed from $FF_{guess}$by step1. And two faulty units which are surrounded by faulty units are adjacent to two units which are predicted as a faulty units by step1.

These are also removed from FF_{guess}. Therefore all four units are removed from FF_{guess}. *Therefore, $FF_{guess} \subset F$.*

According to HADA method, splits are made until we find a subring which contains at most one faulty unit and the *i*th split is performed by d_i(node number: $d_n d_{n-1} \cdots d_2 d_1$). But, the proposed algorithm choose the split method which generate the maximum number of fault-free units using *Find_Dimension_of_Partition()*. This implies that we choose the split method which put the faulty nodes in one of the subring as much as possible.

Theorem 2. If *$|FF_{guess}| \geq 2^{n-1}$ when $MAX(f) \leq 4$ and $n \geq 3$ is true and thee exits a split bit by which (n-1)-cube can be generated with subset of FF_{guess}, then we can find a subring which contains only fault-free nodes.*

Proof : We represent the node number of *n*-cube as $d_n d_{n-1} \cdots d_2 d_1$. We select a partitition bit d_i to make (n-1) cube of 1's and (*n-1*)-cube of 0's[1]. Let's represent the two cubes as $SC0_i$, $SC1_i$. Then there are $\frac{2^n}{2}$

units in each of (*n-1*)-cubes and the number of (*n-1*) cubes is 2 x since 1 $\leq \leq i \leq \leq$ *n*. If $|NF| \geq 2^{n-1}$ and there exists (n-1)-cube $SC*_j$ (* means 0 or 1) which is formed from the subset of FF_{guess} and belongs to the 2 x(*n-1*)-cubes, then all the units in $SC*_j$are fault-free from Lemma 1. This implies the diagnosis is performed correctly[2]. There also exists $\frac{2^n}{2}$edges between nodes generated two (n-1)-cubes $SC0_j$, $SC1_j$. $SC*_j$ can diagnoses the corresponding subcubes using the edges generated by d_j

The proposed algorithm requires small amount of diagnosis timeand test links compared to HADA if the system satisfies the condition of Lemma2. But in the worst case, it requires same amount of diagnosis time and test links.

Example. Consider 4-cube where 0001, 0010, 0110 and 0101 are fault units. In the HADA algorithm, first split is performed by d_1, and 2*3-subcubes which contain one fault unit in each subcube are made. Because each of subcubes can not be diagnosed correctly, second split is performed by d_2, and 4*2-subcubes are made. Then each of 2*2-cubes containstwo faulty units and other 2-cubes contain only fault free units. Finally, diagnosis can be completed. The required diagnosis time and test links are 7 and 32 respectively. But in the proposed algorithm, the first split is performd by d_4 and a 3-subcube $SC1_4$which contains only fault

Table 1. Fault Distribution Cases After One Step of Cube Partition

Hypercube Dimension	HADA			Proposed Algorithm		
	0 : n	1 : n-1	Number of Partitions <= 1	0 : n	1 : n-1	Number of Partitions <= 1
4	793	4848	5641	2197	5641	7838
5	429	2883	3312	1542	5520	7062
6	229	1761	1990	1005	4932	5937
7	145	1027	1172	719	3995	4714
8	52	589	641	408	2930	3338
9	21	272	293	253	2164	2417

Table 2. Fault Distribution Cases of 6-Cube After One Step of Cube Partition

Number of Faults	HADA			Proposed Algorithm		
	0 : n	1 : n-1	Number of Partitions <= 1	0 : n	1 : n-1	Number of Partitions <= 1
2	4924	5076	10000	9676	324	10000
3	2362	7638	10000	7254	2746	10000
4	1122	4986	6108	4222	5463	9685
5	530	3043	3573	2172	6070	8242
6	239	1782	2021	1041	4946	5987
7	89	920	1009	487	3248	3735
8	40	491	531	197	1888	2085
9	10	203	213	89	1054	1143

free units is made. Therefore, only one step of split is enough to complete diagnosis. The required diagnosis time and test links are 5 and 24respectively.

V. Simulation Results

In this section, we present the simulation results on the proposed algorithm. We carry out the experiments by randomly located 10,000 faults pattern with equal fault occurance rate and then we observe the change of fault distributions after the first step of cube partition. Table 1. shows the results for HADA[1] and the proposed algorithm, respectively.

We can see that the proposed algorithm gives better system diagnosability using one or less split. For example, for the six faults 6-cubes, HADA[1] can diagnose for 1,990 cases but the proposed algorithm can diagnose for 5,939 cases. This shows that the proposed algorithm need less number of test levels compared to HADA when |f|=n.

Table 2 shows the simulation result for 6-cube. We perform the simulation under the same conditions as in table 1. Table 2 shows that the proposed algorithm give a better diagnosability compared to HADA's.

Ⅵ. Conclusions

System-level diagnosis is very important technique for multiprocessor systems and real-time system. It must be efficient as well as have a low overhead. Feng[1] proposed HADA and IHADA algorithms for hypercube system.

Feng maps hypercube into ring then subdivide ring until they find a ring which containsone or less faulty unit. But they subdivide ring without syndrome analysis. This requires an unnecessary overhead for ring partition. In this paper, we propose an algorithm using syndrome analysis which predicts fault-free units and generates subcubes using the predicted units. This reduces the number of steps required in Feng's algorithm.

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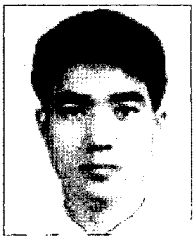

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