Bull. Korean Math. Soc. **43** (2006), No. 3, pp. 589–598

FEKETE-SZEGÖ PROBLEM FOR SUBCLASSES OF STARLIKE FUNCTIONS WITH RESPECT TO SYMMETRIC POINTS

T. N. Shanmugam, C. Ramachandran, and V. Ravichandran

Abstract. In the present investigation, sharp upper bounds of $|a_3 - \mu a_2^2|$ for functions $f(z) = z + a_2z^2 + a_3z^3 + \cdots$ belonging to certain subclasses of starlike and convex functions with respect to symmetric points are obtained. Also certain applications of the main results for subclasses of functions defined by convolution with a normalized analytic function are given. In particular, Fekete-Szegö inequalities for certain classes of functions defined through fractional derivatives are obtained.

1. Introduction

Let $\mathcal{A}$ denote the class of all *analytic* functions $f(z)$ of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k \quad (z \in \Delta := \{z \in \mathbb{C} : |z| < 1\}) \tag{1.1}$$

and $\mathcal{S}$ be the subclass of $\mathcal{A}$ consisting of univalent functions. For two functions $f, g \in \mathcal{A}$, we say that the function $f(z)$ is *subordinate* to $g(z)$ in Δ and write $f \prec g$ or $f(z) \prec g(z)$ $(z \in \Delta)$, if there exists an analytic function $w(z)$ with $w(0) = 0$ and $|w(z)| < 1$ $(z \in \Delta)$, such that $f(z) = g(w(z))$ $(z \in \Delta)$. In particular, if the function g is univalent in Δ, the above subordination is equivalent to $f(0) = g(0)$ and $f(\Delta) \subset g(\Delta)$.

Let $\phi(z)$ be an analytic function with positive real part on Δ with $\phi(0) = 1$, $\phi'(0) > 0$ which maps the unit disk Δ onto a region starlike with respect to 1 which is symmetric with respect to the real axis. Let

Received August 11, 2005. Revised October 2, 2005.
2000 Mathematics Subject Classification: 30C50, 30C45, 30C80.
Key words and phrases: analytic functions, starlike functions, convex functions, subordination, coefficient problem, Fekete-Szegö inequality.
The authors are thankful to the referee for his comments. The research of V. R. is supported by short-term research grant from Universiti Sains Malaysia.

$S^*(\phi)$ be the class of function $f \in \mathcal{S}$ for which $\frac{zf'(z)}{f(z)} \prec \phi(z)$, $(z \in \Delta)$ and $C(\phi)$ be the class of functions $f \in \mathcal{S}$ for which $1+\frac{zf''(z)}{f'(z)} \prec \phi(z)$, $(z \in \Delta)$. These classes were introduced and studied by Ma and Minda [7]. When $\phi(z) = (1+Az)/(1+Bz)$, $(-1 \leq B < A \leq 1)$, the class $S^*(\phi)$ reduces to the class $S^*[A,B]$ studied by Janowski [6]. See also Silverman and Silvia [15]. Ma and Minda [7] have obtained the Fekete-Szegö inequality for the functions in the class $C(\phi)$. Since $f \in C(\phi)$ if and only if $zf' \in S^*(\phi)$, we get the Fekete-Szegö inequality for functions in the class $S^*(\phi)$.

Sakaguchi [14] introduced and studied the class S_s^* of starlike functions with respect to symmetric points. The class $S_s^*(\phi)$ defined below is the generalization of the class S_s^*.

DEFINITION 1.1. [10] Let $\phi(z) = 1 + B_1 z + B_2 z^2 + B_3 z^3 + \cdots$ be a univalent starlike function with respect to 1 which maps the unit disk Δ onto a region in the right half plane which is symmetric with respect to the real axis, and let $B_1 > 0$. The function $f \in \mathcal{A}$ is in the class $S_s^*(\phi)$ if

$$\frac{2zf'(z)}{f(z)-f(-z)} \prec \phi(z).$$

The function $f \in \mathcal{A}$ is in the class $C_s(\phi)$ if

$$\frac{2(zf'(z))'}{f'(z)+f'(-z)} \prec \phi(z).$$

When $\phi(z) = (1+Az)/(1+Bz)$, $(-1 \leq B < A \leq 1)$, we denote the subclasses $S_s^*(\phi)$ and $C_s(\phi)$ by $S_s^*[A,B]$ and $C_s[A,B]$ respectively. For $0 \leq \alpha < 1$, let $S_s^*(\alpha) := S_s^*[1-2\alpha,-1]$ and $C_s(\alpha) := C_s[1-2\alpha,-1]$.

In the present paper, we obtain the Fekete-Szegö inequality for functions in the subclasses $S_s^*(\phi)$ and $C_s(\phi)$. Also we give applications of our results to certain functions defined through convolution (or Hadamard product) and in particular we consider classes $S_s^\lambda(\phi)$ and $C_s^\lambda(\phi)$ defined by fractional derivatives.

To prove our main result, we need the following:

LEMMA 1.2. [7] *If $p(z) = 1 + c_1 z + c_2 z^2 + \cdots$ is an analytic function with positive real part in Δ, then*

$$|c_2 - vc_1^2| \leq \begin{cases} -4v+2 & \text{if} \quad v \leq 0, \\ 2 & \text{if} \quad 0 \leq v \leq 1, \\ 4v-2 & \text{if} \quad v \geq 1. \end{cases}$$

When $v < 0$ or $v > 1$, the equality holds if and only if $p(z)$ is $(1+z)/(1-z)$ or one of its rotations. If $0 < v < 1$, then the equality holds *if and only if* $p(z)$ *is* $(1+z^2)/(1-z^2)$ *or one of its rotations. If* $v=0$, *the equality holds if and only if*

$$p(z) = \left(\frac{1}{2}+\frac{1}{2}\lambda\right)\frac{1+z}{1-z} + \left(\frac{1}{2}-\frac{1}{2}\lambda\right)\frac{1-z}{1+z} \quad (0 \le \lambda \le 1)$$

or one of its rotations. If $v=1$, *the equality holds if and only if* $p(z)$ *is the reciprocal of one of the functions such that the equality holds in the case of* $v=0$.

Also the above upper bound is sharp, and it can be improved as follows when $0<v<1$:

$$|c_2 - vc_1^2| + v|c_1|^2 \le 2 \quad (0 < v \le 1/2)$$

and

$$|c_2 - vc_1^2| + (1-v)|c_1|^2 \le 2 \quad (1/2 < v \le 1).$$

2. Fekete-Szegö problem

Our main result is the following:

THEOREM 2.1. *If* $f(z)$ *given by* (1.1) *belongs to* $S_s^*(\phi)$, *then*

$$|a_3 - \mu a_2^2| \le \begin{cases} \frac{1}{2}\left[B_2 - \frac{\mu}{2}B_1^2\right] & \text{if} \quad \mu \le \sigma_1, \\ \frac{B_1}{2} & \text{if} \quad \sigma_1 \le \mu \le \sigma_2, \\ -\frac{1}{2}\left[B_2 - \frac{\mu}{2}B_1^2\right] & \text{if} \quad \mu \ge \sigma_2, \end{cases}$$

where

$$\sigma_1 := \frac{2(B_2 - B_1)}{B_1^2}, \quad \sigma_2 := \frac{2(B_2 + B_1)}{B_1^2}.$$

The result is sharp.

Proof. For $f \in S_s^*(\phi)$, let

$$p(z) := \frac{2zf'(z)}{f(z)-f(-z)} = 1 + b_1 z + b_2 z^2 + \cdots . \tag{2.1}$$

From (2.1), we obtain

$$2a_2 = b_1 \quad \text{and} \quad 2a_3 = b_2.$$

Since $\phi(z)$ is univalent and $p \prec \phi$, the function

$$p_1(z) = \frac{1+\phi^{-1}(p(z))}{1-\phi^{-1}(p(z))} = 1 + c_1 z + c_2 z^2 + \cdots$$

is analytic and has positive real part in Δ. Thus we have

$$p(z) = \phi\left(\frac{p_1(z)-1}{p_1(z)+1}\right) \tag{2.2}$$

and from this equation (2.2), we obtain

$$b_1 = \frac{1}{2}B_1c_1$$

and

$$b_2 = \frac{1}{2}B_1(c_2 - \frac{1}{2}c_1^2) + \frac{1}{4}B_2c_1^2.$$

Therefore we have

$$a_3 - \mu a_2^2 = \frac{B_1}{4}\left\{c_2 - vc_1^2\right\}, \tag{2.3}$$

where

$$v := \frac{1}{2}\left[1 - \frac{B_2}{B_1} + \frac{\mu}{2}B_1\right].$$

Our result now follows by an application of Lemma 1.2. To show that these bounds are sharp, we define the functions K_{ϕ_n} $(n = 2, 3, \ldots)$ by

$$\frac{2zK'_{\phi_n}(z)}{K_{\phi_n}(z) - K_{\phi_n}(-z)} = \phi(z^{n-1}), \quad K_{\phi_n}(0) = 0 = [K_{\phi_n}]'(0) - 1$$

and the function F^λ and G_λ $(0 \leq \lambda \leq 1)$ by

$$\frac{2zF'_\lambda(z)}{F_\lambda(z) - F_\lambda(-z)} = \phi\left(\frac{z(z+\lambda)}{1+\lambda z}\right), \quad F_\lambda(0) = 0 = [F_\lambda]'(0) - 1$$

and

$$\frac{2zG'_\lambda(z)}{G_\lambda(z) - G_\lambda(-z)} = \phi\left(-\frac{z(z+\lambda)}{1+\lambda z}\right), \quad G_\lambda(0) = 0 = [G_\lambda]'(0) - 1.$$

Clearly the functions $K_{\phi_n}, F_\lambda, G_\lambda \in S_s^*(\phi)$. Also we write $K_\phi := K_{\phi_2}$. If $\mu < \sigma_1$ or $\mu > \sigma_2$, then the equality holds if and only if f is K_ϕ or one of its rotations. When $\sigma_1 < \mu < \sigma_2$, the equality holds if and only if f is K_{ϕ_3} or one of its rotations. If $\mu = \sigma_1$ then the equality holds if and only if f is F_λ or one of its rotations. If $\mu = \sigma_2$ then the equality holds if and only if f is G_λ or one of its rotations. □

If $\sigma_1 \leq \mu \leq \sigma_2$, then, in view of Lemma 1.2, Theorem 2.1 can be improved.

THEOREM 2.2. *Let $f(z)$ given by (1.1) belongs to $S_s^*(\phi)$. Let σ_3 be given by $\sigma_3 := 2B_2/B_1^2$. If $\sigma_1 \le \mu \le \sigma_3$, then*

$$|a_3 - \mu a_2^2| + \frac{1}{B_1^2}\left[2(B_1 - B_2) + \mu B_1^2\right] |a_2|^2 \le \frac{B_1}{2}.$$

If $\sigma_3 \le \mu \le \sigma_2$, then

$$|a_3 - \mu a_2^2| + \frac{1}{B_1^2}\left[2(B_1 + B_2) - \mu B_1^2\right] |a_2|^2 \le \frac{B_1}{2}.$$

EXAMPLE 2.3. Let $-1 \le B < A \le 1$. If $f(z)$ given by (1.1) belongs to $S_s^*[A, B]$, then

$$|a_3 - \mu a_2^2| \le \begin{cases} \dfrac{B-A}{2}\left[B + \dfrac{\mu}{2}(A-B)\right] & \text{if } \mu \le -2\left[\dfrac{1+B}{A-B}\right], \\ \dfrac{A-B}{2} & \text{if } -2\left[\dfrac{1+B}{A-B}\right] \le \mu \le 2\left[\dfrac{1-B}{A-B}\right], \\ \dfrac{A-B}{2}\left[B + \dfrac{\mu}{2}(A-B)\right] & \text{if } \mu \ge 2\left[\dfrac{1-B}{A-B}\right]. \end{cases}$$

In particular, if $f \in S_s^*(\alpha)$, then

$$|a_3 - \mu a_2^2| \le \begin{cases} (1-\alpha)(1-\mu(1-\alpha)) & \text{if} \quad \mu \le 0, \\ (1-\alpha) & \text{if} \quad 0 \le \mu \le \frac{2}{1-\alpha}, \\ -(1-\alpha)(1-\mu(1-\alpha)) & \text{if} \quad \mu \ge \frac{2}{1-\alpha}. \end{cases}$$

The results are sharp.

Since $f \in C_s(\phi)$ if and only if $zf' \in S_s^*(\phi)$, Theorem 2.1, with an obvious change of the parameter μ, leads to the following Corollary.

COROLLARY 2.4. *If $f(z)$ given by (1.1) belongs to $C_s(\phi)$, then*

$$|a_3 - \mu a_2^2| \le \begin{cases} \dfrac{1}{6}\left[B_2 - \dfrac{3}{8}\mu B_1^2\right] & \textit{if} \quad \mu \le \sigma_1, \\ \dfrac{B_1}{6} & \textit{if} \quad \sigma_1 \le \mu \le \sigma_2, \\ -\dfrac{1}{6}\left[B_2 - \dfrac{3}{8}\mu B_1^2\right] & \textit{if} \quad \mu \ge \sigma_2, \end{cases}$$

where

$$\sigma_1 := \frac{8}{3}\frac{(B_2 - B_1)}{B_1^2}, \quad \sigma_2 := \frac{8}{3}\frac{(B_2 + B_1)}{B_1^2}.$$

The result is sharp.

EXAMPLE 2.5. If $f(z)$ given by (1.1) belongs to $C_s[A,B]$, then

$$|a_3-\mu a_2^2| \le \begin{cases} \frac{B-A}{6}\left[B+\frac{3}{8}\mu(A-B)\right] & \text{if } \mu \le -\frac{8}{3}\left[\frac{1+B}{A-B}\right], \\ \frac{A-B}{6} & \text{if } -\frac{8}{3}\left[\frac{1+B}{A-B}\right] \le \mu \le \frac{8}{3}\left[\frac{1-B}{A-B}\right], \\ \frac{A-B}{6}\left[B+\frac{3\mu}{8}(A-B)\right] & \text{if } \mu \ge \frac{8}{3}\left[\frac{1-B}{A-B}\right]. \end{cases}$$

In particular, if $f \in C_s(\alpha)$, then

$$|a_3-\mu a_2^2| \le \begin{cases} \frac{1-\alpha}{3}\left[1-\frac{3}{4}\mu(1-\alpha)\right] & \text{if} \quad \mu \le 0, \\ \frac{1-\alpha}{3} & \text{if} \quad 0 \le \mu \le \frac{8}{3(1-\alpha)}, \\ -\frac{1-\alpha}{3}\left[1-\frac{3}{4}\mu(1-\alpha)\right] & \text{if} \quad \mu \ge \frac{8}{3(1-\alpha)}. \end{cases}$$

The results are sharp.

If $\sigma_1 \le \mu \le \sigma_2$, then, in view of Lemma 1.2, Corollary 2.4 can be improved.

COROLLARY 2.6. *Let $f(z)$ given by (1.1) belongs to $C_s(\phi)$. Let σ_3 be given by $\sigma_3 := \frac{8}{3}\frac{B_2}{B_1^2}$.*
If $\sigma_1 \le \mu \le \sigma_3$, then

$$|a_3-\mu a_2^2| + \frac{8}{3}\frac{1}{B_1^2}\left[B_1 - B_2 + \frac{3}{8}\mu B_1^2\right]|a_2|^2 \le \frac{B_1}{6}.$$

If $\sigma_3 \le \mu \le \sigma_2$, then

$$|a_3-\mu a_2^2| + \frac{8}{3}\frac{1}{B_1^2}\left[B_1 + B_2 - \frac{3}{8}\mu B_1^2\right]|a_2|^2 \le \frac{B_1}{6}.$$

3. Applications to functions defined by fractional derivatives

For two analytic functions $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ and $g(z) = z + \sum_{n=2}^{\infty} g_n z^n$, their convolution (or Hadamard product) is defined to be the function $(f*g)(z)$ given by $(f*g)(z) = z + \sum_{n=2}^{\infty} g_n a_n z^n$. For a fixed $g \in \mathcal{A}$, let $S_s^g(\phi)$ be the class of functions $f \in \mathcal{A}$ for which $(f*g) \in S_s^*(\phi)$ and $C_s^g(\phi)$ be the class of functions $f \in \mathcal{A}$ for which $(f*g) \in C_s(\phi)$.

DEFINITION 3.1 (see [9, 8]; see also [17, 18]). Let $f(z)$ be analytic in a simply connected region of the z-plane containing the origin. The *fractional derivative* of f of order λ is defined by

$$D_z^\lambda f(z) := \frac{1}{\Gamma(1-\lambda)} \frac{d}{dz} \int_0^z \frac{f(\zeta)}{(z-\zeta)^\lambda} d\zeta \quad (0 \leq \lambda < 1),$$

where the multiplicity of $(z-\zeta)^{-\lambda}$ is removed by requiring that $\log(z-\zeta)$ is real for $z - \zeta > 0$.

Using the above Definition 3.1 and its known extensions involving fractional derivatives and fractional integrals, Owa and Srivastava [9] introduced the operator $\Omega^\lambda : \mathcal{A} \to \mathcal{A}$ defined by

$$(\Omega^\lambda f)(z) = \Gamma(2-\lambda) z^\lambda D_z^\lambda f(z), \quad (\lambda \neq 2, 3, 4, \ldots).$$

The classes $S_s^\lambda(\phi)$ and $C_s^\lambda(\phi)$ consist of functions $f \in \mathcal{A}$ for which $\Omega^\lambda f \in S_s^*(\phi)$ and $\Omega^\lambda f \in C_s(\phi)$ respectively. The class $S_s^\lambda(\phi)$ is the special case of the class $S_s^g(\phi)$ and the class $C_s^\lambda(\phi)$ is the special case of the class $C_s^g(\phi)$ when

$$g(z) = z + \sum_{n=2}^{\infty} \frac{\Gamma(n+1)\Gamma(2-\lambda)}{\Gamma(n+1-\lambda)} z^n. \tag{3.1}$$

Since $f \in S_s^g(\phi)$ $(C_s^g(\phi))$ if and only if $f * g \in S_s^*(\phi)$ $(C_s(\phi))$, we obtain the coefficient estimates for functions in the classes $S_s^g(\phi)$ and $C_s^g(\phi)$, from the corresponding estimates for functions in the classes $S_s^*(\phi)$ and $C_s(\phi)$.

Applying Theorem 2.1 for the function $(f * g)(z) = z + g_2 a_2 z^2 + g_3 a_3 z^3 + \cdots$, we get the following theorem after an obvious change of the parameter μ:

THEOREM 3.2. *Let $g(z) = z + \sum_{n=2}^{\infty} g_n z^n$ $(g_n > 0)$. If $f(z)$ given by* (1.1) *belongs to $S_s^g(\phi)$, then*

$$|a_3 - \mu a_2^2| \leq \begin{cases} \dfrac{1}{2g_3}\left(B_2 - \dfrac{\mu g_3}{2g_2^2} B_1^2\right) & \text{if} \quad \mu \leq \sigma_1, \\ \dfrac{B_1}{2g_3} & \text{if} \quad \sigma_1 \leq \mu \leq \sigma_2, \\ -\dfrac{1}{2g_3}\left(B_2 - \dfrac{\mu g_3}{2g_2^2} B_1^2\right) & \text{if} \quad \mu \geq \sigma_2, \end{cases}$$

where

$$\sigma_1 := \frac{2g_2^2}{g_3} \frac{(B_2 - B_1)}{B_1^2}, \quad \sigma_2 := \frac{2g_2^2}{g_3} \frac{(B_2 + B_1)}{B_1^2},$$

The result is sharp.

Corollary 3.3. *Let $g(z) = z + \sum_{n=2}^{\infty} g_n z^n$ $(g_n > 0)$. If $f(z)$ given by (1.1) belongs to $C_s^g(\phi)$, then*

$$|a_3 - \mu a_2^2| \leq \begin{cases} \dfrac{1}{6g_3}\left(B_2 - \dfrac{3}{8}\dfrac{\mu g_3}{g_2^2}B_1^2\right) & \text{if } \mu \leq \sigma_1, \\ \dfrac{B_1}{6g_3} & \text{if } \sigma_1 \leq \mu \leq \sigma_2, \\ -\dfrac{1}{6g_3}\left(B_2 - \dfrac{3}{8}\dfrac{\mu g_3}{g_2^2}B_1^2\right) & \text{if } \mu \geq \sigma_2, \end{cases}$$

where

$$\sigma_1 := \frac{8g_2^2}{3g_3}\frac{(B_2 - B_1)}{B_1^2}, \quad \sigma_2 := \frac{8g_2^2}{3g_3}\frac{(B_2 + B_1)}{B_1^2}.$$

The result is sharp.

Since

$$(\Omega^\lambda f)(z) = z + \sum_{n=2}^{\infty} \frac{\Gamma(n+1)\Gamma(2-\lambda)}{\Gamma(n+1-\lambda)} a_n z^n,$$

we have

$$g_2 := \frac{\Gamma(3)\Gamma(2-\lambda)}{\Gamma(3-\lambda)} = \frac{2}{2-\lambda} \tag{3.2}$$

and

$$g_3 := \frac{\Gamma(4)\Gamma(2-\lambda)}{\Gamma(4-\lambda)} = \frac{6}{(2-\lambda)(3-\lambda)}. \tag{3.3}$$

For g_2 and g_3 given by (3.2) and (3.3), Theorem 3.2 reduces to the following:

Theorem 3.4. *Let $\lambda < 2$. If $f(z)$ given by (1.1) belongs to $S_s^\lambda(\phi)$, then*

$$|a_3 - \mu a_2^2| \leq \begin{cases} \dfrac{(2-\lambda)(3-\lambda)}{12}\left[B_2 - \dfrac{3(2-\lambda)}{4(3-\lambda)}\mu B_1^2\right] & \text{if } \mu \leq \sigma_1, \\ \dfrac{(2-\lambda)(3-\lambda)}{12}B_1 & \text{if } \sigma_1 \leq \mu \leq \sigma_2, \\ -\dfrac{(2-\lambda)(3-\lambda)}{12}\left[B_2 - \dfrac{3(2-\lambda)}{4(3-\lambda)}\mu B_1^2\right] & \text{if } \mu \geq \sigma_2, \end{cases}$$

where

$$\sigma_1 := \frac{4(3-\lambda)}{3(2-\lambda)}\frac{(B_2 - B_1)}{B_1^2}, \quad \sigma_2 := \frac{4(3-\lambda)}{3(2-\lambda)}\frac{(B_2 + B_1)}{B_1^2}.$$

For g_2 and g_3 given by (3.2) and (3.3), Corollary 3.3 reduces to the following:

COROLLARY 3.5. *Let $\lambda < 2$. If $f(z)$ given by (1.1) belongs to $C_s^\lambda(\phi)$, then*

$$|a_3 - \mu a_2^2| \leq \begin{cases} \dfrac{(2-\lambda)(3-\lambda)}{36}\left[B_2 - \dfrac{9(2-\lambda)}{16(3-\lambda)}\mu B_1^2\right] & \text{if } \mu \leq \sigma_1, \\ \dfrac{(2-\lambda)(3-\lambda)}{36} B_1 & \text{if } \sigma_1 \leq \mu \leq \sigma_2, \\ -\dfrac{(2-\lambda)(3-\lambda)}{36}\left[B_2 - \dfrac{9(2-\lambda)}{16(3-\lambda)}\mu B_1^2\right] & \text{if } \mu \geq \sigma_2, \end{cases}$$

where

$$\sigma_1 := \frac{4(3-\lambda)}{(2-\lambda)} \frac{(B_2 - B_1)}{B_1^2}, \quad \sigma_2 := \frac{4(3-\lambda)}{(2-\lambda)} \frac{(B_2 + B_1)}{B_1^2}.$$

References

[1] B. C. Carlson and D. B. Shaffer, *Starlike and prestarlike hypergeometric functions*, SIAM J. Math. Anal. **15** (1984), no. 4, 737–745.

[2] M. Darus and D. K. Thomas, *On the Fekete-Szegö theorem for close-to-convex functions*, Math. Japon. **44** (1996), no. 3, 507–511.

[3] ———, *On the Fekete-Szegö theorem for close-to-convex functions*, Math. Japon. **47** (1998), no. 1, 125–132.

[4] B. A. Frasin and M. Darus, *On the Fekete-Szegö problem*, Int. J. Math. Math. Sci. **24** (2000), no. 9, 577–581.

[5] A. W. Goodman, *Uniformly convex functions*, Ann. Polon. Math. **56** (1991), no. 1, 87–92.

[6] W. Janowski, *Some extremal problems for certain families of analytic functions*, Bull. Acad. Polon. Sci. Ser. Sci. Math. Astronom. **21** (1973), 17–25.

[7] W. Ma and D. Minda, *A unified treatment of some special classes of univalent functions, in: Proceedings of the Conference on Complex Analysis*, Z. Li, F. Ren, L. Yang, and S. Zhang(Eds.), Internat. Press (1994), 157–169.

[8] S. Owa, *On the distortion theorems I*, Kyungpook Math. J. **18** (1978), no. 1, 53–59.

[9] S. Owa and H. M. Srivastava, *Univalent and starlike generalized hypergeometric functions*, Canad. J. Math. **39** (1987), no. 5, 1057–1077.

[10] V. Ravichandran, *Starlike and convex functions with respect to conjugate points*, Acta Math. Acad. Paedagog. Nyházi. (N.S.) **20** (2004), no. 1, 31–37.

[11] V. Ravichandran, M. Darus, M. Hussain Khan, and K. G. Subramanian, *Fekete-Szegö inequality for certain class of analytic functions*, Aust. J. Math. Anal. Appl. **1** (2004), no. 2, Art. 4, 7, pp.

[12] V. Ravichandran, A. Gangadharan, and M. Darus, *Fekete-Szegö inequality for certain class of Bazilevic functions*, Far East J. Math. Sci. (FJMS) **15** (2004), no. 2, 171–180.

[13] F. Rønning, *Uniformly convex functions and a corresponding class of starlike functions*, Proc. Amer. Math. Soc. **118** (1993), no. 1, 189–196.
[14] K. Sakaguchi, *On a certain univalent mapping*, J. Math. Soc. Japan **11** (1959), 72–75.
[15] H. Silverman and E. M. Silvia, *Subclasses of starlike functions subordinate to convex functions*, Canad. J. Math. **37** (1985), no. 1, 48–61.
[16] H. M. Srivastava, A. K. Mishra, and M. K. Das, *The Fekete-Szegö problem for a subclass of close-to-convex functions*, Complex Variables Theory Appl. **44** (2001), no. 2, 145–163.
[17] H. M. Srivastava and S. Owa, *An application of the fractional derivative*, Math. Japon. **29** (1984), no. 3, 383–389.
[18] ———, *Univalent Functions, Fractional Calculus, and Their Applications*, Halsted Press/John Wiley and Songs, Chichester/New York, 1989.
[19] N. Tuneski and M. Darus, *Fekete-Szegö functional for non-Bazilevič functions*, Acta Math. Acad. Paedagog. Nyházi. (N.S.) **18** (2002), no. 2, 63–65.

T. N. Shanmugam and C. Ramachandran, Department of Mathematics, College of Engineering, Anna University, Chennai-600 025, Tamilnadu, India
E-mail: shan@annauniv.edu
crjsp2004@yahoo.com

V. Ravichandran, School of Mathematical Sciences, Universiti Sains Malaysia, 11800 USM Penang, Malaysia
E-mail: vravi@cs.usm.my