Bull. Korean Math. Soc. **43** (2006), No. 3, pp. 627–634

DERIVATIONS OF A WEYL TYPE NON-ASSOCIATIVE ALGEBRA ON A LAURENT EXTENSION

Seul Hee Choi

Abstract. A Weyl type algebra is defined in the book ([4]). A Weyl type non-associative algebra $\overline{WP_{m,n,s}}$ and its restricted subalgebra $\overline{WP_{m,n,s_r}}$ are defined in various papers ([1], [12], [3], [11]). Several authors find all the derivations of an associative (Lie or non-associative) algebra in the papers ([1], [2], [12], [4], [6], [11]). We find all the non-associative algebra derivations of the non-associative algebra $\overline{WP_{0,2,0_B}}$, where $B = \{\partial_0, \partial_1, \partial_2, \partial_{12}, \partial_1^2, \partial_2^2\}$.

1. Preliminaries

Let $\mathbb{F}$ be a field of characteristic zero (not necessarily algebraically closed). Throughout this paper, $\mathbb{N}$ and $\mathbb{Z}$ will denote the non-negative integers and the integers, respectively. Let $\mathbb{F}[x_1, \ldots, x_{m+s}]$ be the polynomial ring with the variables $x_1, \ldots, x_{m+s}$. Suppose that $g_1, \ldots, g_n$ are given polynomials in $\mathbb{F}[x_1, \ldots, x_{m+s}]$. For $n, m, s \in \mathbb{N}$, we define the commutative, associative $\mathbb{F}$-algebra $F_{g_n,m,s} = \mathbb{F}[e^{\pm g_1}, \ldots, e^{\pm g_n}, x_1^{\pm 1}, \ldots, x_m^{\pm 1}, x_{m+1}, \ldots, x_{m+s}]$ in the formal power series ring $\mathbb{F}[[x_1, \ldots, x_{m+s}]]$ which is called a stable algebra with the standard basis ([7]):

$$\{e^{a_1 g_1} \ldots e^{a_n g_n} x_1^{i_1} \cdots x_{m+s}^{i_{m+s}} | a_1, \ldots, a_n, i_1, \ldots, i_m \in \mathbb{Z}, \\ i_{m+1}, \ldots, i_{m+s} \in \mathbb{N}\}$$

and with the obvious addition and the multiplication ([7], [10]), where ∂_w, $1 \leq w \leq m+s$, denotes the usual partial derivative with respect to x_w on $F_{g_n,m,s}$. For partial derivatives $\partial_u, \ldots, \partial_v$ of $F_{g_n,m,s}$, the composition $\partial_u^{j_u} \circ \cdots \circ \partial_v^{j_v}$ of them is denoted $\partial_u^{j_u} \cdots \partial_v^{j_v}$, where $j_u, \ldots, j_v \in \mathbb{N}$. Denote $D = \{\partial_u^{j_u} \cdots \partial_v^{j_v} | j_u, \ldots, j_v \in \mathbb{N}, 1 \leq u, v \leq m+s\}$. We shall

Received November 13, 2005.

2000 Mathematics Subject Classification: Primary 17B40, 17B56.

Key words and phrases: simple, non-associative algebra, Kronecker delta, left identity, annihilator, idempotent, Semi-Lie algebra.

define the vector space $WP(g_n, m, s) = WP(g_n, m, s)_D$ over $\mathbb{F}$ which is spanned by the standard basis

$$\begin{aligned}
&\{e^{a_1 g_1} \cdots e^{a_n g_n} x_1^{i_1} \cdots x_{m+s}^{i_{m+s}} \partial_u^{j_u} \cdots \partial_v^{j_v} | a_1, \ldots, a_n, i_1, \ldots, i_m \in \mathbb{Z}, \\
(1) \quad & i_{m+1}, \ldots, i_{m+s} \in \mathbb{N}, j_u, \ldots, j_v \in \mathbb{N}, 1 \leq u, \ldots, v \leq m+s, \\
&\partial_u^{j_u} \cdots \partial_v^{j_v} \in D\},
\end{aligned}$$

where we take appropriate $g_1, \ldots, g_n$ so that the set (1) is a basis of $WP(g_n, m, s)$. Thus we may define the multiplication $*$ on $WP(g_n, m, s)$ as follows:

$$\begin{aligned}
&e^{a_{11} g_1} \cdots e^{a_{1n} g_n} x_1^{i_{11}} \cdots x_{m+s}^{i_{1,m+s}} \partial_u^{j_u} \cdots \partial_v^{j_v} * e^{a_{21} g_1} \cdots e^{a_{2n} g_n} x_1^{i_{21}} \\
&\cdots x_{m+s}^{i_{2,m+s}} \partial_h^{j_h} \cdots \partial_w^{j_w} = e^{a_{11} g_1} \cdots e^{a_{1n} g_n} x_1^{i_{11}} \cdots x_{m+s}^{i_{1,m+s}} \partial_u^{j_u} \\
(2) \quad &\cdots \partial_v^{j_v} (e^{a_{21} g_1} \cdots e^{a_{2n} g_n} x_1^{i_{21}} \cdots x_{m+s}^{i_{2,m+s}}) \partial_h^{j_h} \cdots \partial_w^{j_w}
\end{aligned}$$

for any basis elements $e^{a_{11} g_1} \cdots e^{a_{1n} g_n} x_1^{i_{11}} \cdots x_{m+s}^{i_{1,m+s}} \partial_u^{j_u} \cdots \partial_v^{j_v}$ and $e^{a_{21} g_1} \cdots e^{a_{2n} g_n} x_1^{i_{21}} \cdots x_{m+s}^{i_{2,m+s}} \partial_h^{j_h} \cdots \partial_w^{j_w} \in WP(g_n, m, s)$. Thus we can define the Weyl-type non-associative algebra $\overline{WP_{g_n,m,s}}$with the multiplication $*$ in (2) and with the set $WN(g_n, m, s)$ ([9], [11]). For $B \subset D$, we define the non-associative subalgebra $\overline{WP_{g_n,m,s_B}}$ of the non-associative algebra $\overline{WP_{g_n,m,s}}$ spanned by

$$\begin{aligned}
&\{e^{a_1 g_1} \cdots e^{a_n g_n} x_1^{i_1} \cdots x_s^{i_s} \partial_u^{j_u} \cdots \partial_v^{j_v} | a_1, \ldots, a_n, i_1, \ldots, i_m \in \mathbb{Z}, \\
(3) \quad & i_{m+1}, \ldots, i_s \in \mathbb{N}, j_u, \ldots, j_v \in \mathbb{N}, \partial_u^{j_u} \cdots \partial_v^{j_v} \in B, \\
&1 \leq u, \ldots, v \leq m+s\}.
\end{aligned}$$

This implies that the algebra $\overline{WP_{g_n,m,s_B}}$ contains the polynomial ring naturally. The simplicity of $\overline{WP_{g_n,m,s_B}}$ is depending on the set B. It is well known that the non-associative algebra $\overline{WP_{g_n,m,s_D}}$ is simple, even though it has the right annihilator ([6], [8]). Throughout the paper, we put $\partial_i \partial_j = \partial_{ij}$. For any $x_1^{i_1} \cdots x_{m+s}^{i_{m+s}} \in \mathbb{F}[x_1, \ldots, x_{m+s}]$, we put $x_1^{i_1} \cdots x_{m+s}^{i_{m+s}} \partial_0 = x_1^{i_1} \cdots x_{m+s}^{i_{m+s}}$, we denote $B_1 = \{\partial_0, \partial_1, \partial_2, \partial_{12}, \partial_1^2, \partial_2^2\}$, and $B_2 = \{\partial_0, \partial_1, \partial_2, \partial_{12}, \partial_1^2\}$. It is easy to prove that the non-associative algebras $\overline{WP_{0,2,0_{B_1}}}$ and $\overline{WP_{0,2,0_{B_2}}}$ are simple. The non-associative algebra $\overline{WP_{g_n,m,s_D}}$ has the left identity 1.

2. Derivations of $\overline{WP_{0,2,0}}_{B_1}$

LEMMA 2.1. *For any derivation D of the non-associative algebra $\overline{WP_{0,2,0}}_{B_1}$ and $x_1^m\partial_1,\dots,\partial_1^2 \in \overline{WP_{0,2,0}}_{B_1}$, the followings hold*

$$(4)\quad D(x_1^m\partial_1) = (1-m)a_{1,0,0}x_1^m\partial_1 - mb_{1,0,0}x_1^{m-1}y\partial_1 + ms_{1,0,0}x_1^{m-1}\partial_1, +a_{2,0,0}x_1^m\partial_2 + a_{3,0,0}x_1^m\partial_{12} + a_{5,0,0}x_1^m\partial_2^2,$$

$$D(\partial_{12}) = c_{1,0,0}\partial_1 + c_{2,0,0}\partial_2 + c_{3,0,0}\partial_{12} + c_{4,0,0}\partial_1^2 + c_{5,0,0}\partial_2^2,$$

$$(5)\quad D(\partial_1^2) = d_{1,0,0}\partial_1 + d_{2,0,0}\partial_2 + d_{3,0,0}\partial_{12} + d_{4,0,0}\partial_1^2 + d_{5,0,0}\partial_2^2,$$

where $a_{1,0,0},\dots,d_{5,0,0} \in \mathbb{F}$ and $m \in \mathbb{Z}$. Similarly, we have the formulas of $D(x_2^n\partial_2)$ and $D(\partial_2^2)$, where $n \in \mathbb{Z}$.

Proof. Note that the right annihilator of $\sum_{u=1}^{2}\partial_u$ is spanned by B_1 and $x_u\partial_u$, $1 \le u \le 2$, is local multiplicative identity of the algebra. Using the above facts, by induction on k of $x_u^k\partial_u$, $1 \le u \le 2$, we can prove the lemma. So let us omit the details of its proof. □

LEMMA 2.2. *For any derivation D of the non-associative algebra $\overline{WP_{0,2,0}}_{B_1}$ and $x_1,\dots,x_2\partial_1^2 \in \overline{WP_{0,2,0}}_{B_1}$, the followings hold*

$$\begin{aligned}
(6)\ D(x_1) &= -a_{1,0,0}x_1 - b_{1,0,0}x_2 + s_{1,0,0},\\
D(x_1\partial_2) &= -a_{1,0,0}x_1\partial_2 - b_{1,0,0}x_2\partial_2 + b_{2,0,0}x_1\partial_2 + s_{1,0,0}\partial_2 + b_{1,0,0}x_1\partial_1\\
&\quad + b_{3,0,0}x_1\partial_{12} + b_{4,0,0}x_1\partial_1^2,\\
D(x_1\partial_{12}) &= -a_{1,0,0}x_1\partial_{12} - b_{1,0,0}x_2\partial_{12} + c_{3,0,0}x_1\partial_{12} + s_{1,0,0}\partial_{12}\\
&\quad + c_{1,0,0}x_1\partial_1 + c_{2,0,0}x_1\partial_2 + c_{4,0,0}x_1\partial_1^2 + c_{5,0,0}x_1\partial_2^2,\\
D(x_2\partial_{12}) &= -a_{2,0,0}x_1\partial_{12} - b_{2,0,0}x_2\partial_{12} + c_{3,0,0}x_2\partial_{12} + t_{2,0,0}\partial_{12}\\
&\quad + c_{1,0,0}x_2\partial_1 + c_{2,0,0}x_2\partial_2 + c_{4,0,0}x_2\partial_1^2 + c_{5,0,0}x_2\partial_2^2,\\
D(x_1\partial_1^2) &= -a_{1,0,0}x_1\partial_1^2 - b_{1,0,0}x_2\partial_1^2 + d_{4,0,0}x_1\partial_1^2 + s_{1,0,0}\partial_1^2 + d_{1,0,0}x_1\partial_1\\
&\quad +d_{2,0,0}x_1\partial_2 + d_{3,0,0}x_1\partial_{12} + d_{5,0,0}x_1\partial_2^2,\\
D(x_2\partial_1^2) &= -b_{2,0,0}x_2\partial_1^2 - a_{2,0,0}x_1\partial_1^2 + d_{4,0,0}x_2\partial_1^2 + t_{2,0,0}\partial_1^2 + d_{1,0,0}x_2\partial_1\\
&\quad +d_{2,0,0}x_2\partial_2 + d_{3,0,0}x_2\partial_{12} + d_{5,0,0}x_2\partial_2^2,
\end{aligned}$$

where $a_{1,0,0},\dots,d_{5,0,0} \in \mathbb{F}$. We have similar formulas of $D(x_2)$, $D(x_2\partial_1)$, $D(x_2\partial_{12})$, $D(x_1\partial_2^2)$, and $D(x_2\partial_2^2)$ in (6).

Proof. The proof of the lemma comes from the similar comments of Lemma 2.1, so omitted. □

LEMMA 2.3. *For any derivation D of the non-associative algebra $\overline{WP_{0,2,0}}_{B_1}$ and for the bases $x_1^m x_2^n$, $x_1^m x_2^n \partial_1$ in $\overline{WP_{0,2,0}}_{B_1}$, the followings hold*

$$
\begin{aligned}
(7)\quad & D(x_1^m x_2^n) \\
&= -ma_{1,0,0}x_1^m x_2^n - na_{2,0,0}x_1^{m+1}x_2^{n-1} - mb_{1,0,0}x_1^{m-1}x_2^{n+1} \\
&\quad - nb_{2,0,0}x_1^m x_2^n + mb_{3,0,0}x_1^{m-1}x_2^n + m(m-1)b_{4,0,0}x_1^{m-2}x_2^{n+1} \\
&\quad + ms_{1,0,0}x_1^{m-1}x_2^n + nt_{2,0,0}x_1^m x_2^{n-1}, \\
& D(x_1^m x_2^n \partial_1) \\
&= (1-m)a_{1,0,0}x_1^m x_2^n \partial_1 - na_{2,0,0}x_1^{m+1}x_2^{n-1}\partial_1 \\
&\quad - mb_{1,0,0}x_1^{m-1}x_2^{n+1}\partial_1 - nb_{2,0,0}x_1^m x_2^n \partial_1 + mb_{3,0,0}x_1^{m-1}x_2^n\partial_1 \\
&\quad + m(m-1)b_{4,0,0}x_1^{m-2}x_2^{n+1}\partial_1 + ms_{1,0,0}x_1^{m-1}x_2^n\partial_1 + nt_{2,0,0}x_1^m x_2^{n-1}\partial_1 \\
&\quad + a_{2,0,0}x_1^m x_2^n\partial_2 + a_{3,0,0}x_1^m x_2^n\partial_{12} + a_{5,0,0}x_1^m x_2^n\partial_2^2,
\end{aligned}
$$

where $a_{1,0,0}, \ldots, t_{2,0,0} \in \mathbb{F}$. We have the following similar formulas of $D(x_1^m x_2^n \partial_2)$, $D(x_1^m x_2^n \partial_{12})$, $D(x_1^m x_2^n \partial_1^2)$, $D(x_1^m x_2^n \partial_2^2)$ for any bases $x_1^m x_2^n \partial_2$, $x_1^m x_2^n \partial_{12}$, $x_1^m x_2^n \partial_1^2$ and $x_1^m x_2^n \partial_2^2$ in $\overline{WP_{0,0,2}}_{B_1}$ as (7).

Proof. The proof of the lemma comes from the similar comments of the proof of Lemma 2.2, so omitted. □

LEMMA 2.4. *If there is a derivation D of the non-associative algebra $\overline{WP_{0,2,0}}_{B_1}$ such that*

$$
\begin{aligned}
D(x_1^m x_2^n \partial_i) \;=\; & \delta_{\partial_i,\partial_1} a_{3,0,0}x_1^m x_2^n\partial_{12} + \delta_{\partial_i,\partial_1} a_{5,0,0}x_1^m x_2^n\partial_2^2 \\
& + \delta_{\partial_i,\partial_2} b_{3,0,0}x_1^m x_2^n\partial_{12} + \delta_{\partial_i,\partial_2} b_{4,0}x_1^m x_2^n\partial_1^2 \\
& + \delta_{\partial_i,\partial_{12}} c_{1,0,0}x_1^m x_2^n\partial_1 + \delta_{\partial_i,\partial_{12}} c_{2,0,0}x_1^m x_2^n\partial_2 \\
& + \delta_{\partial_i,\partial_{12}} c_{3,0,0}x_1^m x_2^n\partial_{12} + \delta_{\partial_i,\partial_{12}} c_{4,0,0}x_1^m x_2^n\partial_1^2 \\
& + \delta_{\partial_i,\partial_{12}} c_{5,0,0}x_1^m x_2^n\partial_2^2 + \delta_{\partial_i,\partial_1^2} d_{1,0,0}x_1^m x_2^n\partial_1 \\
& + \delta_{\partial_i,\partial_1^2} d_{2,0,0}x_1^m x_2^n\partial_2 + \delta_{\partial_i,\partial_1^2} d_{3,0,0}x_1^m x_2^n\partial_{12} \\
& + \delta_{\partial_i,\partial_1^2} d_{4,0,0}x_1^m x_2^n\partial_1^2 + \delta_{\partial_i,\partial_1^2} d_{5,0,0}x_1^m x_2^n\partial_2^2 \\
& + \delta_{\partial_i,\partial_2^2} h_{1,0,0}x_1^m x_2^n\partial_1 + \delta_{\partial_i,\partial_2^2} h_{2,0,0}x_1^m x_2^n\partial_2 \\
& + \delta_{\partial_i,\partial_2^2} h_{3,0,0}x_1^m x_2^n\partial_{12} + \delta_{\partial_i,\partial_2^2} h_{4,0,0}x_1^m x_2^n\partial_1^2 \\
& + \delta_{\partial_i,\partial_2^2} h_{5,0,0}x_1^m x_2^n\partial_2^2,
\end{aligned}
$$

then $a_{3,0,0} = \cdots = h_{5,0,0} = 0$, i.e., $D(x_1^m x_2^n \partial_i) = 0$ for $x_1^m x_2^n \partial_i \in \overline{WP_{0,2,0}}_{B_1}$, where $\delta_{i,j}$ is the Kronecker delta, and $\partial_i \in B_1$.

Proof. Let D be the derivation in the lemma. For any elements $x_1^{m_1}x_2^{n_1}\partial_1$ and $x_1^{m_2}x_2^{n_2}\partial_2$, $m_1, m_2, n_1, n_2 \in \mathbb{Z}$, we have that

$$(8)\qquad D(x_1^{m_1}x_2^{n_1}\partial_1 * x_1^{m_2}x_2^{n_2}\partial_2) = m_2 D(x_1^{m_1+m_2-1}x_2^{n_1+n_2}\partial_2).$$

The left side of (8) is

$$\begin{aligned}(9)\qquad & D(x_1^{m_1}x_2^{n_1}\partial_1) * x_1^{m_2}x_2^{n_2}\partial_2 + x_1^{m_1}x_2^{n_1}\partial_1 * D(x_1^{m_2}x_2^{n_2}\partial_2)\\ = \;& (a_{3,0,0}x_1^{m_1}x_2^{n_1}\partial_{12} + a_{5,0,0}x_1^{m_1}x_2^{n_1}\partial_2^2 * x_1^{m_2}x_2^{n_2}\partial_2 + x_1^{m_1}x_2^{n_1}\partial_1 *\\ & (b_{3,0,0}x_1^{m_2}x_2^{n_2}\partial_{12} + b_{4,0,0}x_1^{m_2}x_2^{n_2}\partial_1^2)\\ = \;& m_2n_2a_{3,0,0}x_1^{m_1+m_2-1}x_2^{n_1+n_2-1}\partial_2\\ & +n_2(n_2-1)a_{5,0,0}x_1^{m_1+m_2}x_2^{n_1+n_2-2}\partial_2\\ & +m_2b_{3,0,0}x_1^{m_1+m_2-1}x_2^{n_1+n_2}\partial_{12} + m_2b_{4,0,0}x_1^{m_1+m_2-1}x_2^{n_1+n_2}\partial_1^2\end{aligned}$$

and the right side of (8) is

$$(10)\qquad m_2b_{3,0,0}x_1^{m_1+m_2-1}x_2^{n_1+n_2}\partial_{12} + m_2b_{4,0,0}x_1^{m_1+m_2-1}x_2^{n_1+n_2}\partial_1^2.$$

By (9) and (10), we have that $a_{3,0,0} = a_{5,0,0} = 0$. By

$$D(x_1^{m_2}x_2^{n_2}\partial_2 * x_1^{m_1}x_2^{n_1}\partial_1) = n_1 D(x_1^{m_1+m_2}x_2^{n_1+n_2-1}\partial_1),$$

we also have that $(b_{3,0}x_1^{m_2}x_2^{n_2}\partial_{12}+b_{4,0,0}x_1^{m_2}x_2^{n_2}\partial_1^2)*x_1^{m_1}x_2^{n_1}\partial_1+x_1^{m_2}x_2^{n_2}\partial_2$ $*0 = 0$. This implies that $b_{3,0} = b_{4,0} = 0$. By

$$D(x_1^{m_1}x_2^{n_1}\partial_{12} * x_1^{m_2}x_2^{n_2}\partial_2) = m_2n_2 D(x_1^{m_1+m_2-1}x_2^{n_1+n_2-1}\partial_1),$$

we have that

$$\begin{aligned} & (c_{1,0,0}x_1^{m_1}x_2^{n_1}\partial_1 + c_{2,0,0}x_1^{m_1}x_2^{n_1}\partial_2 + c_{3,0,0}x_1^{m_1}x_2^{n_1}\partial_{12}\\ & +c_{4,0,0}x_1^{m_1}x_2^{n_1}\partial_1^2 + c_{5,0,0}x_1^{m_1}x_2^{n_1}\partial_2^2) * x_1^{m_2}x_2^{n_2}\partial_2 + x_1^{m_1}x_2^{n_1}\partial_{12} * 0\\ = \;& m_2c_{1,0,0}x_1^{m_1+m_2-1}x_2^{n_1+n_2}\partial_2\\ & +n_2c_{2,0,0}x_1^{m_1+m_2}x_2^{n_1+n_2-1}\partial_2 + m_2n_2c_{3,0,0}x_1^{m_1+m_2-1}x_2^{n_1+n_2-1}\partial_2\\ & +m_2(m_2-1)c_{4,0,0}x_1^{m_1+m_2-2}x_2^{n_1+n_2}\partial_2\\ & +n_2(n_2-1)c_{5,0,0}x_1^{m_1+m_2}x_2^{n_1+n_2-2}\partial_2\\ = \;& 0.\end{aligned}$$

This implies that $c_{i,0,0} = 0$, $1 \le i \le 5$. Similarly, we can prove that $d_{i,0,0} = h_{i,0,0} = 0$, $1 \le i \le 5$. This implies that D is the zero derivation of $\overline{WP_{0,2,0}}_{B_1}$. This completes the proof of lemma. □

Note 1. By Lemma 2.4, for any basis element $x_1^m x_2^n\partial_u$, $\partial_u \in B_1$, of $\overline{WP_{0,2,0}}_{B_1}$, if we define $\mathbb{F}$-linear maps D_v, $1 \le v \le 2$, $D_{v,w}$, $1 \le v \ne w \le$

2, and D_{4+v}, $1 \le v \le 2$, as follows:

$$D_v(x_1^{n_1}x_2^{n_2}\partial_u) = (\delta_{u,v} - n_u)x_1^{n_1}x_2^{n_2}\partial_u \text{ for } 1 \le v \le 2$$
$$D_{v,w}(x_1^{n_1}x_2^{n_2}\partial_u) = -n_w x_v^{n_v+1}x_w^{n_w-1}\partial_u + \delta_{u,v}x_1^{n_1}x_2^{n_2}\partial_w$$
$$\text{for } 1 \le v \ne w \le 2,$$
$$D_{4+v}(x_1^{n_1}x_2^{n_2}\partial_u) = n_v x_v^{n_v-1}x_k^{n_k}\partial_u \text{ for } 1 \le v \le 2,$$

where $\delta_{i,j}$ is the Kronecker delta, then the $\mathbb{F}$-linear maps D_v, $1 \le v \le 2$, $D_{v,w}$, $1 \le v \ne w \le 2$, and D_{4+v}, $1 \le v \le 2$, of $\overline{WP_{0,2,0}}_{B_1}$ can be linearly extended to derivations of $\overline{WP_{0,2,0}}_{B_1}$ and the derivation of Lemma 2.3 is the linear sum of these derivations.

THEOREM 2.1. *For any $D \in Der(\overline{WN_{0,2,0}}_{B_1})$, D is linear sum of the derivations D_v, $1 \le v \le 2$, $D_{v,w}$, $1 \le v \ne w \le 2$, and D_{4+v}, $1 \le v \le 2$, which are defined in Note 1.*

Proof. By Lemma 2.3, for any basis $x_1^m x_2^n\partial_u$, $\partial_u \in B_1$, $D(x_1^m x_2^n\partial_u)$ is one of the forms of (7). By Lemma 2.4, D can be written as the linear sum of D_v, $1 \le v \le 2$, $D_{v,w}$, $1 \le v \ne w \le 2$, and D_{4+v}, $1 \le v \le 2$. This completes the proof of the theorem. □

COROLLARY 2.1. *The dimension $Dim(Der(\overline{WN_{0,2,0}}_{B_1}))$ of*

$$Der(\overline{WN_{0,2,0}}_{B_1})$$

is 6.

Proof. The proof of the corollary is straightforward by Theorem 2.1. □

3. Derivations of $\overline{WP_{0,2,0}}_{B_2}$

Note that the non-associative algebra $\overline{WP_{0,2,0}}_{B_2}$ is a simple subalgebra of $\overline{WP_{0,2,0}}_{B_1}$. Since the algebra $\overline{WP_{0,2,0}}_{B_2}$ enjoys the similar results of Lemma 2.1-4, we have the following note.

Note 2. By the similar results of Lemma 2.4, for any basis element $x_1^m x_2^n\partial_u$, $\partial_u \in B_2$, of $\overline{WP_{0,2,0}}_{B_2}$, if we define $\mathbb{F}$-linear maps D_v, $1 \le v \le 2$, $D_{v,w}$, $1 \le v \ne w \le 2$, and D_{4+v}, $1 \le v \le 2$, as follows:

$$D_v(x_1^{n_1}x_2^{n_2}\partial_u) = (\delta_{u,v} - n_u)x_1^{n_1}x_2^{n_2}\partial_u \text{ for } 1 \le v \le 2,$$
$$D_{v,w}(x_1^{n_1}x_2^{n_2}\partial_u) = -n_w x_v^{n_v+1}x_w^{n_w-1}\partial_u + \delta_{u,v}x_1^{n_1}x_2^{n_2}\partial_w$$
$$\text{for } 1 \le v \ne w \le 2,$$
$$D_{4+v}(x_1^{n_1}x_2^{n_2}\partial_u) = n_v x_v^{n_v-1}x_k^{n_k}\partial_u \text{ for } 1 \le v \le 2,$$

where $\delta_{i,j}$ is the Kronecker delta, then the $\mathbb{F}$-linear maps D_v, $1 \leq v \leq 2$, $D_{v,w}$, $1 \leq v \neq w \leq 2$, and D_{4+v}, $1 \leq v \leq 2$, of $\overline{WP_{0,2,0}}_{B_2}$ can be linearly extended to derivations of $\overline{WP_{0,2,0}}_{B_2}$ as Note 1.

THEOREM 3.1. *For any* $D \in Der(\overline{WN_{0,2,0}}_{B_2})$, D *is linear sum of the derivations* D_v, $1 \leq v \leq 2$, $D_{v,w}$, $1 \leq v \neq w \leq 2$, *and* D_{4+v}, $1 \leq v \leq 2$, *which are defined in Notes.*

Proof. The proof of the theorem is similar to the proof of Theorem 2.1, so omitted. □

If we put $B_3 = \{\partial_0, \partial_1, \partial_2, \partial_{12}, \partial_2^2\}$, then we define the non-associative algebra $\overline{WP_{0,2,0}}_{B_3}$ which is a simple subalgebra of $\overline{WP_{0,2,0}}_{B_1}$.

PROPOSITION 3.1. *The non-associative algebra* $\overline{WN_{0,2,0}}_{B_2}$ *is isomorphic to the non-associative algebra* $\overline{WN_{0,2,0}}_{B_3}$ *as non-associative algebras.*

Proof. The proof of the proposition is easy, so omitted. □

COROLLARY 3.1. *The additive abelian group* $Der(\overline{WN_{0,2,0}}_{B_2})$ *is isomorphic to the additive abelian group* $Der(\overline{WN_{0,2,0}}_{B_3})$ *as abelian groups.*

Proof. The proof of the corollary is straightforward by Theorem 3.1 and Proposition 3.1. □

COROLLARY 3.2. *The dimension* $Dim(Der(\overline{WN_{0,2,0}}_{B_2}))$ *(resp.* Dim $(Der(\overline{WN_{0,2,0}}_{B_3})))$ *of* $\overline{WN_{0,2,0}}_{B_2}$ *(resp.* $\overline{WN_{0,2,0}}_{B_3}$*) is 6.*

Proof. The proof of the corollary is straightforward by Theorem 3.1. □

References

[1] H. M. Ahmadi, K.-B. Nam, and J. Pakianathan, *Lie admissible non-associative algebras*, Algebra Colloq. **12** (2005), no. 1, 113–120.

[2] R. Block, *On torsion-free abelian groups and Lie algebras*, Proc. Amer. Math. Soc. **9** (1958), 613–620.

[3] S. H. Choi and K.-B. Nam, *Derivations of a restricted Weyl type algebra I*, Rocky Mountain J. Math., Accepted, 2005.

[4] J. Diximier, *Enveloping Algebras*, AMS, 1996.

[5] J. E. Humphreys, *Introduction to Lie Algebras and Representation Theory*, Springer-Verlag, New York, 1978.

[6] T. Ikeda, N. Kawamoto, and K. Nam, *A class of simple subalgebras of generalized Witt algebras*, Groups—Korea '98 (Pusan), de Gruyter, Berlin, 2000, 189–202.

[7] V. G. Kac, *Description of filtered Lie algebra with which graded Lie algebras of Cartan type are associated*, Izv. Akad. Nauk SSSR Ser. Mat. **38** (1974), 800–834.
[8] A. I. Kostrikin and I. R. Šafarevič, *Graded Lie algebras of finite characteristic*, Math. USSR Izv. **3** (1970), no. 2, 237–240.
[9] K.-S. Lee and K.-B. Nam, *Some W-type algebras I*, J. Appl. Algebra Discrete Struct. **2** (2004), no. 1, 39–46.
[10] K.-B. Nam, *Generalized W and H type Lie algebras*, Algebra Colloq. **6** (1999), no. 3, 329–340.
[11] K.-B. Nam and S. H. Choi, *Automorphism group of non-associative algebras* $\overline{WN_{2,0,0_1}}$, J. Comput. Math. Optim. **1** (2005), no. 1, 35–44.
[12] K.-B. Nam, S. H. Choi, M.-O. Wang, *Weyl-type non-associative algebras III*, J. Appl. Algebra Discrete Struct. **3** (2005), no. 2, 91–100.

Department of Mathematics, Jeonju University, Chon-ju 560-759, Korea
E-mail: chois@jj.ac.kr