Study on the Ability Level Test of Mathematics Application[1]

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The background and the existing problems in teaching of mathematics application were analyzed. Based on the mathematics knowledge which from the simple application, complicate application, and synthetically application to the mathematics modeling, the ability level test questions of mathematics application was worded out to help the teaching and learning in mathematics application.

1. THE BACKGROUND OF TEACHING OF MATHEMATICS APPLICATION

With the wide application of mathematics in the society, the society requires each citizen to possess certain the application consciousness and ability of mathematics. So, it seems very important to train the students' application consciousness and ability of mathematics at basic education stage.

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In the mathematics education of the world of the 20th century, the application of mathematics has gone through three times reforms of the mathematics education too. In 1980's, mathematics courses of various countries are reformed one after another, the common characteristic of the future mathematics course goal established is:

1) Help students to establish the correct mathematics view;
2) Train student's basic mathematics accomplishment;
3) Help students to improve the thinking ability of mathematics; and
4) Train student's ability of applied mathematics to solve certain practical problem and set up simple mathematics modeling (*cf.* Feng & Zhang, 2000; Feng, Zhang & Liu, 2000; Shao, 2002; Wang & Ye, 2004; Ye, 2002).

So it already became the common characteristics to emphasize mathematics application in mathematics education of various countries as well as China.

In the mathematics education in China, the method of teaching-learning of mathematics application has been changed of several times (*cf.* Xu, 2003, pp. 2–40; Zhang, 2003, pp. 2–30). In 1992, the specialist of the mathematics education of our country appealed for paying attention to the application of mathematics in the middle school strongly (*cf.* Ye, 2002, pp. 5–28). The new mathematics course standard issued recently has already regarded it as an important content that has trained student's mathematics quality.

It is traditional advantage of our mathematics education that we have strict train and strong foundations. For example, we join mathematics test of 21 countries and regions, the score of mathematics of the continent is 80 marks on average, rank first. But the score of science test dispose 15 marks. Computing capability is strong, but application ability is weak. It has increased the word problems of mathematics knowledge in the college entrance examination paper in recent years, but the score rate of these topics is lower than other topics.

Because the theory is a foundation of application, it is essential definitely to learn knowledge of mathematics well for application mathematics. Without understanding and grasping to the mathematics knowledge, it is far from being application at all. But it is not equal to say that if there is knowledge, the knowledge can be applied naturally. The examination of our country has strengthened foundation, but it weak the application consciousness and ability of mathematics knowledge at the same time. Mathematics is applied in two main respects: First, the inside application of mathematics, namely applies existing mathematics knowledge and way of thinking of mathematics to solve the new mathematics problem; Second, the application of mathematics in the social life and production. Both of these respects are important. The application of mathematics is the second respect here.

2. THE EXISTING PROBLEM OF MATHEMATICS APPLICATION TEACHING

Because of the development of the mathematics oneself, social and student, the new mathematics course standard has already emphasized on strengthening the application of mathematics, the college entrance examination has strengthened the application of mathematics knowledge, the new teaching material has increased a lot of word problems, the teacher pays attention to the teaching of knowledge application of mathematics too, students' application consciousness and ability improve a lot than before. But the ability of student's applied mathematics knowledge is unsatisfactory, it can be found out from college entrance examination of mathematics.

Sun (2001)[2] did questionnaire investigation:

1) Teacher's view: 73% of the teachers think that it is necessary to train consciousness and ability of applied mathematics in the high school, 57.4% of the teachers think that sacrificing foundation is in order to emphasize on application. 51.2% of the teachers think that strengthening mathematics application in teaching is for a good mark in the examination for students.

2) Teacher's view on the teaching material: 63.4% teacher think the proportion of application in teaching material is too little, 81% of the teachers think the content outmoded, a lot of word problems to have nothing to do with reality, 52.6% of the teachers think that word problems are narrow, 49.8% of the teachers think that are simpler.

3) The teaching way: 79.1% of the teachers intensify training word problems to students before examination, only 10.3% of teacher who have connected application when teaching the concept and theorem. It is directed against the difficulty of mathematics application, 30% of the teachers have not taken any measure, although 60.4% of the teachers have taken measures, but little effect. Only 8% of the teachers have led students to the society practice and let student to solve the practical problem with mathematics knowledge.

4) The reasons of students solving problems to be difficulty. 36.4% teacher think student's foundation is not steady and strong, 51.2% of the teachers do not think students can understand the topic, 82.5% of the teachers think that it is universal difficulty for students that they are short of mathematics modeling ability. 30.8% of the teachers feel that students' self- confidence is not enough.

In order to investigate the mathematics modeling ability of students of Senior One[3],

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[2] A pre-service teacher (a student of Yangzhou University, Jiangsu Province)

Ban (2002) tested one class each from two upper middle schools in Chongqing[4]. The number of students in each class was 60. At the same time, we did corresponding questionnaire investigation. The result of investigation was:

1) 50% of the classmates do not like solving word problems;
2) To the simple word problem, students have the idea, but to more difficult word problem they do not know what to do, even the topics can not be understood;
3) Because of uninterested of word problems, student spent little time on word problem at ordinary times;
4) They do not take care of the application of mathematics knowledge in life;
5) 40% classmates' think it is the greatest obstacle of mathematics modeling is short of mathematics knowledge, 40% of the classmates think the greatest obstacle is comprehensive ability, 20% classmates think the greatest obstacle is computing capability to be bad and lack of experience of life.

Through the above analysis, it can be found out that exist big problem for mathematics knowledge application in our country's basic education. The teacher's respect: these of the teacher's idea, ability, and teaching way have not been directed against students' individual difference. Students' respect: lack of mathematics knowledge enough, poor understanding ability and calculating ability. The teaching material's respect: proportion of word problems is simply little, the content is outmoded, applied range of knowledge is too narrow. So we think it is the best to teach different content according to different students, different period, and different environment in word problem teaching.

According to everybody's different original cognitive structure, mathematics knowledge and hobby we may teach by different method. The students of different grades have different original cognitive structure and mathematics knowledge structure, even if the same grade, there are different approach and ability of solving problems for everyone.

According to different environment of student themselves, we may teach by choosing different practical problem. The word problems come from the practical problem, so the background knowledge of actual life contributes to understand the word problems.

How do we teach according to different student, period, and environment? How could we know students' original cognitive structure and mathematics knowledge structure?

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[3] "Senior One" is the first year class of the upper middle school (Grades 10– 12), *i.e.*, Grade 10.
[4] Ban, as an upper middle school teacher, used the test materials previously in the Experimental Middle School of Tianjin.

3. LEVEL TEST OF MATHEMATICS APPLIED ABILITY

By use of the level test of mathematics applied ability, students know on which level themselves mathematics knowledge, cognitive structure and ability of solving practical problem, then they are interest in the word problems and confident of studying, the teachers select different word problems and different way to give lessons to different students.

We have collected a large amount of relevant materials (including many kind of teaching material of the high school, the high school student's mathematics knowledge application contests of "Fangzheng Cup" of Beijing and "Jingqiao Cup" of Shanghai), visited experts and first line teacher, and tested ability of mathematics application for the student of Senior one. We have worked out five level test of mathematics knowledge application so as to test ability of mathematics knowledge application. The principle of working out the level test is: from the simple application problem, to the complicated application problem, to the integrated application problems, to giving the scene of problem without data finally. We tried to make the level test live, actually, little to draw up artificially. Every level needs corresponding mathematics knowledge but not to need too much mathematics knowledge, it needs the ability of understanding, analyzing and solving the problem. For every level test, it took two hours to finish. Because the fifth needed to collect the data and consult the materials, so it needed more time.

3.1 The first level: State the real language with the figure or the algebra language.

This level does not have the difficulty of reading understanding, so it may be solved if there is simple basic mathematics knowledge.

1) It is1000 meters from Xiao Ming's family to the school, it is 800 meters from Xiao Hua's family to the school, how far is it from Xiao Ming's family to Xiao Hua's family?
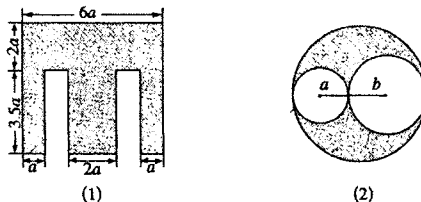

Figure 1. Two shaded areas

2) There is a five-floor, every floor have 10 steps. Hand have a ruler which can

measure 2 meters long only, how could know the height of the fifth floor?

3) The first—level football match, there are 12 teams in all, every two teams should carry on the match, how many matches does it need going on altogether?

4) Calculate the area of each shaded area in Figure 1.

5) A bus leaves from the station and begin to drive at the consistent velocity after driving one section in higher speed. After a while, the automobile reaches the next stop. The automobile begins to accelerate after the passengers get on or off the bus and drives at the consistent velocity again some time later. In Figure 2 which one picture reflects the automobile's speed change within this period of time approximate?
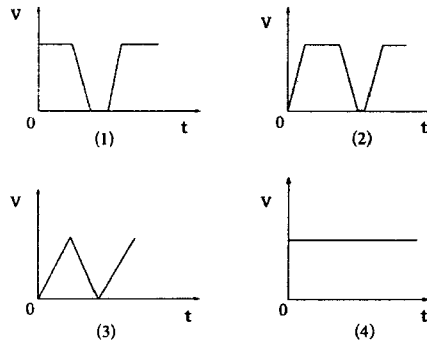

Figure 2. Graphs

6) A scaling ladder is 25 meters long, such as Figure 3 to lean against the wall; it is 7 meters from ladder bottom to the wall.

(1) How high is it from top of this ladder to ground?

(2) If the top of the ladder glides 4 meters, then does the bottom of the ladder slip 4 meters in the horizontal direction?
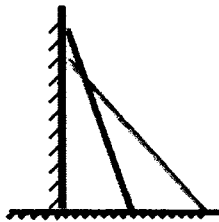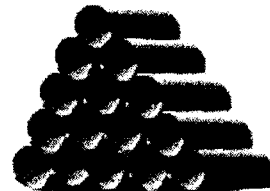

Figure 3. A sliding ladder   *Figure* 4. 15 empty oil drums

7) One shop marks a price for certain clothing after to raise 40% according to the cost price, and sells by the price of 80%. As a result each one still makes a profit of 15 yuans, how much is the cost of each one of this kind of clothing?

8) It shows as Figure 4, 15 empty oil drum (bottom diameter of each empty oil drum is 50 centimeters) pile together. To build one canopy, how high does the canopy take at least?

3.2 The second level: simple word problems

Word's understanding of the second level is slightly difficult, but quantitative relation is obvious. The result is clear and the problems do not need too much analysis.

Let students explain some actual results, describe some real phenomena and solve some application problems imitating.

1) The diameter of a log is 400mm. The log is processed into rectangle post. Which kinds of sawing make the timber discarded least?

2) One family (father, mother and children) wants to go to some place for tour. The first travel agency says: "if father buys one ticket, other persons can enjoy the preferential treatment of the half-price ticket"; the second travel agency says: "family's travel can be regarded as the group ticket, the discount is 2/3." If the original price of these two travel agencies is the same, for different child in the family, calculate the charges (give formula) of two travel agencies separately and discuss which travel agency is more preferential.

3) Two men buy grain twice in the same grain shop (twice price is different). The first bought 100 kilograms of the grain each time; the second bought the grain with 100 yuans each time. If stipulating, whose average unit price purchasing the grain twice is low, whose purchasing way is worthwhile. Please judge the purchasing way of two men and give reasons.

4) A company has offered two kinds of communication business, "global system for mobile communications": monthly rent is 50 yuans, it cost 0.40 yuan every one minute. "Shenzhou Card": Not paying the monthly rent, it cost 0.6 yuan for one minute; long distance is 0.8 yuan for one minute. Which way should be chosen?

5) In a shop, if the price of goods that is purchased by 8 yuans each one is 10 yuans for sale, the shop would 200 pieces each day. Now it takes the method by raising prices and reducing purchasing quantity to increase profits. It is known that the goods sales will reduce 10 pieces, if the sale price is raised 0.5 yuan each one. How much the sale price is determined, the profits will be maximums and what is the profit?

6) It is predicted according to weather station, centers of the typhoon is B city from

straight A of 300 kilometers of the east. The typhoon moves toward the northwest at speed of 25 kilometers per hour. It would be influenced within the area from the typhoon center of 250 kilometers. When beginning and how many hours the typhoon will influence A city? How long is its duration?

7) In summer vacation, the school organizes 48 students to participate the summer camp; it is going to the water garden to take the pleasure-boat. The boat lease lists as the following table. Please design a scheme of leasing and make the rent paid at least (forbid overloading).

Table 1. Rental rates of boats

Boat size	Limit number of people for every boat	Rent
Big Boat	5	30 yuans
Regular Boat	3	20 yuans

3.3 The third level: The scene is founded and the problem is the only conclusion.

The teacher offers the problems and leads students to discuss. The student choose and set up model by teacher's enlighten independently, finish to calculate and to verification of the result by student themselves.

1) The indoor temperature of every room of the hotel of Shanghai are adjusted in unite by the control room in the ground floor. One worker finds that indoor temperature of instrument showing and indoor actual temperature are difference, and he does not change well. The reason is that the distance from high-story room to the control room is very long and three electric wires of the three-phase electricity are not the same long because of the turn. So it causes the resistance of three electric wires to be different and the deviation appeared on the instrument finally. How do they measure the resistance of the three wires? Universal instrument can't measure the resistance of the wires, which one is put on the tenth floor; the other end is placed on the control room of ground floor. What shall we do about this?

2) There are two clothing factories to produce the same kind of clothing. The first factory produces 900 sets of clothes every month, the proportion of time of producing the jackets and the trousers is 2:1. The second factory produces1200 sets of clothes every month, the proportion of time of producing the jackets and the trousers is 3:2. If two factories cooperate, please arrange producing scheme to make the output exceed sum of production capacity of two factories former, how many suits of clothes are produced every month.

3) Mr. Li plans to purchase a commodity apartment and needs the loan of 80,000 yuans from bank. The monthly profit of the bank loan is 0.01(the compound interest). The deadline of the loan is 25 years. Mr. Li can balance of 950 yuans in the stability income every month. If he plans to loan monthly, does he have repaying abilities?

4) The school employs 30 workers to make 200 chairs and 100 desks. The proportion of time of producing the desks and the chairs is 3:2. How should the 30 workers be divided into two groups (one group make the desk, one group make the chair) to enable finishing all tasks fastest?

5) According to the statistical data, the production of energy of our country has been developed very fast since 1985. The following is several statistics of the production of energy (a hundred million tons of standard coal) of our country: 860 million tons in 1985, 1,040 million tons in 1990, and 1,290 million tons in 1995. Relevant experts predict: The production of energy of our country will exceed 1,610 million tons by 2000. Please provide a simple model to prove the prediction of relevant experts whether is rational.

6) The workshop has a batch of 180 centimeters of steel tube. Now for making part, the steel tube should be cut into three kinds of different length: 70 centimeters, 52 centimeters, and 35 centimeters. Because the production task stipulates, the require-ment of these three kinds of material is 100, 150, and 100 respectively. We know, it will produce "the leftover bits" unavoidably while the steel tube is cut. Considering the view of economizing materials, how about way of cutting should be adopted that make the total leftover bits to reach the minimum under finishing the task?

3.4   The fourth level: Students find and put forward some practical problems independently.

The problems that need to suppose can be solved. This level needs high ability of reading understanding of the words. By use of knowledge and commonly modeling method, Students set up model, calculate and verify the result by themselves discussing. And then according to the actual meaning and concrete background of the problems, students revise and appraise to the model.

1) Build a not-covering cylinder type net pond that its volume is constant.
   (1) How to choose size of the pond, it makes the material to be the most save?
   (2) If the material cost is 30 yuans per square meter to bottom of the pond, the material cost is 20 yuans per square meter to wall of the pond. What kind of size makes the cost of the pond lowest?

2) There is a high-story building before one residential building. Somebody measures the angle of elevation of CD of high-story building to be α and the angle of depression of CD to be β on A floor of residential building. He measures the angle of elevation of CD of high-story building to be θ on B floor residential building. It is known $AB = a$. What is high of high-story building and distance among the residential building and the high-story building CD?

3) Design a right four-prism refrigerator, which has one freezer and one cool room. The freezer is made up of three drawers, in order to prevent the mutual taint of odor of food. How the size of appearance and the proportion of the freezer and the cool room make the material to be the most save?

4) Passengers wait for having tickets checked up in the waiting room of the station. The passengers are increasing according to certain speed and the speed of checking up tickets will be certain. If opening one checking up tickets, it needs half an hour to checking up passenger who have tickets waiting and make them to enter the station; If opening two checking up tickets at the same time, it needs ten minutes only to make all passenger to enter the station. Now there is one transit newly train to carry passenger. If all the passengers must have tickets checked up and enter the station within 5 minutes, how many checking up tickets does this station open at the same time at least?

5) One wolf, one sheep and one basket cabbage are on the bank in river. One ferry wants them to go to the other bank of the river, but because his ship is very small, one can be taken only each time. Because of the obvious reason, if the wolf is together with the sheep or the sheep is together with the cabbage, it needs man to look after. How does the ferry take them over the river.

3.5 The fifth level: This level needs to collect the wider data, look for the relation and set up modeling.

The teachers only offer the scene of problem and coach consultation. According to the scene, students collect information, even assume and design some known conditions by themselves, put forward the varied solution and draw the conclusion. The analysis of error or stability of the modeling is finished by student independently.

1) (Estimating population) Collect relevant data to estimate the numbers of people 18 years old of 2000 of our country.

2) Such as Figure 5, there is a river MN and a television tower AB in one side of the river. One person stands in P of the other side of river. He has one protractor and one measure tape (measure length not exceed 5 meters) of length. Please design one measurement scheme (no pass river), and provide the formula to calculate

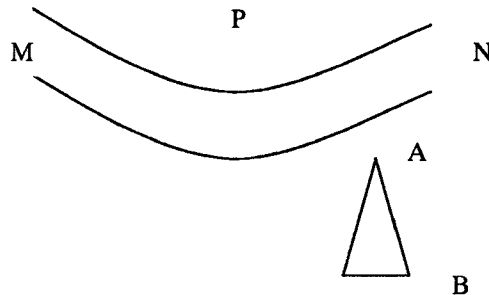height AB of building and distance PA. It is hoped that the measured data is as little as possible in your scheme.

Figure 5. The river MN and a television tower AB

3) (Washing optimization model) When washing clothes, after soap or washing powder by rubbing with the hands, clothes take filth that needs rinsing with water. If there is water of a certain amount now, please set up mathematics model to analysis how to arrange the procedure of washing (how many times of rinsing, how much water each time) to make the clothes cleanest by using these water?

4. EXISTING PROBLEM OF THE LEVEL TEST

Because of urgent time, the level test of mathematics applied ability has not been practiced yet. Our future work is to solicit suggestion of the in-service teachers, carry on test student, modify and perfect the level test to make it correspond to reality even more and reflect mathematics applied ability of student. It plays an important role to mathematics applied teaching in the future.

REFERENCES

Ban, Chunhong (2003). *The Test the investigation once of mathematics modeling ability of the student in Senior One* (in Chinese). Website:
http://www.21maths.com/sxjm/zxsjm/jmlw/200307/294.html

Feng, Yongming & Zhang, Qifan (2000). On mathematics modeling of Middle School (in Chinese). *Journal of Mathematics Education* 9(2).

Feng, Yongming; Zhang, Qifan; Liu, Fenwen (2000). Discussion of mathematics modeling teaching of high school (in Chinese). *Bulletin of Mathematics (Wuhan)* **2000**(13), 35–40.

Shao, Dongsheng (2002). *Teaching research and practice of mathematics modeling of high school.*

Master Thesis. Fuzhou, Fujian: Fujian Normal University.

Sun, Duo (2001). *To Trains High School Student's Consciousness and Ability of Mathematics Application*. Master Thesis. Yangzhou, Jiangsu: Yangzhou University.

Wu, Changjiang (2003). *The mathematics word problems of high school* (in Chinese). Shanghai: Shanghai University Press.

Xu, Binyan (2003). *The Mathematics Course and Teaching Theory* (in Chinese). Hangzhou: Zhejiang Education Publishing House.

Ye, Qixiao (2002). *Mathematics modeling of middle school* (in Chinese). Changsha: Hunan Education Press.

Zhang, Siming (2003). *The Practical Exploration of Subject Study of Mathematics* (in Chinese). Beijing: Higher Education Press.