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WEIGHTED COMPOSITION OPERATORS BETWEEN BERGMAN-TYPE SPACES

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Abstract. In this paper, we characterize the boundedness and compactness of weighted composition operators $\psi C_\varphi f = \psi f o \varphi$ acting between Bergman-type spaces.

1. Introduction

Let $\mathbb{D}$ be the open unit disk in the complex plane $\mathbb{C}$. Denote by $\mathbb{H}(\mathbb{D})$ the space of holomorphic functions on $\mathbb{D}$. A weighted composition operator $\psi C_\varphi(f)(z) = \psi(z)f(\varphi(z))$, for all $z \in \mathbb{D}$, where φ and ψ are holomorphic functions defined in $\mathbb{D}$ such that $\varphi(\mathbb{D}) \subset \mathbb{D}$. When $\psi = 1$, we just have the composition operator C_φ defined by $C_\varphi(f) = f o \varphi$ and when $\varphi(z) = z$ we have the multiplication operator M_ψ defined by $M_\psi(f) = \psi f$. During the last century, composition operators have been studied extensively on spaces of analytic functions with the aim to explore the connection between the behavior of C_φ and function theoretic properties of φ. During the past few decades this subject has undergone explosive growth. As a consequence of the Littlewood Subordination principle [10] it is known that every analytic self map φ induces a bounded composition on Hardy and weighted Bergman spaces of the unit disk. However characterizing the compact composition operators acting on Hardy spaces of the disk was a difficult problem. Commendable work in this direction was done by Schwartz [14], Shapiro and Taylor [15], MacCluer and Shapiro [11] and Shapiro [16]. Many other important properties of C_φ have also been studied on these spaces. We refer

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to recent monographs [16] and [4] for an over all view of the whole spectrum of present knowledge concerning composition operators. Weighted composition operators appear naturally in different contexts. For example, Singh and Sharma [19] related the boundedness of composition operators on Hardy space of the upper half plane with the boundedness of weighted composition operators on the Hardy space of the open unit disk $\mathbb{D}$. Weighted composition operators also played an important role in the study of compact composition operators on Hardy spaces and Bergman spaces of unbounded domains (see for example [12] and [18] for more details.) Isometries in many Banach spaces of analytic functions are just weighted composition operators, for example see [6] and [8].

Recently, several authors have studied weighted composition operators on different spaces of analytic functions. For example, one can refer to [1], [2] and [3] for study of these operators on Hardy spaces, [9] and [22] for disk algebra, [21] and [23] for Bloch-type spaces and [5] and [12] for weighted Bergman spaces. In this paper we characterize boundedness and compactness of weighted composition operators between Bergman-type spaces.

2. Preliminaries

In this section we review the basic concepts of weighted Bergman spaces A^p_α and Bergman-type spaces, denoted by $A^{-\alpha}$ and $A_0^{-\alpha}$ which are closely related to the weighted Bergman spaces A^p_α and are sometimes called growth spaces. We also collect some essential facts that will be needed throughout the paper.

2.1. Weighted Bergman spaces

Let $dA(z)$ be the area measure on $\mathbb{D}$ normalized so that area of $\mathbb{D}$ is 1. For each $\alpha \in (-1, \infty)$, we set $d\lambda_\alpha(z) = (\alpha+1)(1-|z|^2)^\alpha dA(z), z \in \mathbb{D}$. Then $d\lambda_\alpha$ is a probability measure on $\mathbb{D}$. For $0 < p < \infty$ the weighted Bergman space A^p_α is defined as

$$A^p_\alpha = \{f \in \mathbb{H}(\mathbb{D}) : ||f||_{A^p_\alpha} = \Big(\int_{\mathbb{D}} |f(z)|^p d\lambda_\alpha(z)\Big)^{1/p} < \infty\}.$$

Note that $||f||_{A^p_\alpha}$ is a true norm only if $1 \le p < \infty$ and in this case A^p_α is a Banach space. For $0 < p < 1$, A^p_α is a non-locally convex topological vector space and $d(f,g) = ||f-g||^p_{A^p_\alpha}$ is a complete metric for it.
The following lemma tells us how fast an arbitrary function from A^p_α grows near the boundary. The growth of functions in the weighted

Bergman spaces is essential in our study. To this end, the following sharp estimate will be useful. (see [7], p. 53.)

LEMMA 2.1. *Let $f \in A^p_\alpha$. Then for every z in $\mathbb{D}$, we have*

$$|f(z)| \leq \frac{||f||_{A^p_\alpha}}{(1-|z|^2)^{(2+\alpha)/p}}$$

with equality if and only if f is a constant multiple of the function

$$k_a(z) = \Big(\frac{1-|z|^2}{(1-\bar{a}z)^2}\Big)^{(2+\alpha)/p}. \tag{2.1}$$

It can be easily shown that $||k_a||^p_{A^p_\alpha} = 1$. Since polynomials are dense in A^p_α, it is an immediate consequence of Lemma 2.1 that for $f \in A^p_\alpha$,

$$|f(z)| = o\Big(\frac{||f||_{A^p_\alpha}}{(1-|z|^2)^{(2+\alpha)/p}}\Big) \text{ as } |z| \to 1, \tag{2.2}$$

which means that the boundary growth is not as fast as permitted by Lemma 2.1.

2.2. The Growth Spaces $A^{-\alpha}$ and $A_0^{-\alpha}$

For any $\alpha > 0$, the space $A^{-\alpha}$ consists of analytic functions f in $\mathbb{D}$ such that

$$||f||_{A^{-\alpha}} = \sup\{(1-|z|^2)^\alpha |f(z)| : z \in \mathbb{D}\} < \infty.$$

Each $A^{-\alpha}$ is a non-separable Banach space with the norm defined above and contains all bounded analytic functions on $\mathbb{D}$. The closure in $A^{-\alpha}$ of the set of polynomials will be denoted by $A_0^{-\alpha}$, which is a separable Banach space and consists of exactly those functions f in $A^{-\alpha}$ with

$$\lim_{z\to 1^-}(1-|z|^2)^\alpha |f(z)| = 0.$$

For general background on weighted Bergman spaces A^p_α and Bergman-type spaces, $A^{-\alpha}$ and $A_0^{-\alpha}$, one may consult [7] and [24] and the references therein.

3. Weighted composition operators

In this section we study weighted composition operators between Bergman-type spaces. W. Smith in [20] characterized the boundedness and compactness of composition operators between weighted Bergman spaces, whereas S. Ohno, K. Stroethoff and R. Zhao [21] characterized boundedness and compactness of composition operators between weighted Bloch spaces.

THEOREM 3.1. *Let $1 \le p < \infty, -1 < \beta < \infty$ and $\alpha > 0$. Let φ and ψ be holomorphic maps on $\mathbb{D}$ such that $\varphi(\mathbb{D}) \subset \mathbb{D}$. Then the weighted composition operator $\psi C_\varphi : A^p_\beta \to A^{-\alpha}$ is bounded if and only if*

$$\sup_{z\in\mathbb{D}} \frac{(1-|z|^2)^\alpha |\psi(z)|}{(1-|\varphi(z)|^2)^{(2+\beta)/p}} < \infty. \tag{3.1}$$

PROOF. First suppose that

$$M = \sup_{z\in\mathbb{D}} \frac{(1-|z|^2)^\alpha |\psi(z)|}{(1-|\varphi(z)|^2)^{(2+\beta)/p}} < \infty.$$

By Lemma 2.1, we have

$$|f(z)| \le \frac{||f||_{A^p_\beta}}{(1-|z|^2)^{(2+\beta)/p}}$$

for all $z \in \mathbb{D}$, independent of $f \in A^p_\beta$. Thus for $z \in \mathbb{D}$, we get

$$\begin{aligned} ||\psi C_\varphi f||_{A^{-\alpha}} &= \sup_{z\in\mathbb{D}} (1-|z|^2)^\alpha |\psi C_\varphi f(z)| \\ &= \sup_{z\in\mathbb{D}} (1-|z|^2)^\alpha |\psi(z) f(\varphi(z))| \\ &\le \sup_{z\in\mathbb{D}} \frac{(1-|z|^2)^\alpha |\psi(z)|}{(1-|\varphi(z)|^2)^{(2+\beta)/p}} ||f||_{A^p_\beta} \\ &= M||f||_{A^p_\beta}, \end{aligned}$$

hence $\psi C_\varphi : A^p_\beta \to A^{-\alpha}$ is bounded. Conversely, suppose $\psi C_\varphi : A^p_\beta \to A^{-\alpha}$ is bounded. Fix a point z_0 in $\mathbb{D}$ and let $w = \varphi(z_0)$. Consider the function

$$f_w(z) = \Big(\frac{1-|w|^2}{(1-\overline{w}z)^2}\Big)^{(2+\beta)/p}.$$

Then $||f_w||_{A^p_\beta} = 1$. Since $\psi C_\varphi : A^p_\beta \to A^{-\alpha}$ is bounded, there is a constant C such that

$$||\psi C_\varphi f_w||_{A^{-\alpha}} \le C||f_w||_{A^p_\beta} = C,$$

hence for each point $z \in \mathbb{D}$ we have

$$|\psi(z)|(1-|z|^2)^\alpha |f_w(\varphi(z))| \le C.$$

In particular, when $z = z_0$ we get

$$|\psi(z_0)|(1-|z_0|^2)^\alpha \Big(\frac{1-|\varphi(z_0)|^2}{(1-|\varphi(z_0)|^2)^2}\Big)^{(2+\beta)/p} \le C,$$

whence

$$\frac{(1-|z_0|^2)^\alpha|\psi(z_0)|}{(1-|\varphi(z_0)|^2)^{(2+\beta)/p}} \le C.$$

Since $z_0 \in \mathbb{D}$ was arbitrary, the result follows. □

COROLLARY 3.2. *Let $1 \le p < \infty, -1 < \beta < \infty, \alpha > 0$ and φ be holomorphic map on $\mathbb{D}$ such that $\varphi(\mathbb{D}) \subset \mathbb{D}$. Then the composition operator $C_\varphi : A^p_\beta \to A^{-\alpha}$ is bounded if and only if*

$$\sup_{z\in\mathbb{D}} \frac{(1-|z|^2)^\alpha}{(1-|\varphi(z)|^2)^{(2+\beta)/p}} < \infty.$$

COROLLARY 3.3. *Let $1 \le p < \infty, -1 < \beta < \infty, \alpha > 0$ and ψ be holomorphic map on $\mathbb{D}$. Then the multiplication operator $M_\psi : A^p_\beta \to A^{-\alpha}$ is bounded if and only if*

$$\sup_{z\in\mathbb{D}} (1-|z|^2)^{(p\alpha-\beta-2)/p}|\psi(z)| < \infty.$$

THEOREM 3.4. *Let $1 \le p < \infty, -1 < \beta < \infty$ and $\alpha > 0$. Let φ and ψ be holomorphic maps on $\mathbb{D}$ such that $\varphi(\mathbb{D}) \subset \mathbb{D}$. Then the weighted composition operator $\psi C_\varphi : A^p_\beta \to A^{-\alpha}$ is compact if and only if*

$$\lim_{r\to 1} \sup_{\{z:|\varphi(z)|>r\}} \frac{(1-|z|^2)^\alpha|\psi(z)|}{(1-|\varphi(z)|^2)^{(2+\beta)/p}} = 0. \tag{3.2}$$

PROOF. Let $\{f_n\}$ be a bounded sequence in A^p_β that converges to zero uniformly on compact subsets of $\mathbb{D}$. Let $M = \sup_n ||f_n||_{A^p_\beta} < \infty$. Given $\varepsilon > 0$, there exist an r such that if $|\varphi(z)| > r$, then

$$\frac{(1-|z|^2)^\alpha|\psi(z)|}{(1-|\varphi(z)|^2)^{(2+\beta)/p}} < \varepsilon.$$

By Lemma 2.1 we have

$$|f_n(z)| \le \frac{||f_n||_{A^p_\beta}}{(1-|z|^2)^{(2+\beta)/p}}.$$

Thus for $z \in \mathbb{D}$, we have

$$\begin{aligned}(1-|z|^2)^\alpha|\psi C_\varphi f_n(z)| &= (1-|z|^2)^\alpha|\psi(z)|f_n(\varphi(z))| \\ &\le \frac{(1-|z|^2)^\alpha|\psi(z)|}{(1-|\varphi(z)|^2)^{(2+\beta)/p}}||f_n||_{A^p_\beta} \\ &\le \varepsilon M,\end{aligned}$$

for all n. On the other hand, since $f_n \to 0$ uniformly on $\{w : |w| \leq r\}$, there exist an n_0 such that, if $|\varphi(z)| \leq r$ and $n \geq n_0$ then $|f_n(\varphi(z))| < \varepsilon$. Also Theorem 3.1 implies that $\psi \in A^{-\alpha}$. Thus we have

$$N = \sup_{z\in\mathbb{D}}(1-|z|^2)^\alpha|\psi(z)| < \infty$$

and hence

$$\begin{aligned}(1-|z|^2)^\alpha|\psi C_\varphi f_n(z)| &= (1-|z|^2)^\alpha|\psi(z)||f_n(\varphi(z))|\\ &\leq N\varepsilon.\end{aligned}$$

Conversely, suppose that $\psi C_\varphi : A^p_\beta \to A^{-\alpha}$ is compact and (3.2) does not holds. Then there exist a positive number δ and a sequence $\{z_n\}$ in $\mathbb{D}$ such that $|\varphi(z_n)| \to 1$ and

$$\frac{(1-|z_n|^2)^\alpha|\psi(z_n)|}{(1-|\varphi(z_n)|^2)^{(2+\beta)/p}} \geq \delta,$$

for all n. For each n, let $w_n = \varphi(z_n)$ and consider the function f_n as

$$f_n(z) = \Big(\frac{1-|w_n|^2}{(1-\overline{w}_n z)^2}\Big)^{(2+\beta)/p}, \; z \in \mathbb{D}.$$

Then f_n is norm bounded and $f_n \to 0$ uniformly on compact subsets of $\mathbb{D}$, it follows that a subsequence of $\{\psi C_\varphi f_n\}$ tends to 0 in $A^{-\alpha}$. On the other hand,

$$\begin{aligned}||\psi C_\varphi f_n||_{A^{-\alpha}} &\geq (1-|z_n|^2)^\alpha|\psi C_\varphi f_n(z_n)|\\ &= (1-|z_n|^2)^\alpha|\psi(z_n)f_n(\varphi(z_n))|\\ &= \frac{(1-|z_n|^2)^\alpha|\psi(z_n)|}{(1-|\varphi(z_n)|^2)^{(2+\beta)/p}}\\ &\geq \delta,\end{aligned}$$

which is absurd. Hence we are done. □

COROLLARY 3.5. *Let $1 \leq p < \infty, -1 < \beta < \infty, \alpha > 0$ and φ be holomorphic map on $\mathbb{D}$ such that $\varphi(\mathbb{D}) \subset \mathbb{D}$. Then the composition operator $C_\varphi : A^p_\beta \to A^{-\alpha}$ is compact if and only if*

$$\lim_{r\to 1}\sup_{\{z:|\varphi(z)|>r\}}\frac{(1-|z|^2)^\alpha}{(1-|\varphi(z)|^2)^{(2+\beta)/p}} = 0.$$

COROLLARY 3.6. *Let $1 \leq p < \infty, -1 < \beta < \infty, \alpha > 0$ and ψ be holomorphic map on $\mathbb{D}$. Then the multiplication operator $M_\psi : A^p_\beta \to$*

$A^{-\alpha}$ *is compact if and only if*

$$\lim_{r\to 1}\sup_{|z|>r}(1-|z|^2)^{(p\alpha-\beta-2)/p}|\psi(z)|=0.$$

THEOREM 3.7. *Let* $1\le p<\infty, -1<\beta<\infty$ *and* $\alpha>0$ *and* φ *and* ψ *be holomorphic maps on* $\mathbb{D}$ *such that* $\varphi(\mathbb{D})\subset\mathbb{D}$. *Then the weighted composition operator* $\psi C_\varphi : A^p_\beta \to A_0^{-\alpha}$ *is bounded if and only if*

(*i*) $\displaystyle\sup_{z\in\mathbb{D}}\frac{(1-|z|^2)^\alpha|\psi(z)|}{(1-|\varphi(z)|^2)^{(2+\beta)/p}}<\infty$

(*ii*) $\psi\in A_0^{-\alpha}$.

PROOF. First, suppose $\psi C_\varphi : A^p_\beta\to A_0^{-\alpha}$ is bounded. Then (*i*) can be proved exactly in the same way as in the proof of Theorem 3.1. By taking $f(z)=c$, we get $\psi\in A_0^{-\alpha}$. Conversely suppose that (*i*) and (*ii*) are satisfied. Let $\varepsilon>0$ and $f\in A^p_\beta$. Then by (2.2)

$$|f(z)|=o\Big(\frac{||f||_{A^p_\beta}}{(1-|z|^2)^{(2+\beta)/p}}\Big)\ \text{as}\ |z|\to 1$$

and so by (*i*) there is $\delta_1\in(0,1)$ such that for $z\in\mathbb{D}$ with $|z|>\delta_1$, we can find a constant $C_1>0$ such that

$$(1-|z|^2)^\alpha|\psi(z)f(\varphi(z))|<\varepsilon\frac{(1-|z|^2)^\alpha|\psi(z)|}{(1-|\varphi(z)|^2)^{(2+\beta)/p}}||f||_{A^p_\beta}$$

$$\le C_1\varepsilon. \tag{3.3}$$

On the other hand since by (*ii*) $\psi\in A_0^{-\alpha}$, for the above $\varepsilon>0$, there is $\delta_2\in(0,1)$ such that $|z|>\delta_2$, implies

$$(1-|z|^2)^\alpha|\psi(z)|<\varepsilon.$$

Thus for $|\varphi(z)|\le\delta_1$, if $|z|>\delta_2$, we have a constant $C_2>0$ such that

$$(1-|z|^2)^\alpha|\psi(z)f(\varphi(z))|\le\frac{(1-|z|^2)^\alpha|\psi(z)|}{(1-{\delta_1}^2)^{(2+\beta)/p}}||f||_{A^p_\beta}$$

$$\le C_2\varepsilon. \tag{3.4}$$

By combining (3.3) and (3.4), we see that whenever $|z|>\delta_2$ we have

$$(1-|z|^2)^\alpha|\psi(z)f(\varphi(z))|\le\max(C_1,C_2)\varepsilon.$$

This means that

$$\lim_{|z|\to 1}(1-|z|^2)^\alpha|\psi C_\varphi f(z)|=0.$$

Thus $\psi C_\varphi f \in A_0^{-\alpha}$. This completes the proof. □

COROLLARY 3.8. *Let $1 \le p < \infty, -1 < \beta < \infty, \alpha > 0$ and φ be holomorphic maps on $\mathbb{D}$ such that $\varphi(\mathbb{D}) \subset \mathbb{D}$. Then the composition operator $C_\varphi : A_\beta^p \to A_0^{-\alpha}$ is bounded if and only if*

$$\sup_{z\in\mathbb{D}} \frac{(1-|z|^2)^\alpha}{(1-|\varphi(z)|^2)^{(2+\beta)/p}} < \infty$$

COROLLARY 3.9. *Let $1 \le p < \infty, -1 < \beta < \infty, \alpha > 0$ and ψ be holomorphic map on $\mathbb{D}$. Then the multiplication operator $M_\psi : A_\beta^p \to A_0^{-\alpha}$ is bounded if and only if*

(*i*) $\sup_{z\in\mathbb{D}} (1-|z|^2)^{(p\alpha-\beta-2)/p}|\psi(z)| < \infty$

(*ii*) $\psi \in A_0^{-\alpha}$.

The following characterization can be proved on similar lines as Lemma 5.2 in [21].

LEMMA 3.10. *A closed set K in $A_0^{-\alpha}$ is compact if and only if it is bounded and satisfies*

$$\lim_{|z|\to 1} \sup_{f\in K} (1-|z|^2)^\alpha |f(z)| = 0.$$

THEOREM 3.11. *Let $1 \le p < \infty, -1 < \beta < \infty$ and $\alpha > 0$. Let φ and ψ be holomorphic maps on $\mathbb{D}$ such that $\varphi(\mathbb{D}) \subset \mathbb{D}$. Then the weighted composition operator $\psi C_\varphi : A_\beta^p \to A_0^{-\alpha}$ is compact if and only if*

$$\lim_{|z|\to 1} \frac{(1-|z|^2)^\alpha |\psi(z)|}{(1-|\varphi(z)|^2)^{(2+\beta)/p}} = 0. \tag{3.5}$$

PROOF. By Lemma 3.10, the set $\{\psi C_\varphi f : f \in A_\beta^p, ||f||_{A_\beta^p} \le 1\}$ has compact closure in $A_0^{-\alpha}$ if and only if

$$\lim_{|z|\to 1} \sup\{(1-|z|^2)^\alpha |(\psi C_\varphi)(z)| : f \in A_\beta^p, ||f||_{A_\beta^p} \le M\} = 0,$$

for some $M > 0$. Suppose that $f \in A_0^{-\alpha}$ is such that $||f||_{A_\beta^p} \le 1$, and ψ and φ satisfies (3.5). Then

$$\begin{aligned}(1-|z|^2)^\alpha |(\psi C_\varphi)(z)| &= (1-|z|^2)^\alpha |\psi(z) f(\varphi(z))| \\ &\le \frac{(1-|z|^2)^\alpha |\psi(z)|}{(1-|\varphi(z)|^2)^{(2+\beta)/p}}.\end{aligned}$$

Thus

$$\sup\{(1-|z|^2)^\alpha|(\psi C_\varphi)(z)| : f \in A^p_\beta, ||f||_{A^p_\beta} \leq 1\} \leq \frac{(1-|z|^2)^\alpha|\psi(z)|}{(1-|\varphi(z)|^2)^{(2+\beta)/p}}$$

and it follows that

$$\lim_{|z|\to 1}\sup\{(1-|z|^2)^\alpha|(\psi C_\varphi)(z)| : f \in A^p_\beta, ||f||_{A^p_\beta} \leq 1\} = 0.$$

Hence $\psi C_\varphi : A^p_\beta \to A_0^{-\alpha}$ is compact. Conversely, suppose that $\psi C_\varphi : A^p_\beta \to A_0^{-\alpha}$ is compact. Using the same test as in the proof of Theorem 3.6, we see that

$$\lim_{|\varphi(z)|\to 1}\frac{(1-|z|^2)^\alpha|\psi(z)|}{(1-|\varphi(z)|^2)^{(2+\beta)/p}} = 0. \tag{3.6}$$

Since $\psi C_\varphi : A^p_\beta \to A_0^{-\alpha}$ is bounded, Theorem 3.7 implies that $\psi \in A_0^{-\alpha}$. It is easy to show that $\psi \in A_0^{-\alpha}$ and (3.6) is equivalent to (3.5). □

COROLLARY 3.12. *Let $1 \leq p < \infty, -1 < \beta < \infty, \alpha > 0$ and φ be holomorphic map on $\mathbb{D}$ such that $\varphi(\mathbb{D}) \subset \mathbb{D}$. Then the composition operator $C_\varphi : A^p_\beta \to A_0^{-\alpha}$ is compact if and only if*

$$\lim_{|z|\to 1}\frac{(1-|z|^2)^\alpha}{(1-|\varphi(z)|^2)^{(2+\beta)/p}} = 0.$$

COROLLARY 3.13. *Let $1 \leq p < \infty, -1 < \beta < \infty, \alpha > 0$ and ψ be holomorphic map on $\mathbb{D}$. Then the multiplication operator $M_\psi : A^p_\beta \to A_0^{-\alpha}$ is bounded if and only if*

(i) $\lim_{|z|\to 1}(1-|z|^2)^{(p\alpha-\beta-2)/p}|\psi(z)| = 0$

(ii) $\psi \in A_0^{-\alpha}$.

References

[1] K. R. M. Attele, *Multipliers of composition operators,* Tokyo J. Math. **15** (1992), 185–198.

[2] M. D. Contreras and A. G. Hernandez-Diaz, *Weighted composition operators on Hardy spaces*, J. Math. Anal. Appl. **263** (2001), 224–233.

[3] ———, *Weighted composition operators on spaces of functions with derivatiae in a Hardy space*, J. Operator Theory **52** (2004), 173–184.

[4] C. C. Cowen and B. D. MacCluer, *Composition operators on spaces of analytic functions,* CRC Press Boca Raton, New York, 1995.

[5] Z. Cuckovic and R. Zhao, *Weighted composition operators on the Bergman space*, J. London Math. Soc. **70** (2004), 499–511.

[6] F. Forelli, *The isometries of H^p spaces*, Canad. J Math. **16** (1964), 721–728.

[7] H. Hedenmalm, B. Korenblum and K. Zhu, *Theory of Bergman spaces*, Springer, New York, Berlin, etc. 2000.

[8] K. Hoffman, *Banach spaces of analytic functions*, Dover Publications, Inc., 1988.

[9] H. Kamowitz, *Compact operators of the form uC_φ*, Pacific J. Math. **80** (1979), 205–211.

[10] J. E. Littlewood, *On inequalities in the theory of functions*, Proc. London Math. Soc. **23** (1925).

[11] B. D. MacCluer and J. H. Shapiro, *Angular derivatives and compact composition operators on Hardy and Bergman spaces*, Can. J. Math. **38** (1986), 878–906.

[12] V. Matache, *Compact composition operators on Hardy spaces of a half-plane*, Proc. Amer. Math. Soc. **127** (1999), 1483–1491.

[13] G. Mirzakarimi and K. Seddighi, *Weighted composition operators on Bergman and Dirichlet spaces*, Georgian. Math. J. **4** (1997), 373–383.

[14] H. J. Schwartz, *Composition operators on H^p*, Thesis, University of Toledo, 1969.

[15] J. H. Shapiro and P. D. Taylor, *Compact, nuclear and Hilbert-Schmidt composition operators on H^2*, Indiana Univ. Math J. **23** (1973), 471–496.

[16] J. H. Shapiro, *The essential norm of a composition operator*, Ann. Math. **125** (1987), 375–404.

[17] ———, *Composition operators and classical function theory*, Springer-Verlag, New York. 1993.

[18] J. H. Shapiro and W. Smith, *Hardy spaces that support no compact composition operators*, J. Funct. Anal. (to appear).

[19] R. K. Singh and S. D. Sharma, *Composition operators on a functional Hilbert space*, Bull. Austral. Math. Soc. **20** (1979), 277–284.

[20] W. Smith, *Composition operators between Bergman and Hardy spaces*, Trans. Amer. Math. Soc. **348** (1996), 2331–2348.

[21] S. Ohno, K. Stroethoff, and R. Zhao, *Weighted composition operators between Bloch-type spaces*, Rocky Mountain J. Math. **33** (2003), 191–215.

[22] S. Ohno and H. Takagi, *Some properties of weighted composition operators on algebras of analytic functions*, J. Nonlinear Convex Anal. **2** (2001), 369–380.

[23] S. Ohno and R. Zhao, *Weighted composition operators on the Bloch spaces*, Bull. Austral. Math. Soc. **63** (2001), 177–185.

[24] K. Zhu, *Operator theory in function spaces*, Marcel Dekker, New York, 1990.

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