Journal of Electrical Engineering & Technology, Vol. 1, No. 3, pp. 327~331, 2006 327

Analysis of Torque Characteristics for the Single-phase Induction Motor Considering Space Harmonics

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Abstract - This paper presents the analysis method for the torque characteristic of the 1-phase induction motor considering space harmonics in the air gap. The equivalent circuit method is used, where the circuit constants are obtained by classic theory. In addition, the space harmonic components in air gap magneto motive force are analyzed and added to the equivalent circuit to obtain accurate torque characteristics in low speed regions. Each torque component due to the harmonics is calculated and the total torque characteristic is obtained and compared with the measurement result.

Keywords: Pull-up torque, Single-phase induction motor, Space harmonics, Starting torque

1. Introduction

It is more difficult to estimate the starting and the pull-up torque of the 1-phase induction motor than it is for the 3-phase one. This is due to the several differences that exist between them. First, the 1-phase induction motor has unbalanced impedance and therefore it doesn't create the unidirectional rotational magnetic field when the rotor moves. Second, generally it has skewed rotor bars to reduce harmful effects from the space harmonics [1]. Third, most 1-phase induction motors have concentric winding and not short pitch winding.

Two methods have recently been used to analyze induction motors, which are the numerical approach such as FEA and the equivalent circuit method. However, both methods have several limits when estimating the starting performance of the 1-phase motor sufficiently due to the above three differences; as follows.

First of all, the FEA as a numerical method is being widely used with very high accuracy in electric machine analysis. In particular, a time harmonic approach is very profitable from the viewpoint of solving time. But it is not applicable to the 1-phase motor because of the first difference, that is, the unbalanced and eccentric magnetic field. As such, the tedious time stepping FEA should be performed. Second, the effects of the skewed rotor bar can be considered by only the 3D or the quasi-3D analysis [2-3], which is very expensive in many aspects. In many cases, undoubtedly the 2D FEA is being used for the 3-phase induction motor, but it can be explained by the third difference. The 3-phase type has conventionally fixed distributed winding and short pitch winding. Therefore, its harmonic components are depressed considerably and the analysis results of the 2D FEA do not provide a great deviation from those of the 3D analysis. However, in case of the concentric winding of a 1-phase induction motor, the machine designer should determine the winding turns for each slot. Moreover the winding doesn't have short pitch. So the harmonic components are controlled by only the rotor skewing and the winding specification from the designer's experience. Consequently for accurate analysis of a 1-phase induction motor, the 3D time stepping analysis should be performed. The 2D FEA may exaggerate the harmonic effects severely in the low speed region because the cusp or crawling phenomena takes place in the region due to the space harmonics.

The equivalent circuit, as a classic tool for analyzing the induction motor, consists of several electric constants. It has advantages to analyze the 1-phase induction motor with the 3 differences. The first difference, the unbalance impedance and eccentric field is solved simply by the revolving field theory [4]. The second difference, the skewed rotor bar is also easily considered by putting the skew leakage into the leakage reactance and the third one can be dealt with short pitch factor. But the magnetizing reactance, X_m of classic circuit constants is predicted by considering the only fundamental component in the air gap. This means that every harmonic component is neglected in the electromagnetic energy conversion process contrary to the 2D FEA; therefore the extraordinary phenomena due to the space harmonics are rarely obtainable by using the conventional equivalent methods.

To fill up the weak points of the equivalent circuit, this paper presents a modified circuit considering space harmonics. The space harmonics components are extracted

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Received: December 9, 2005 ; Accepted: February 18, 2006

from air gap MMF distribution produced by the stator current and then the equivalent circuit including the harmonics is constructed. The performances are calculated by the circuit analysis and the effects to the low speed characteristics due to the harmonics are discussed.

2. Analysis Model

A commercial 1-phase induction motor is selected for analysis and the specification is given by Table 1 while its overall shape and winding pattern are depicted in Fig. 1. The stator core has 14 slots per pole, and 2 slots among them are small.

Table 1. Specification of analysis model

Item	Value	Item	Value
Voltage	220V	Air gap	0.3mm
Frequency	60Hz	Run-Cap.	0.5uF
No. of Poles	2	Start Method	PTC
O.D of Stator	130mm	I.D of Stator	90mm
Coil turns(main)	798	Coil turns(aux.)	458

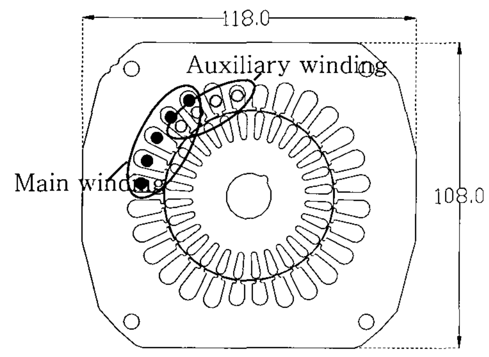

Fig. 1. Analysis model

3. Equivalent Circuit Construction

3.1 MMF Distribution in Air Gap

The number of slots and the coils in the slots determine the distribution of MMF in the air gap. In assumption of infinite permeability of the stator and rotor core, each MMF waveform of the main and the auxiliary winding of the analysis model are shown in Fig. 2 in normalized form. They contain one fundamental and numerous harmonic components. By the Fourier expansion, the harmonic components can be obtained and the distribution factors of (1), K_{wn} are indicated in Fig. 3, where n is the harmonic number in odd integers. And α_n is n^th^ harmonic's coefficient of the Fourier series. It is known that the auxiliary winding's harmonics are higher than those of the main winding. The fundamental components make the positive power conversion, but the harmonics contributes to extraordinary phenomena, such as the cusps and the dips in the speed-torque characteristics.

$$\frac{\alpha_n}{\alpha_1} = \frac{K_{wn}}{nK_{w1}} \tag{1}$$

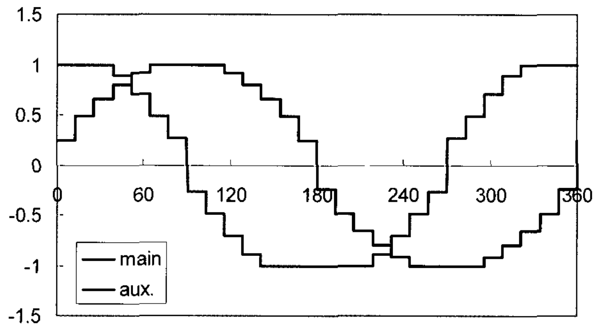

Fig. 2. MMF distribution of the analysis model (normalized)

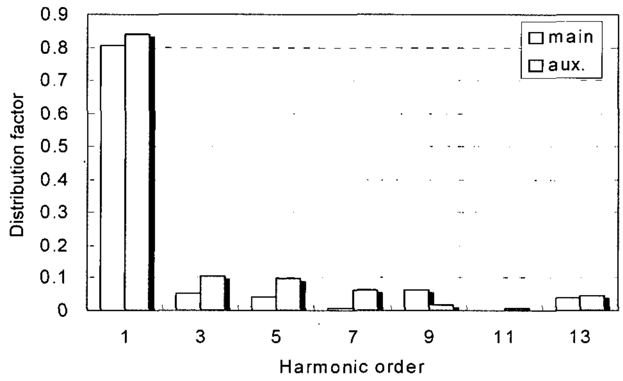

Fig. 3. Space harmonics of the analysis model(normalized)

3.2 Equivalent Circuit with Space Harmonics

The equivalent circuit for a single phase induction motor can be derived from the revolving field theory. The general circuit of pure 1-phase without auxiliary winding can be expanded to consider the space harmonics and are shown in Fig. 4, where X_l is the leakage reactance including the skew effect. s is the slip and $E_{f,n}$ and $E_{b,n}$ are the n^{th} forward and backward induced electromotive forces respectively. The n^th^ magnetizing reactance $X_{M,n}$, rotor's leakage reactance $X_{2,n}$ and resistance $r_{2,n}$ are obtained by (2) and (3), where the fundamental ones have classic form [5].

$$X_{M,n} = X_{M,1}\left(\frac{K_{wn}}{K_{w1}}\right)^2 \tag{2}$$

$$X_{2,n} = X_{2,1}\left(\frac{K_{wn}}{K_{w1}}\right)^2, \quad r_{2,n} = r_{2,1}\left(\frac{K_{wn}}{K_{w1}}\right)^2 \tag{3}$$

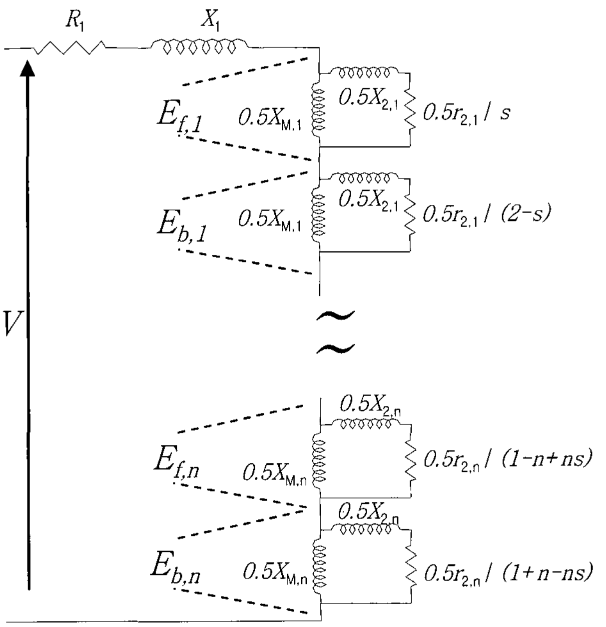

Fig. 4. Equivalent circuit for pure 1-phase induction motor with space harmonics

The circuit in Fig. 4 should be modified for the analysis model having auxiliary winding and it is represented in Fig. 5.

In Fig. 5, the leakage impedances of main and auxiliary parts Z_1, Z_{1a} are easily referred to in the classic texts [3]. And Z_c is the reactance of running capacitor C_r, given by (4).

$$Z_c = \frac{1}{j\omega C_R}, \tag{4}$$

And a_n is the nth harmonic's turn ratio and can be obtained from distribution factor, K_w in Fig. 4 and given by (5).

$$a_n = \frac{N_a K_{wa,n}}{N_m K_{wm,n}} \tag{5}$$

N_a and N_m in (5) are the practical turn numbers of auxiliary and main winding respectively. The equivalent forward and backward impedances $Z_{f,n}$ and $Z_{b,n}$ including the harmonics are represented by (6) and (7).

$$Z_{f,n} = R_{f,n} + jX_{f,n}, \tag{6}$$

$$Z_{b,n} = R_{b,n} + jX_{b,n}, \tag{7}$$

In (6) and (7), each resistance and reactance and its coefficients are as follows.

$$R_{f,n}, R_{b,n} = \frac{0.5K_{r,n} r_{2,n}/(1\mp n \pm ns)}{[(r_{2,n}/X_{0,n})/(1\mp n\pm ns)]^2+1}, \tag{9}$$

$$X_{f,n}, X_{b,n} =$$
$$\frac{0.5X_0 K_{p,n}[(r_{2,n}/X_{0,n})/(1\mp n\pm ns)]^2 + 0.5K_{p,n}X_{2,n}}{[(r_{2,n}/X_{0,n})/(1\mp n\pm ns)]^2+1}, \tag{10}$$

$$X_{0,n} = X_{2,n} + X_{M,n}, \tag{11}$$

$$K_{p,n} = \frac{X_{M,n}}{X_{M,n}+X_{2,n}}. \tag{12}$$

The (+) sign is for the forward components and the (-) sign is for the backward ones respectively in (9) and (10).

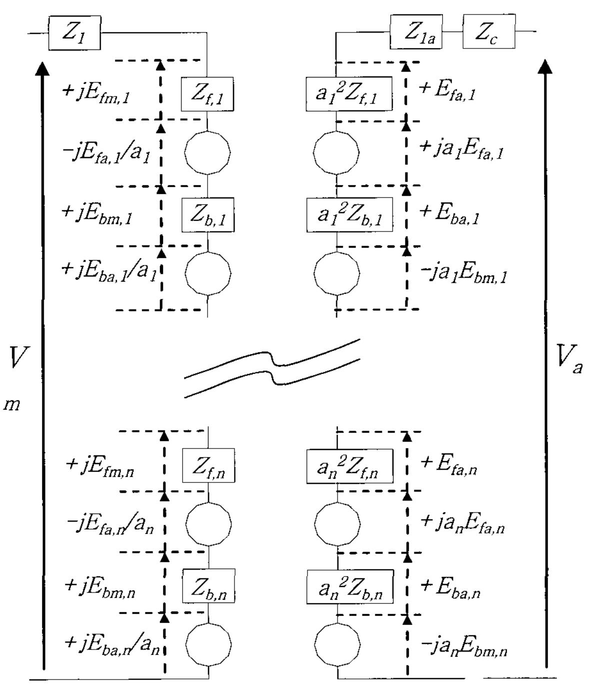

Fig. 5. Equivalent circuit for 1-phase induction motor with auxiliary winding

From Figure 5, the voltage equations of the main and the auxiliary winding are deduced by (12) and (13). By the equations, each winding current I_m and I_a can be obtained.

$$V_m = I_m[Z_1 + \sum(Z_{f,n}+Z_{b,n})] - jI_a\sum a_n(Z_{f,n}-Z_{b,n}) \tag{12}$$

$$\begin{aligned} V_a &= jI_m\sum a_n(Z_{f,n}-Z_{b,n}) \\ &+ jI_a[Z_{1a} + Z_c + \sum a_n{}^2(Z_{f,n}+Z_{b,n})] \end{aligned} \tag{13}$$

The output powers related with each of the harmonics are also calculated by using the magnitude and the phase difference θ

of each of the winding currents and given by (14).

$$P_{o,n} = (R_{f,n} I^2_{f,n} - R_{b,n} I^2_{b,n})(1-s) \tag{14}$$

where,

$$I^2_{f,n} = |I_m|^2 + a_n^2 |I_a|^2 + 2a_n \cos(90-\theta)|I_m I_a| \tag{15}$$

$$I^2_{b,n} = |I_m|^2 + a_n^2 |I_a|^2 + 2a_n \cos(90+\theta)|I_m I_a| \tag{16}$$

The respective harmonic and the total torque can be obtained easily from (14) and represented by (17).

$$T = \sum T_n = \sum P_{o,n} / \omega \tag{17}$$

4. Analysis Results

The torque characteristics of the analysis model are calculated by the proposed circuit and each torque component due to the space harmonics that are represented in Fig. 6. It shows that the 3rd, 5th and 9th harmonic components are relatively large and it corresponds with the harmonics of air gap MMF in Fig. 3. The total torque combining the fundamental and harmonic components is also calculated and depicted in Fig. 7, where it is compared with the toque characteristic neglecting space harmonics. It is shown that the torque increases monotonically with speed in case of ignoring the harmonics, but when they are considered, several dips appear in the low speed region. Because the value of starting torque is very close to that of torque dip due to the 9th harmonics, it is expected that a critical problem does not occur in this model. However, if making a worse modification in the analysis model, the 9th component probably makes both dip and abnormal starting in high load torque. So much attention should be paid to determination of winding specification to avoid the unexpected phenomenon such as crawling in the starting region.

The measuring system is in very unstable state in the low speed region, therefore measurement of every torque variation due to harmonics is nearly impossible. In general, the starting and the pull-up torque are measured and they are presented in Table 2. It can be known that the pull-up torque 1.9Nm of measurements is produced by the 9th harmonic compared with the simulations. And they are in good agreement with the simulation data in Fig. 7.

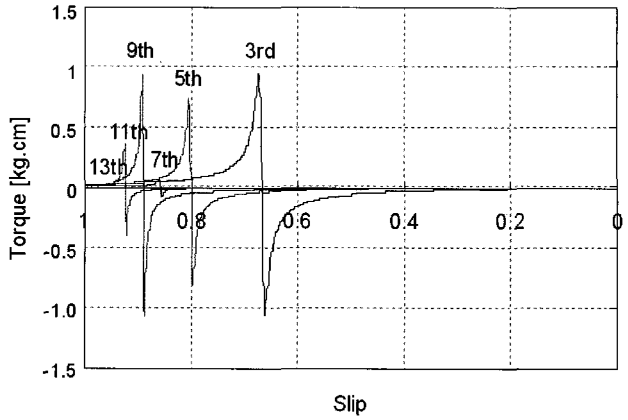

Fig. 6. Torque components according to the space harmonics

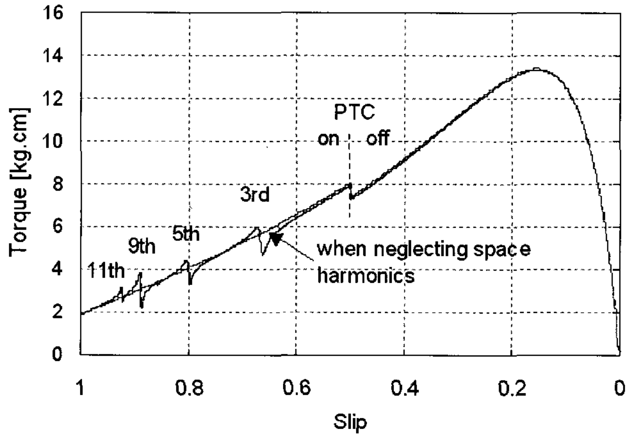

Fig. 7. Total torque versus slip

Table 2. Specification of analysis model

Measured Item	Kg.cm
Starting Torque	2.1
Pull-up Torque	1.9

5. Conclusion

To include the space harmonics in the air gap of a 1-phase induction motor, this paper presents an expanded equivalent circuit from the conventional one based on revolving theory. Using the proposed method, the harmonic torque characteristics of a commercial motor are calculated and compared with the measurements. It is useful for precise configuration of stator winding for safe starting.

Acknowledgements

This paper was supported by research funds provided by Kunsan National University.

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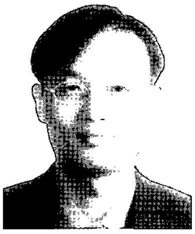

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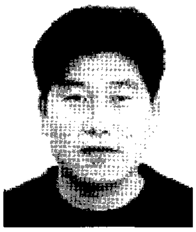

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