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DEHN SURGERY AND A-POLYNOMIAL FOR KNOTS

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ABSTRACT. The Property P conjecture states that the 3-manifold Y_r obtained by Dehn surgery on a non-trivial knot in S^3 with surgery coefficient $r \in \mathbf{Q}$ has the non-trivial fundamental group (so not simply connected). Recently Kronheimer and Mrowka provided a proof of the Property P conjecture for the case $r = \pm 2$ that was the only remaining case to be established for the conjecture. In particular, their results show that the two phenomena of having a cyclic fundamental group and having a homomorphism with non-cyclic image in $SU(2)$ are quite different for 3-manifolds obtained by Dehn fillings. In this paper we extend their results to some other Dehn surgeries via the A-polynomial, and provide more evidence of the ubiquity of the above mentioned phenomena.

1. Introduction and main results

The Property P conjecture says that the 3-manifold $Y_r(K)$ obtained by Dehn surgery on a non-trivial knot K in S^3 with surgery coefficient $r \in \mathbf{Q}$ has the non-trivial fundamental group (so not simply connected), for the case $|r| \leq 2$. Recently Kronheimer and Mrowka provided at least two different proofs of the Property P conjecture in [5] and [6]. In fact, they proved a much stronger version of the Property P conjecture as follows in [6].

THEOREM 1.1. (Kronheimer and Mrowka) *Let K be a non-trivial knot in S^3, and let $Y_r(K)$ be the 3-manifold obtained by Dehn surgery on K with surgery coefficient $r \in \mathbf{Q}$. If $|r| \leq 2$, then there exists a homomorphism $\rho : \pi_1(Y_r(K)) \to SU(2)$ with non-cyclic image. In particular, this implies that $\pi_1(Y_r(K))$ is not cyclic.*

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According to [6], the fact that $Y_r(K)$ cannot have a cyclic fundamental group was already known for all cases except $r = \pm 2$. In a little more detail, Gabai [4] and Kronheimer and Mrowka [5] proved for $r = 0$ and $r = \pm 1$ respectively. The torus knot case is due to Moser [8]. All other cases are consequences of the well-known cyclic surgery theorem of Culler, Gordon, Luecke and Shalen [2]. The aim of this short paper is to extend the result of Theorem 1.1 to some other Dehn surgeries via the notion of the A-polynomial.

The recent results of Kronheimer, Mrowka, Ozsvath, and Szabo [7] show that the resulting 3-manifold $Y_3(K)$ and $Y_4(K)$ of the Dehn surgery on a non-trivial knot K with surgery coefficient 3 and 4 cannot yield a lens space. However, it is not obvious whether or not the fundamental groups of $Y_3(K)$ and $Y_4(K)$ admit homomorphisms to $SU(2)$ with non-cyclic image. Moreover, according to [6] Dunfield has provided an example of a non-trivial knot in S^3 on which a Dehn filling has a fundamental group which is not cyclic but admits no homomorphism to $SU(2)$ with non-cyclic image. In this paper we present more evidence showing that the two phenomena of having a cyclic fundamental group and having a homomorphism with non-cyclic image in $SU(2)$ are quite different for 3-manifolds obtained by Dehn fillings. We do this using a variant of the A-polynomial of a knot in an homology 3-sphere, first introduced by Cooper et al. in [1], which describes the variety of characters of $SL_2(\mathbf{C})$ representations of the fundamental group of the knot complement.

In order to explain our result, we first recall the definition of the A-polynomial of a knot as in [1] and [3]. To do so, let N denote the exterior of a knot K in S^3 (or a homology 3-sphere). Then the boundary ∂N of N is a torus, and its fundamental group $\pi_1(\partial N) = \mathbf{Z}^2$ has a natural meridian and longitude basis μ and λ. Let $\rho : \pi_1(N) \to SL_2(\mathbf{C})$ be a representation. Then the restriction of ρ to $\pi_1(\partial N)$ can be conjugated so that

$$\rho(\mu) = \begin{pmatrix} M & 0 \\ 0 & M^{-1} \end{pmatrix} \quad \text{and} \quad \rho(\lambda) = \begin{pmatrix} L & 0 \\ 0 & L^{-1} \end{pmatrix},$$

since $\rho(\mu)$ and $\rho(\lambda)$ are two commuting matrices in $SL_2(\mathbf{C})$. The possible eigenvalues (M, L) over $\mathbf{C}^* \times \mathbf{C}^*$ of ρ form an complex algebraic subvariety of $\mathbf{C}^* \times \mathbf{C}^*$, as ρ varies. *The A-polynomial* is now defined to be the defining polynomial $f(M, L)$ for the 1-dimensional part of this subvariety. Thus it is just a plane curve of which each point corresponds to the restriction of each representation of $\pi_1(N)$ to $\pi_1(\partial N)$.

In this paper we will use the representations $\rho : \pi_1(N) \to SU(2)$ rather than the representations into $SL_2(\mathbf{C})$. So in this case the eigenvalues (M, L) of $(\rho(\mu), \rho(\lambda))$ form a real 1-dimensional subset of the unit torus in $\mathbf{C}^* \times \mathbf{C}^*$ which can be described by the restriction of the A-polynomial to the unit torus. We will call the restriction of the A-polynomial to the unit torus the *real $A_{\mathbf{R}}$-polynomial* of the knot K. More precisely, the real $A_{\mathbf{R}}$-polynomial g is given by

$$g(\alpha, \beta) = f(e^{i\alpha}, e^{i\beta}), \quad (\alpha, \beta) \in [-\pi, \pi] \times [-\pi, \pi].$$

In general, g may not be a polynomial of α and β, but it is obviously doubly periodic of two variables with period 2π. Since every representation $\rho : \pi_1(N) \to SU(2)$ induces a representation into $SL_2(\mathbf{C})$, we can say that the zero locus of the real $A_{\mathbf{R}}$-polynomial in the unit torus describes all the possible points which may arise as representations of $\pi_1(N)$ into $SU(2)$. This observation is one of the starting points of this paper.

From now on, α and β will be regarded as numbers in the square $[-\pi, \pi] \times [-\pi, \pi]$ modulo 2π, unless stated otherwise.

For each $-\pi \le t \le \pi$, let L_t be the closed line segment

$$L_t = \{(\alpha, \beta) \in [-\pi, \pi] \times [-\pi, \pi] \mid \alpha = t, -\pi \le \beta \le \pi\},$$

and let L_t^* be the open line segment obtained by removing the endpoints. For any small neighborhood W_k of each point $(\frac{2\pi k}{p}, \pi)$, let us W_k^* denote the intersection of $W_k \backslash \{\beta = \pi\}$ with the sector emanating from the point $(\frac{2\pi k}{p}, \pi)$ bounded by two line segment $L_{\frac{2\pi k}{p}}$ and the part of $p\alpha + q\beta = q\pi$. (See Figure 1.)

The following notation will be used in later discussions, for the sake of convenience. Let S denote the subset of the square $[-\pi, \pi] \times [-\pi, \pi]$ satisfying the property that S is invariant under the involution $s \mapsto -s$ modulo 2π. Then we define the set $\mathcal{R}^w(N \mid S)$ by a subset of the representation variety $\mathcal{R}^w(N)$ of flat $SU(2)$-connections with determinant w such that the pair (α, β) defined above lies in the subset S modulo 2π.

Then our main result can be stated as follows.

THEOREM 1.2. *Let K be a non-trivial knot in S^3, and let $Y_r(K)$ be the 3-manifold obtained by Dehn surgery on K with surgery coefficient $r = \frac{p}{q} \in \mathbf{Q}$, where p is a positive integer and q is an integer. Assume that the following two conditions hold:*

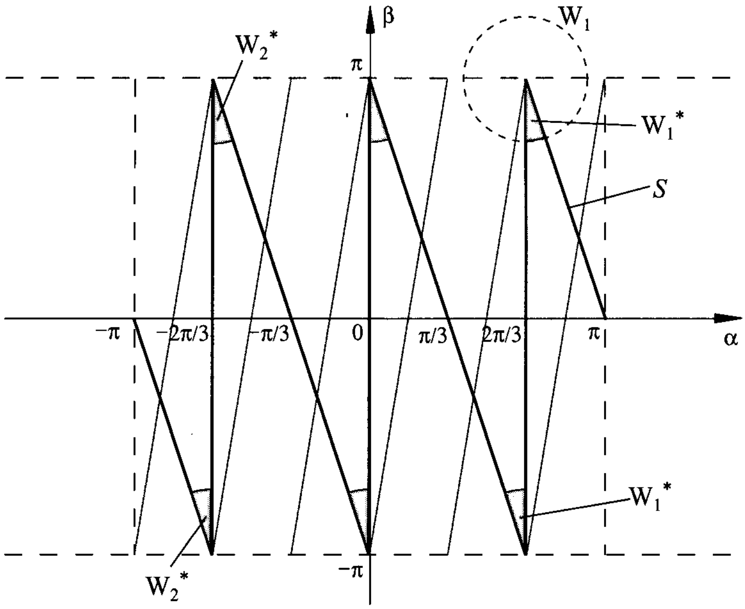

FIGURE 1. Here S is drawn for the case $\frac{p}{q} = 3$. The thin lines with slope $+6$ represent all possible parameters (α, β) for the representations of $\pi_1(N)$ into $SU(2)$, i.e., the real $A_{\mathbf{R}}$-polynomial for the right-handed trefoil knot 3_1.

(1) *The $A_{\mathbf{R}}$-polynomial does not have any solution over the open line segment $L^*_{\frac{2\pi k}{p}}$ for all $k = 0, 1, 2, \ldots, p-1$.* (Here $\frac{2\pi k}{p}$ should be regarded as a number in $[-\pi, \pi]$ modulo 2π.)

(2) *With the definition of W^*_k above the representation variety $\mathcal{R}^w(N \mid W^*_k)$ is empty.*

Then there exists a homomorphism $\rho : \pi_1(Y_r(K)) \to SU(2)$ with non-cyclic image. In particular, $\pi_1(Y_r(K))$ is not cyclic.

In case of $p = 1$ or 2, it can always be shown that the $A_{\mathbf{R}}$-polynomial does not have any solution over the open line segments L^*_0, $L^*_{\pm\pi}$, and the second hypothesis is satisfied by Lemma 12 in [6]. Hence we can give another verification of Theorem 1.1.

For concrete new examples, we can take the right-handed trefoil knot $K = 3_1$ in S^3 with $r = 3$. Then its A-polynomial is known to be $M^6 + L = 0$ in $\mathbf{C}^* \times \mathbf{C}^*$. (Here we did not include a factor of $L - 1$

coming from reducible representations. See the Appendix of [1].) Thus its corresponding real $A_{\mathbf{R}}$-polynomial is given by $e^{6i\alpha} + e^{i\beta} = 0$ (or $\beta = 6\alpha + \pi \bmod 2\pi\mathbf{Z}^2$) for $(\alpha, \beta) \in [-\pi, \pi] \times [-\pi, \pi]$. It is easy to show that the $A_{\mathbf{R}}$-polynomial does not have any solution on the open line segment $L^*_{\pm\frac{2\pi}{3}}$. Moreover, at each point $(\frac{2\pi k}{p}, \pi)$, the slope of $\beta = 6\alpha + \pi$ is 6. Hence the second hypothesis of Theorem 1.2 is automatically satisfied. (See Figure 1.) It is easy to see that the same argument above also works for the case $r = 6$ (and many other surgery coefficients). Hence it follows from Theorem 1.2 that $\pi_1(Y_3(3_1))$ and $\pi_1(Y_6(3_1))$ admit homomorphisms into $SU(2)$ with non-cyclic image. This answers a question of Kronheimer and Mrowka in [6].

Unfortunately, the real $A_{\mathbf{R}}$-polynomial does have a solution on the open line segment $L^*_{\frac{\pi}{2}}$. Hence it cannot be applied to decide whether there exists a homomorphism of $\pi_1(Y_4(3_1))$ into $SU(2)$ with non-cyclic image. On the other hand, it is well known that the surgery with coefficient $+5$ on the right-handed trefoil knot produces a lens space $L(5, 4)$. (e.g., see Section 7 in [9], and here the orientation of the lens space is taken so that $L(p, q)$ is obtained by $\frac{p}{q}$-surgery on the unknot in S^3.) Since the $A_{\mathbf{R}}$-polynomial $\beta = 6\alpha + \pi$ has a solution on the open line segment $L^*_{\frac{2\pi k}{5}}$ for integers $0 \leq k \leq 4$, we see that the first hypothesis of Theorem 1.2 cannot be removed.

This paper is organized as follows. In Section 2, we give some basic preliminaries on the representation varieties and its connection to gauge theory. In Section 3, we give a proof of Theorem 1.2.

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2. Representation varieties via gauge theory

We begin with some basic set-ups which are necessary for later discussion. The notations and definitions of this section are largely taken from the paper [6].

As before, let Y denote a compact connected 3-manifold (possibly with boundary). Let w be a unitary line bundle over Y, and let E be a unitary rank 2 vector bundle whose determinant line bundle $\det(E)$ is isomorphic to w under an isomorphism ψ. Let $\mathfrak{g}_E$ denote the bundle whose sections are the traceless, skew-hermitian endomorphisms of

E, and $\mathcal{A}$ be the affine space of $SO(3)$ connections in $\mathfrak{g}_E$. Let $\mathcal{G}$ be the gauge group of unitary automorphisms of E with determinant equal to 1. We use $\mathcal{B}^w(Y)$ for the quotient space $\mathcal{A}/\mathcal{G}$. Then we write $\mathcal{R}^w(Y)$ in $\mathcal{B}^w(Y)$ for the space of $\mathcal{G}$-orbits of flat connections. This is called the *representation variety* of flat connections with determinant w. Then the representation variety $\mathcal{R}^w(Y)$ is isomorphic to the space of representations $\rho : \pi_1(Y) \to SO(3)$ with $w_2(\rho) = c_1(w)$ modulo 2. Here $w_2(\rho)$ means the second Stiefel-Whitney class of the $SO(3)$-vector bundle associated to the representation ρ, and $c_1(w)$ means the first Chern class of the line bundle w. It is important to note that if $c_1(w) = 0$ modulo 2 every representation of $\pi_1(N) \to SO(3)$ with $c_1(w) = w_2(\rho)$ modulo 2 lifts to a representation into $SU(2)$. Thus in this case the representation variety $\mathcal{R}^w(Y)$ is isomorphic to the space of homomorphisms $\rho : \pi_1(Y) \to SU(2)$ up to conjugation.

Let ι denote an embedding of a solid torus $S^1 \times D^2$ into Y. Choose a trivialization of w over the image $\iota(S^1 \times D^2)$. Then under this trivialization each connection A in $\mathcal{A}(Y)$ gives rise to a unique connection B_A in E over the image $\iota(S^1 \times D^2)$ so that $\det(B_A)$ is the product connection in w over the image $\iota(S^1 \times D^2)$. Then using the holonomy of B_A along loops parallel to the core of the embedding and a smooth class function $\phi : SU(2) \to \mathbf{R}$, we can obtain a perturbed representation variety $\mathcal{R}^w_{(\iota,\phi)}(Y)$ as in [6].

Next we assume that N denotes the complementary manifold with a single torus boundary given by $N = Y \backslash (\iota(S^1 \times D^2))^\circ$. Let z_0 denote a base point on the boundary ∂D^2 of the disk D^2. Then we have a natural longitude a and meridian b of the solid torus $\iota(S^1 \times D^2)$ given by

$$a = \iota(S^1 \times \{z_0\}),\ b = \iota(\{1\} \times \partial D^2).$$

Under the above mentioned trivialization of w over $\iota(S^1 \times D^2)$, the restriction of E to ∂N is reduced to an $SU(2)$-bundle. Given a connection A on $\mathfrak{g}_E$ that is flat on ∂N, let B_A denote the unique flat $SU(2)$ connection in E over ∂N. Now using a determinant 1 isomorphism between the fiber of E over the base point $\iota(1, z_0)$ we obtain the holonomy of B_A along the two loops μ and λ given by

$$\mathrm{Hol}_a(B_A) = \begin{pmatrix} e^{i\alpha} & 0 \\ 0 & e^{-i\alpha} \end{pmatrix} \quad \text{and} \quad \mathrm{Hol}_b(B_A) = \begin{pmatrix} e^{i\beta} & 0 \\ 0 & e^{-i\beta} \end{pmatrix}.$$

This implies that corresponding to the flat connection A we can obtain a representation $\rho : \pi_1(\partial N) \to SU(2)$ given by

$$\rho(a) = \begin{pmatrix} e^{i\alpha} & 0 \\ 0 & e^{-i\alpha} \end{pmatrix} \quad \text{and} \quad \rho(b) = \begin{pmatrix} e^{i\beta} & 0 \\ 0 & e^{-i\beta} \end{pmatrix}.$$

Note that the pair $(\alpha(A), \beta(A))$ in $[-\pi, \pi] \times [-\pi, \pi]$ is uniquely determined by A up to the ambiguity of sign: the points (α, β) and $(-\alpha, -\beta)$ correspond to the same flat connection over N.

Clearly a class function ϕ on $SU(2)$ corresponds to a function $f : \mathbf{R} \to \mathbf{R}$ by the relation

$$f(t) = \phi\left(\begin{pmatrix} e^{it} & 0 \\ 0 & e^{-it} \end{pmatrix}\right).$$

Then the function satisfies $f(t + 2\pi) = f(t)$ and $f(t) = f(-t)$. With these said, the following proposition (Lemma 7 in [6]) will be crucially used in this paper.

Proposition 2.1. *Let $f : \mathbf{R} \to \mathbf{R}$ be a function corresponding to ϕ as above. Then the restriction from Y to N gives rise to a bijection between $\mathcal{R}^w_{(\iota,\phi)}(Y)$ and*

$$\mathcal{R}^w(N \,|\, \beta = -f'(\alpha)).$$

3. Proof of Theorem 1.2

The aim of this section is to prove Theorem 1.2.

To do so, as in Section 3 of [6], we take Y as a 3-manifold obtained by Dehn surgery on a non-trivial knot K in S^3 with surgery coefficient 0, and let $w \to Y$ be a line bundle with $c_1(w)$ a generator for $H^2(Y; \mathbf{Z}) = \mathbf{Z}$. Let ι denote an embedding of $S^1 \times D^2$ to Y whose core is a curve representing a generator for $H_1(Y; \mathbf{Z})$ (that is, the non-trivial knot K). Let N denote the manifold with torus boundary

$$N = Y \backslash (\iota(S^1 \times D^2))^\circ,$$

and let a (resp. b) denote the core curve (resp. meridian curve) of the solid torus $\iota(S^1 \times D^2)$ so that a (resp. b) is isotopic to the meridian μ (resp. the longitude λ) of the tubular neighborhood of the non-trivial knot K in S^3. Let s denote an isotopy class of essential closed curve on the torus ∂N (which is called the *slope*) given by $s = [pa + qb] = [p\mu + q\lambda]$. We denote by $Y_s(K)$ the manifold obtained by Dehn filling with slope

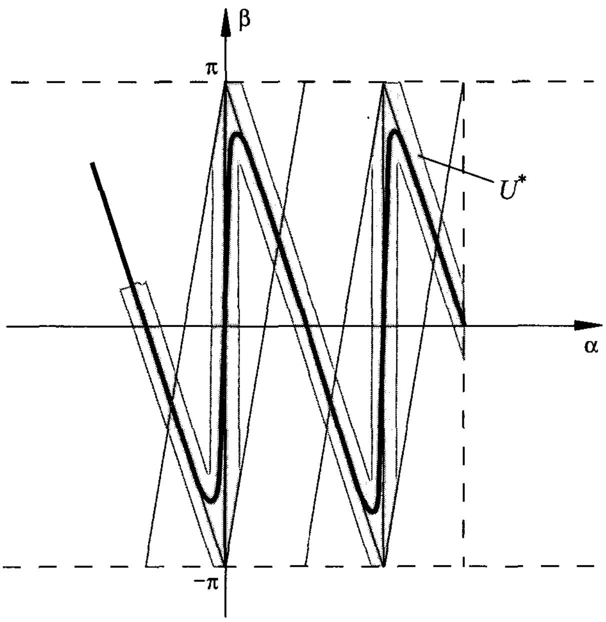

FIGURE 2. Here U^* denotes a sufficiently small open neighborhood of S^*, and the thicken curve inside U^* denotes an odd function $-g$ with period 2π.

s from the manifold N. In particular, Y is just $Y_b(K)$ which is the 3-manifold obtained by 0-Dehn surgery along K from S^3, and usually it is denoted by $Y_0(K)$. Further $Y_a(K)$ is just S^3.

Then we need the following proposition analogous to Proposition 9 in [6].

PROPOSITION 3.1. *Let s be as above, and assume that the following three conditions hold:*

1. *Neither $\pi_1(Y_a(K))$ nor $\pi_1(Y_s(K))$ admits a homomorphism into $SU(2)$ with non-cyclic image.*
2. *The $A_{\mathbf{R}}$-polynomial does not have any solution over the open line segment $L^*_{\frac{2\pi k}{p}}$ for all $k = 0, 1, 2, \ldots, p-1$.*
3. *With the definition of W^*_k above the representation variety $\mathcal{R}^w(N \mid W^*_k)$ is empty.*

Then there is a holonomy perturbation (ι, ϕ) for the manifold Y such that the perturbed representation variety $\mathcal{R}^w_{(\iota,\phi)}(Y)$ is empty.

Proof. The proof is a slight modification of the proof of Proposition 9 in [6]. So we shall use the same notations there.

As in [6], let w_a denote the line bundle over $Y_a(K)$, and let w_s and $\tilde{w}_s$ denote the line bundles over $Y_s(K)$. (We do not use the definition of these line bundles seriously, so we do not give more precise definitions in this paper.) Since the second cohomology $H^2(Y_a(K);\mathbf{Z}) = 0$ and so $c_1(w_a) = 0 \bmod 2$, every representation of $\pi_1(Y_a(K))$ into $SO(3)$ lifts to a representation into $SU(2)$. Similarly $c_1(\tilde{w}_a) = 0 \bmod 2$ (see p.5 in [6])). Thus every element of $\mathcal{R}^{\tilde{w}_s}(Y_s)$ corresponds to a homomorphism of $\pi_1(Y_s(K))$ into $SU(2)$. Moreover, the restriction of the representations to N gives rise to the following identifications

$$\mathcal{R}^{w_a}(Y_a(K)) \to \mathcal{R}^{w}(N \mid \alpha = 0)$$
$$\mathcal{R}^{w_s}(Y_s(K)) \to \mathcal{R}^{w}(N \mid p\alpha + q\beta = 0)$$
$$\mathcal{R}^{\tilde{w}_s}(Y_s(K)) \to \mathcal{R}^{w}(N \mid p\alpha + q\beta = q\pi).$$

Since $H_1(N;\mathbf{Z}) = \mathbf{Z}$, it is also true that there must exist reducibles in the representation variety $\mathcal{R}^w(N)$. But, as in Lemma 11 of [6], it turns out that the pair (α,β) for the reducible elements should lie on the line $\beta = \pi$ on the square $[-\pi,\pi]\times[-\pi,\pi]$. Since $Y_a(K)$ is just S^3, by hypothesis (1) the representation variety $\mathcal{R}^w(Y_a(K))$ consists of a single reducible, and its corresponding pair (α,β) should be $(0,\pi)$ on the square $[-\pi,\pi]\times[-\pi,\pi]$. This implies that $\mathcal{R}^w(N \mid L_0^*)$ and $\mathcal{R}^w(N \mid L_\pi^*)$ are all empty. Similarly the reducible elements in $\mathcal{R}^{\tilde{w}_s}(Y_s(K))$ should be represented by the points $(\frac{2\pi k}{p},\pi)$ on the square $[-\pi,\pi]\times[-\pi,\pi]$ for $k = 0,1,2,\dots,p-1$. This together with the hypothesis (1) implies that $\mathcal{R}^w(N \mid p\alpha + q\beta = q\pi)$ consists of only the points $(\frac{2\pi k}{p},\pi)$ on the square $[-\pi,\pi]\times[-\pi,\pi]$ for $k = 0,1,2,\dots,p-1$.

Now let S_1 be the piecewise linear arc on the square $[0,\pi]\times[-\pi,\pi]$ with vertices at the points

$$z_1 = (0,\pi)$$
$$z_2 = \left(\frac{2\pi}{p},-\pi\right),\ z_3 = \left(\frac{2\pi}{p},\pi\right)$$
$$\dots$$
$$z_{2[\frac{p}{2}]} = \left(\frac{2\pi[\frac{p}{2}]}{p},-\pi\right),\ z_{2[\frac{p}{2}]+1} = \left(\frac{2\pi[\frac{p}{2}]}{p},\pi\right)$$
$$z_{2[\frac{p}{2}]+2} = \left(\pi,(1-\frac{p}{q})\pi\right),\ z_{2[\frac{p}{2}]+3} = (\pi,0).$$

Let S be the piecewise linear arc on the square $[-\pi,\pi]\times[-\pi,\pi]$ obtained by reflecting S_1 about the origin and joining the reflected one to S_1 by the line segment L_0. (See Figure 2.) Let S^* denote the complement in S of the points whose β coordinates are π. Let U^* denote any symmetric neighborhood of S^* about the origin. Note from the hypothesis (2) and (3) that the representation variety $\mathcal{R}^w(N\,|\,S^*)$ is empty. Hence, using the compactness of $\mathcal{R}^w(N)$ we can choose a symmetric neighborhood U^* of S^* so that $\mathcal{R}^w(N\,|\,U^*)$ is empty. (See Figure 2 again.)

Next we choose a smooth odd function g with period 2π such that the graph of $-g$ on the interval $[-\pi,\pi]$ is contained in U^*. (See Figure 2.) This implies the existence of a corresponding class function ϕ and a periodic even function $f:\mathbf{R}\to\mathbf{R}$ with $f'=g$ such that

$$\mathcal{R}^w_{(\iota,\phi)}(Y)=\mathcal{R}^w(N\,|\,\beta=-f'(\alpha)).$$

But the right hand side is empty, since it is contained in the empty set by hypothesis of the theorem. This completes the proof of Proposition 3.1. □

Now the proof of Theorem 1.2 follows from the exactly same argument in [6]. In a little more detail, if K is a non-trivial knot in S^3 that contradicts to the conclusion of Theorem 1.2, then the manifold $Y_0(K)$ admits a taut foliation and is not $S^1\times S^2$ [4]. $Y_0(K)$ can also be embedded as a separating hypersurface in a closed symplectic 4-manifolds X which satisfies the Witten's conjecture relating the Seiberg-Witten invariants to the Donaldson's invariants by Proposition 7 in [6]. Thus we see that the Donaldson's invariants are non-trivial. On the other hand, since the perturbed representation variety $\mathcal{R}^w_{(\iota,\phi)}(Y_0(K))$ is empty, the Donaldson's invariants must be trivial. This is clearly a contradiction, which completes the proof of Theorem 1.2.

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